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OPTIMIZATION OF CONVENTIONAL IRREVERSIBLE CASCADE REFRIGERATION SYSTEMS

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Lucrarea propune un model matematic destinat optimizării instalațiilor frigorifice cu comprimare mecanică de vapori în cascadă dublă. Optimizarea unei instalații frigorifice presupune găsirea regimului funcțional optim, caracterizat de un coeficient de performanță frigorifică maxim. Pentru elaborarea modelului matematic ciclurile termodinamice inversate corespunzătoare celor două cascade, superioară, şi respectiv, inferioară, sunt considerate exo- şi endo-ireversibile. Pe baza modelului matematic s-a efectuat un studiu de optimizare, în urma căruia s-au obținut o infinitate de regimuri optime de funcționare, dintre care, doar unul singur este caracterizat de un coeficient de performanță frigorifică maxim maximorum.

This paper proposes a mathematical model devoted to the optimization of conventional double cascade refrigeration systems. The optimization involves finding the optimum operation regime corresponding to a maximum coefficient of performance COP. The thermodynamic cycles belonging to the top and bottom cascades, respectively, are of reversed endo- and exo-irreversible type. An optimization analysis is performed. An infinity of optimum operation regimes are obtained but only one of them leads to a maximum-maximorum COP.

Keywords: conventional double cascade refrigeration systems, irreversible processes, optimal operation regime, coefficient of performance.

1. Introduction

Conventional cascade refrigeration systems are obtained by coupling two different single stage vapor compression refrigeration systems. The two refrigeration systems are thermally coupled through an intermediate condenser-evaporator heat exchanger. Being a more economical solution than the multiple stages vapor compression refrigeration systems, the conventional cascade refrigeration systems are usually used to obtain low temperatures (-50÷-90)°C, [1, 2].

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The optimization of a refrigeration system involves finding the optimum operation regime, corresponding to a maximum coefficient of performance (COP) [3÷8]. A maximum COP means minimum energy consumption, for an imposed cooling load. Starting from this objective function, for almost all refrigeration systems with mechanical vapor compression, the optimal performance is obtained for a certain distribution of heat exchangers conductances, on one hand, and for certain values of temperature differences between the working fluid and heat sources, on the other hand.

Thus, this paper presents a mathematical model, which aims to find the optimum operating regime of a conventional double cascade refrigeration system when the cooling load is imposed. The temperature – entropy (T-S) diagram of the generalized thermodynamic cycle of this refrigeration system is shown in Fig.1.

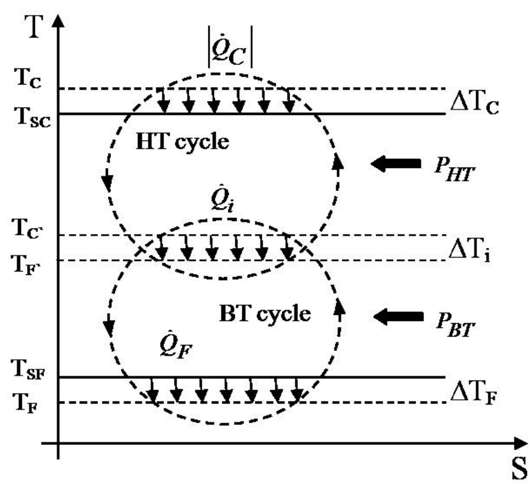


Fig.1. The generalized thermodynamic cycle of the conventional double cascade refrigeration system in temperature – entropy (T-S) diagram

There are many papers that deal with the optimization of real cascade refrigeration systems. The approach differs from author to author, by using different thermodynamic analysis methods like the entropy generation minimization method [3, 4] and finite time thermodynamics techniques [5÷8]. Some authors considered in their optimization method different working fluids, aiming for optimal condensing temperature at the intermediate heat exchanger [9] or for optimum pairs of working fluids [10].

2. Mathematical model

To develop the mathematical model the following hypotheses are considered: the operation regime of the double cascade refrigeration systems is stationary;

thermodynamic cycles are exo- irreversible due to the heat exchange at finite temperature differences and endo-irreversible due to the internal irreversibilities; the losses between the cold and hot parts of the machine are not taken into account; the intermediary heat exchanger that ensures the thermal coupling of the two cascades is adiabatically insulated; the heat transfer is conductive - convective.

Considering the average temperatures of working fluids in the three heat exchangers of the conventional double cascade refrigeration system, T_F and $T_{C'}$ for the low temperature stage (BT cycle) and $T_{F'}$ and T_C for the high temperature stage (HT cycle). The low and high source temperatures, T_{SF} and T_{SC}, respectively, are constant. Thus, the temperature differences in the heat exchangers can be written:

$$\begin{aligned} \Delta T_C &= T_C - T_{SC} > 0 \\ \Delta T_i &= T_{C'} - T_{F'} > 0 \\ \Delta T_F &= T_{SF} - T_F > 0 \end{aligned} \tag{1}$$

Energy and entropy balance equations, for both high and low temperature cascades are used, as follows:

Heat balance equations:

High temperature cascade:

for the condenser: $\left|\dot{Q}_C\right| = K_C \cdot \Delta T_C$ (2)

for the evaporator: $\dot{Q}_i = K_i \cdot \Delta T_i$ (3)

Low temperature cascade:

for the condenser: $\dot{Q}_i = K_i \cdot \Delta T_i$ (4)

for the evaporator: $\dot{Q}_F = K_F \cdot \Delta T_F$ (5)

Equations of energy balance per cycles:

High temperature cascade: $P_{HT} = \left|\dot{Q}_C\right| - \dot{Q}_i$ (6)

Low temperature cascade: $P_{BT} = \dot{Q}_i - \dot{Q}_F$ (7)

where P_{HT} and P_{BT} are the compressor power input in the two cascades.

By summing up relations (6) and (7), the total consumed power is obtained:

$$P_T = P_{HT} + P_{BT} = \left|\dot{Q}_C\right| - \dot{Q}_i + \dot{Q}_i - \dot{Q}_F = \left|\dot{Q}_C\right| - \dot{Q}_F \tag{8}$$

Entropy balance equations for endo-irreversible cycles:

High temperature cascade: $-\dfrac{\left|\dot{Q}_C\right|}{T_C} + \dfrac{\dot{Q}_i}{T_{F'}} + \dot{S}_{iH} = 0$ (9)

$$\text{Low temperature cascade:} -\frac{\dot{Q}_i}{T_{C'}} + \frac{\dot{Q}_F}{T_F} + \dot{S}_{iB} = 0 \tag{10}$$

In the above eqs., K_F, K_i, and K_C are the heat exchangers conductances for the low temperature cascade evaporator, intermediate heat exchanger (condenser-evaporator) and high temperature cascade condenser, respectively, while the terms $\dot{S}_{iH}$ and $\dot{S}_{iB}$, in the entropy balance equations, are the entropy sources dues to the internal irreversibilities of the HT and BT cycles, respectively.

The conventional double cascade energetic coefficient of performance can be defined as:

$$COP = \frac{\text{"}useful\ energy\text{"}}{\text{"}consumed\ energy\text{"}} = \frac{\dot{Q}_F}{P_T} \tag{11}$$

Thus, the objective function of this optimization model is the maximization of the COP. For an imposed cooling load ($\dot{Q}_F = ct.$), the maximization of COP involves, as observed in equation (11), the minimization of the total power input. Thus, the expression of the total consumed power (P_T), eq. (8), will be processed bellow. From eq. (5), T_F can be expressed as $T_F = T_{SF} - \frac{\dot{Q}_F}{K_F}$. Using T_F together with eqs. (4) and (1), in eq. (10), after some algebra it results:

$$T_{F'} / T_{C'} = 1 - \left(A + \dot{S}_{iB}\right) / K_i \tag{12}$$

In eq. (12) the following notation was used $A = \dot{Q}_F \Big/ \left(T_{SF} - \frac{\dot{Q}_F}{K_F}\right)$.

If in eq. (12) one considers eq. (1), it results the temperature difference between the two working fluids in the intermediate condenser-evaporator heat exchanger, as follow:

$$\Delta T_i = \left\{1 / \left[1 - \left(A + \dot{S}_{iB}\right) / K_i\right] - 1\right\} \cdot T_{F'} \tag{13}$$

Combining eqs. (2), (3), (8) and (12), one finds:

$$\frac{T_{SC}}{T_C} = \frac{1 - \left(A + \dot{S}_{iB}\right) \cdot \left(1/K_i + 1/K_C\right) - \dot{S}_{iH} / K_C \cdot \left(1 - \left(A + \dot{S}_{iB}\right) / K_i\right)}{1 - \left(A + \dot{S}_{iB}\right) / K_i} \tag{14}$$

Using eq. (1) and (2), eq. (8) can be written as follows:

$$P_T = K_C \cdot \Delta T_C - \dot{Q}_F = K_C \cdot \left(T_C - T_{SC}\right) - \dot{Q}_F = K_C \cdot T_{SC} \cdot \left(\frac{T_C}{T_{SC}} - 1\right) - \dot{Q}_F \tag{15}$$

Thus, based on eqs. (14) and (15), the total power consumption is expressed as:

$$P_T = K_C \cdot T_{SC} \cdot \left[\frac{1-\left(A+\dot{S}_{iB}\right)/K_i}{1-\left(A+\dot{S}_{iB}\right)\cdot\left(1/K_i+1/K_C\right)-\dot{S}_{iH}/K_C\cdot\left(1-\left(A+\dot{S}_{iB}\right)/K_i\right)} - 1 \right] - \dot{Q}_F \tag{16}$$

After mathematical processing, eq. (16) can be written as:

$$P_T = T_{SC} \cdot E - \dot{Q}_F \tag{17}$$

where:

$$E = \left[1+\dot{S}_{iH}\cdot\left(\frac{1}{A+\dot{S}_{iB}}-\frac{1}{K_i}\right)\right] \Bigg/ \left[\left(\frac{1}{A+\dot{S}_{iB}}-\frac{1}{K_i}\right)\cdot\left(1-\frac{\dot{S}_{iH}}{K_C}\right)-\frac{1}{K_C}\right] \tag{18}$$

Knowing that $A = f(K_F)$, from eq. (18) it results that $E = f(K_C, K_i, K_F)$.

As seen from eqs. (17) and (18), at imposed $\dot{Q}_F$, T_{SF}, T_{SC}, $\dot{S}_{iH}$, $\dot{S}_{iB}$, the minimum of P_T is obtained when the expression E is minimum.

If we impose a constant value of the overall heat exchanger conductance, i.e. $K_T = K_i + K_C + K_F = ct.$, then K_i can be expressed as:

$$K_i = K_T - K_C - K_F \tag{19}$$

By substituting the eq. (19) into eq. (17), it results that $E = f(K_C, K_F)$. The optimal values of K_F (K_{Fopt}) and K_C (K_{Copt}), for which expression E reaches a minimum, will be found as solutions of eqs. $\frac{\partial E}{\partial K_F} = 0$ and $\frac{\partial E}{\partial K_C} = 0$.

Thus, eq. $\frac{\partial E}{\partial K_F} = 0$, after mathematical processing, becomes:

$$\frac{\frac{\partial A}{\partial K_F}}{\left(A+\dot{S}_{iB}\right)^2} = \frac{\frac{\partial K_i}{\partial K_F}}{K_i^2} \tag{20}$$

Solving $\frac{\partial A}{\partial K_F}$ and $\frac{\partial K_i}{\partial K_F}$, from eq. (19), and replacing them in eq.(20) gives:

$$\frac{K_i}{K_{Fopt}} = \frac{A_{opt}+\dot{S}_{iB}}{A_{opt}} \tag{21}$$

If A is substituted in eq. (21), after mathematical processing it results:

$$K_i = K_{Fopt} + \dot{S}_{iB} \cdot \frac{T_{SF}\cdot K_{Fopt} - \dot{Q}_F}{\dot{Q}_F} \tag{22}$$

Next, $\frac{\partial E}{\partial K_C} = 0$ is computed, giving:

$$-\frac{\frac{\partial K_i}{\partial K_C}}{K_i^2}=\frac{1}{K_{Copt}^2}\cdot\left[1+\dot{S}_{iH}\cdot\left(\frac{1}{A+\dot{S}_{iB}}-\frac{1}{K_i}\right)\right]^2 \tag{23}$$

From relation (19), $\frac{\partial K_i}{\partial K_C}=-1$. With this relation, eq. (23) becomes:

$$\frac{1}{K_i}=\frac{1}{K_{Copt}}\cdot\left[1+\dot{S}_{iH}\cdot\left(\frac{1}{A+\dot{S}_{iB}}-\frac{1}{K_i}\right)\right] \tag{24}$$

From relation (24) it results: $K_{Copt}=K_i\cdot\left[1+\dot{S}_{iH}\cdot\left(\frac{1}{A+\dot{S}_{iB}}-\frac{1}{K_i}\right)\right]$ (25)

Combining eqs. (22) and (25), after mathematical processing it is obtained:

$$K_{Copt}=K_{Fopt}\cdot\left[1+\frac{1}{A_{opt}}\cdot\left(\dot{S}_{iB}+\dot{S}_{iH}\cdot\left(1-\frac{A_{opt}}{K_{Fopt}}\right)\right)\right] \tag{26}$$

If eqs. (22) and (26) are substituted in eq. (19) it results K_{Fopt}:

$$K_{Fopt}=\frac{K_T+2\cdot\dot{S}_i}{3+T_{SF}/\dot{Q}_F\cdot\left(\dot{S}_i+\dot{S}_{iB}\right)} \tag{27}$$

where $\dot{S}_i=\dot{S}_{iH}+\dot{S}_{iB}$.

Starting from eq. (19) and using eqs. (26) and (27), K_{iopt} is found as:

$$K_{iopt}=K_T-(K_{Fopt}+K_{Copt}) \tag{28}$$

If eqs. (17) and (18) are substituted in eq. (11), dividing by $\dot{Q}_F$ and using the optimal heat exchanger conductances values, it results the maximum coefficient of performance:

$$COP_{\max}=\frac{1}{\frac{T_{SC}}{\dot{Q}_F}\cdot\left[\frac{1+\dot{S}_{iH}\cdot\left(\frac{1}{A_{opt}+\dot{S}_{iB}}-\frac{1}{K_{iopt}}\right)}{\left(\frac{1}{A_{opt}+\dot{S}_{iB}}-\frac{1}{K_{iopt}}\right)\cdot\left(1-\frac{\dot{S}_{iH}}{K_{Copt}}\right)-\frac{1}{K_{Copt}}}\right]-1} \tag{29}$$

The optimum temperature difference for lower cascade evaporator is obtained from relationship (5), corresponding to K_{Fopt}:

$$\Delta T_{Fopt}=\dot{Q}_F/K_{Fopt} \tag{30}$$

Similarly, the optimum temperature difference for the upper cascade condenser results by combining eqs. (2), (8), (11) and (17) for K_{Fopt}, K_{Copt} and $COP_{\max}$ values:

$$\Delta T_{Copt} = \dot{Q}_F \cdot (1/COP_{\max} + 1)/K_{Copt} \tag{31}$$

Under these conditions, it is found the optimal operating regime, characterized by:

$$Parameters \begin{Bmatrix} \dot{Q}_F \\ T_{SC} \\ T_{SF} \\ T_{F'} \\ K_T \\ \dot{S}_{iH} \\ \dot{S}_{iB} \end{Bmatrix} \rightarrow Optimum\ Variabiles \begin{Bmatrix} K_{Fopt} \\ K_{iopt} \\ K_{Copt} \\ COP_{max} \\ \Delta T_{Fopt} \\ \Delta T_{Copt} \\ \Delta T_{iopt} \end{Bmatrix}$$

In other words, the optimum operating variables are obtained for a given set of parameters, which generally represents the initial data in the design work.

3. Optimization study

In order to see how the mathematical model proposed in this paper provides reliable results, a conventional double cascade refrigeration system was considered, whose operating conditions are characterized by the following variable values: the cooling load $\dot{Q}_F = 100 \div 10000$ W; cold source temperature: $T_{SF} = 200$ K; hot source temperature: $T_{SC} = 308$ K; overall heat exchanger conductance: $K_T = 1300$ W/K; evaporating temperature for high stage (HT cycle): $T_{F'} = 247$ K; internal entropy sources, corresponding to the BT cycle $\dot{S}_{iB} = 0.5$ W/K and to the HT cycle $\dot{S}_{iH} = 0.7$ W/K, respectively.

A computer program was developed according to this mathematical model. Parts of the results are presented below (Figs. 2÷4).

Figure 2 presents the optimum heat exchanger conductances distribution function of the cooling load. It is noted that once the cooling load increases, at its low values $\dot{Q}_F \in (100 \div 2000)$ W, K_{Copt} strongly decreases and K_{Fopt} strongly increases, while K_{iopt} increases slowly. Next, for values of $\dot{Q}_F \in (2000 \div 10000)$ W, the heat exchanger conductances distribution tends to respect the principle of equipartition [11, 12]. Over $\dot{Q}_F = 43300\,\text{W}$ the equipartition of exchanger conductances distribution is established.

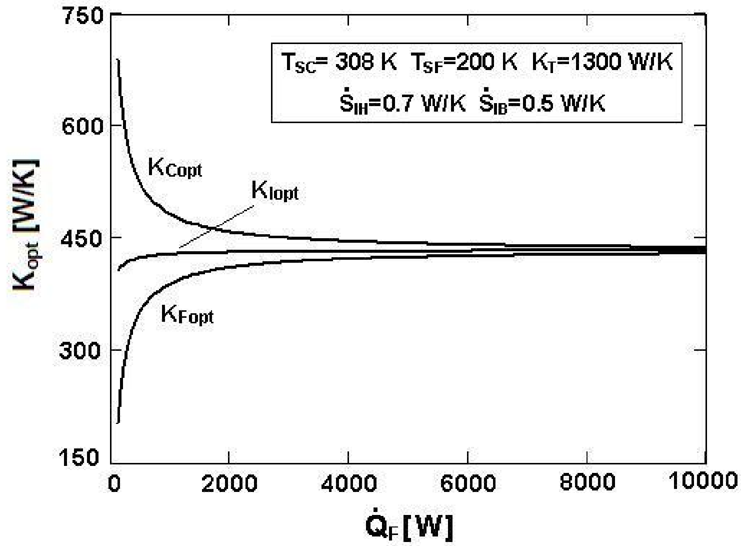


Fig.2. Optimum heat exchanger conductances as function of the cooling load

Figure 3 shows the variation of the maximum coefficient of performance, corresponding to the optimum distribution of heat exchangers conductances (K_{Fopt}, K_{iopt} and K_{Copt}), function of the cooling load. Each value of the coefficient of performance is a maximum value for an imposed set of variables. Also, it can be observed that there are an infinity of operating regimes at maximum COP, but there is only one operating regime characterized by an maximum-maximorum COP ($COP_{max}^{max} = 1.18$ [-], corresponding to $\dot{Q}_F = 2400\,\text{W}$).

Based on the same thermodynamic analysis method, the existence of a maximum-maximorum energetic coefficient of performances operating regime was also obtained in case of the optimization of power plants [13].

Figure 4 shows the variation of the the optimum temperature differences at the low temperature stage evaporator (ΔT_{Fopt}), at the high temperature stage condenser (ΔT_{Copt}), and between the two working fluids in the intermediate condenser-evaporator heat exchanger (ΔT_{iopt}) respectively, with respect to the cooling load. On can observe that all three temperature differences ΔT_{Fopt}, ΔT_{iopt} and ΔT_{Copt}, respectively, increase once the cooling load increases. It results that ΔT_{iopt} is larger then ΔT_{Fopt} and that ΔT_{Copt} is greater than ΔT_{iopt}. For the operating regime with the maximum-maximorum *COP* the optimal values are: $\Delta T_{Fopt} = 5.79$ K, $\Delta T_{iopt} = 7.6$ K and $\Delta T_{Copt} = 9.8$ K.

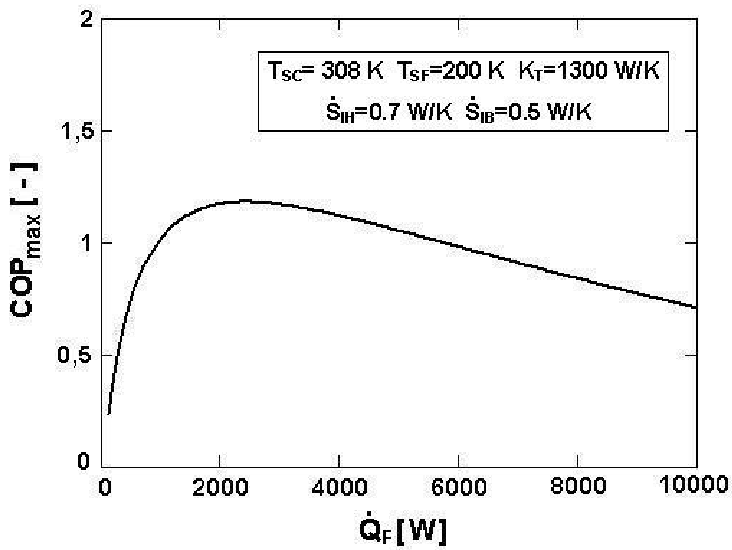


Fig.3. Maximum *COP* as function of the cooling load

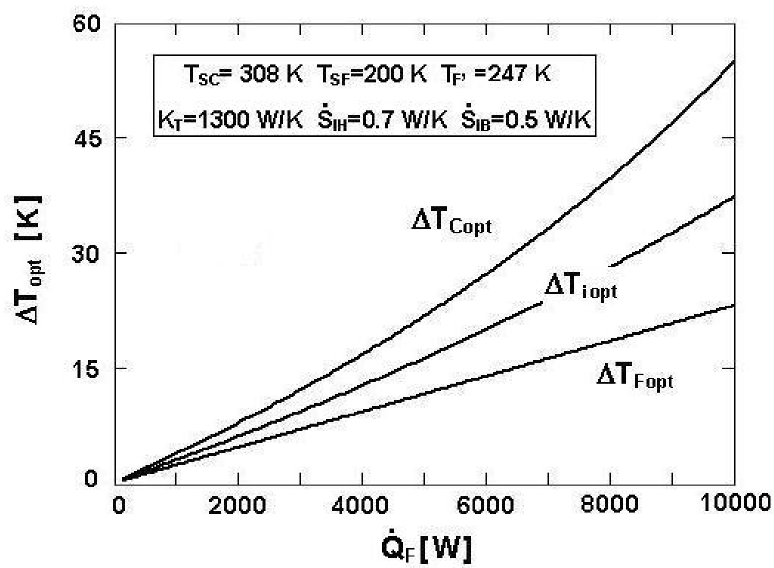
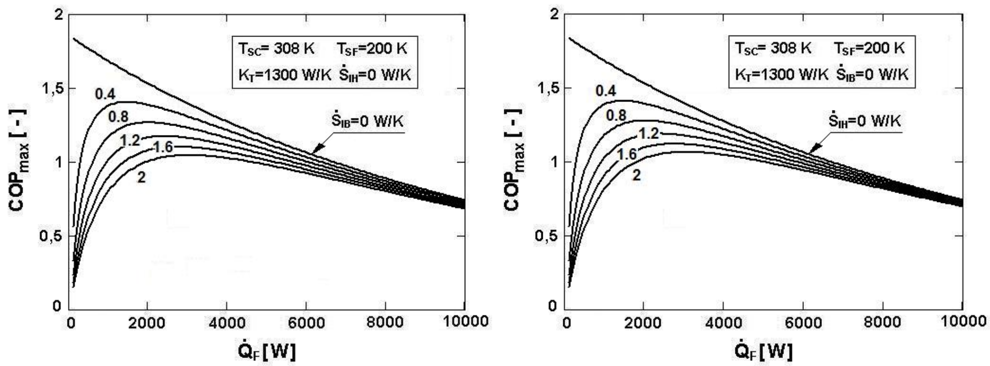


Fig.4 Optimum temperature differences as function of the cooling load



(a) (b)

Fig. 5 Influence of $\dot{S}_{iB}$ and $\dot{S}_{iH}$ on maximum COP as a function of the cooling load (a) influence of $\dot{S}_{iB}$; (b) influence of $\dot{S}_{iH}$.

Figures 5 (a) and 5 (b) present the influence of the internal irreversibility sources, $\dot{S}_{iB}$ and $\dot{S}_{iH}$, on maximum COP as a function of the cooling load. It results that the refrigeration system, only in a endo-irreversible case ($\dot{S}_{iB} > 0$; $\dot{S}_{iH} > 0$), has an operating regime for which COP is maximum– maximorum. In this case COP_{max}^{max}, values decrease with the increase in the internal irreversibility sources, corresponding to an increasing cooling load. Thus, the mathematical model proposed in this paper confirms the real operating case, where the internal irreversibilities increase with the increasing of the cooling load [1, 2].

Comparing the influence of $\dot{S}_{iB}$ (Fig. 5 a) with the influence of $\dot{S}_{iH}$ (Fig. 5 b), it results that the two internal irreversibility sources have a symmetric influence on the maximum COP performance. Thus, both internal irreversibility sources of the two stages cycle in a conventional cascade refrigeration system have the same influence on the operating regime.

4. Conclusions

The paper has presented a mathematical model, which aims to find the optimum operating regime of a conventional double cascade refrigeration system for imposed cooling load ($\dot{Q}_F = ct.$) and overall heat exchanger conductance ($K_T = ct.$). The main hypotheses of the mathematical model are the following: stationary operating regime; exo- and endo-irreversible thermodynamic cycles, due to the heat exchange at finite temperature differences and due to the internal irreversibilities, respectively; no heat losses between the cold and hot ends of the system; adiabatic thermal coupling in the intermediary heat exchanger; conductive - convective heat transfer between the working fluid and heat sources. Generalized thermodynamic cycle of the conventional cascade system is presented in Fig. 1.

The search for the optimum operating regime involves finding the optimal constructive parameters (heat exchanger conductances distribution) and functional parameters (temperature differences between the working fluids and the heat sources) for which the coefficient of performance (COP) is maximum.

A computer program was developed according to the mathematical model. Some results are presented in the paper. Optimum parameter values are obtained for a specified set of variables which represent the initial set of data for the design activity.

For the imposed set of variables ($\dot{Q}_F \in (100 \div 10000)$W; $T_{SF} = 200$K; $T_{SC} = 308$K; $K_T = 1300$W/K; $\dot{S}_{iB} = 0.5$W/K and $\dot{S}_{iH} = 0.7$W/K), in Fig. 2 is presented the optimal distribution of the heat exchanger conductances (K_{Copt}, K_{Fopt} and K_{iopt}) which leads to a maximum coefficient of performance. If $\dot{Q}_F \in (2000 \div 10000)$ W, the heat exchanger conductances distribution tends to respect the principle of equipartition. Over $\dot{Q}_F = 43300$ W the equipartition of exchanger conductances distribution is established.

From Fig. 3 it results that there is an infinity of operating regimes at maximum COP, but there is only one operating regime characterized by an maximum-maximorum COP ($COP_{max}^{max} = 1.18$ [-] for $\dot{Q}_F = 2400$W). Maximum COP, in imposed cooling load conditions, corresponds to an economical operating regime, characterized by a minimum power consumption.

The optimal temperature differences (ΔT_{Fopt}, ΔT_{Copt} and ΔT_{iopt}), between working fluid and heat sources, between the working fluids in the intermediate condenser-evaporator heat exchanger, increase with the increase in the cooling load; ΔT_{iopt} experiences a greater increase than ΔT_{Fopt} and ΔT_{Copt} is greater than ΔT_{iopt} (Fig. 4); for the maximum-maximorum COP operating regime their optimal values are: $\Delta T_{Fopt} = 5.79$K, $\Delta T_{iopt} = 7.6$K and $\Delta T_{Copt} = 9.8$K.

The results (Figs. $2 \div 4$) point out that all constructive and functional parameters are directly and evenly (Fig. 5) influenced by the internal irreversibility sources ($\dot{S}_{iB}$ and $\dot{S}_{iH}$) corresponding to the two stages of the conventional cascade refrigeration system. Thus, it results that in order to obtain a correct information using the here presented mathematical model it is very important to appreciate the correct values of the internal irreversibility sources.

The results obtained by this mathematical model are in good concordance with the real operating regimes where the internal irreversibilities increase with the increase of the cooling load.

The mathematical model proposed in this paper does not take into consideration the properties of the working fluids corresponding to high and low temperature stages. In real operating conditions, besides irreversibilities dues to the imperfections of the thermodynamic processes, the pair of working fluids has a major influence on the conventional double cascade refrigeration system COP. In these conditions, future work will be conducted in this direction for completing and improving the here proposed mathematical model.

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