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MULTIFRACTAL ANALYSIS OF THE DYNAMICS OF THE ROMANIAN EXCHANGE RATE ROL-USD DURING THE TRANSITION PERIOD

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În această lucrare prezentăm un studiu asupra condiţiilor pe care trebuie să le satisfacă o serie temporală pentru ca rezultatul unei analize multifractale a acesteia să fie satisfăcătoare. Analiza de acest tip este importantă deoarece în multe situaţii practice, cum este şi cazul unei serii financiare, este necesar să se stabilească dacă seria are caracter mono sau multifractal. Rezultatele pentru serii temporale scurte (limitate de condiţii practice) sunt, în general, afectate de erori care conduc la lărgirea şi la translaţia spectrului de singularitate. Am arătat că analiza multifractală dă rezultate bune pentru o serie temporală de cel puţin 4000 de puncte. În cazul seriilor mai scurte, rezultate îmbunătăţite considerabil se obţin în urma unui proces artificial de lungire constând în repetarea seriei temporale, după un proces prealabil de eliminare a tendinţelor dominante.

In this work we present a study on the conditions that a time series has to fulfil in order that a multifractal analysis produces reliable results. The analysis of this type is very important because in many practical situations, particularly the present case of a financial time series, the first usually addresed question is whether the data under study are monofractal or multifractal. The results for short time sries (as limited by practical constraints) can be (and indeed are) affected by errors leading to a broadening and a translation of the singularity spectrum. We find that our multifractal analysis gives reliable results for a time series longer than 4000 points. If the available time series is (much) shorter, considerably improved results are expected via a lenghtening procedure consisting in the repeating of the available time series. Clearly, a carefully detrended procedure has to be previously applied in order to avoid artificially introduced fluctuation that might alter the singularity spectrum.

Keywords: multifractal analysis, singularity spectrum, fractal dimension, Hurst exponent

1. Introduction

About ten years ago, the physics community discovered that methods of physics such as statistical physics and chaotic dynamics are well suited for the

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analysis of social, economic and financial phenomena [1,2]. This led to the emergence of the new multidisciplinary field of econophysics that meanwhile developed specific methods for short and long run predictions for the financial and economic evolution [3].

One of the most popular methods in the field is the multifractal analysis because the real data from different financial markets is known to exhibit self-similar properties. Multifractals were introduced in the field of economics [4,5] to surpass the shortcomings of classical theories that predict the impossibility of occurrence of precipitous events. When the dimension of a time series is non-integer, this is associated with two specific features: inhomogeneity – extreme fluctuations at irregular intervals, and scaling symmetries – definite relationships between fluctuations over different separation distances. In some cases, such as exchange rates, the underlying structural equations give rise to fractality [6]. The specific analysis methods are based on the Hurst exponent, correlation functions and frequency spectrum, or on more sophisticated ones, like wavelet transforms or Hölder exponent spectrum [7-9].

In this paper we focus upon the time series of the sampled daily exchange rate ROL-USD over more than five years. The observed exponential-like growth is a consequence of the strong inflationist trend a pattern that usually occurs where things are out of order. The economic systems under transition represent a very interesting field of research, particularly for the occurrence of events that are very improbable or even catastrophic under normal circumstances.

This paper estimates the conditions that a time series has to fulfill in order that a multifractal analysis produces reasonably reliable results. It is shown that a minimum length of the financial series under analysis is a stringent requirement. This result is obtained by comparing the output of multifractal computation, particularly of the singularity spectrum for a well known signal (brown noise) and our financial time series for various lengths thereof. On the basis of the analysis, a proposal for improved results in the case of short time series is suggested.

2. Theoretical background

In 1971 Renyi [10] defined an order q information

$$I_q(r) = \frac{1}{1-q} \ln Z_q(r) \tag{1}$$

where the order q partition function is defined by

$$Z_q(r) = \sum_{i=1}^{N(r)} p_i^q \tag{2}$$

with q any real number including zero. The summ is over the $N(r)$ cells that cover the phase space attractor.

An infinite series of generalized dimensions of various orders was introduced by Grassberger in 1983 [11], defined by

$$D_q = -\lim_{r\to 0}\frac{I_q(r)}{\ln r} = -\frac{1}{1-q}\lim_{r\to 0}\frac{\ln Z_q(r)}{\ln r}. \qquad (3)$$

The low integer order generalized dimensions have the following meaning: D_0 is the box counting dimension, D_1 is the information dimension and D_2 is the correlation dimension [12].

The time series of many various real processes have a clear fractal structure. In applications, particularly in econophysics, it is very important to determine their fractal characteristics. There are two methods of computing a singularity spectrum for a time series: by detrended fluctuation analysis (MFDFA) [13,14] and by wavelet transform modulus maxima (WTMM) [15,16]. The analysis of the LEU/USD exchange rate during the transition epoch of Romanian economy by the first method was already presented [17]. In this paper, we shall use the second method for a similar analysis.

First we shall introduce the main ideas. The wavelet transform associates to the signal function $f(t)$ another function $T_\psi(b,a)$ according to the relationship

$$T_\psi(b,a) = \frac{1}{a}\int_{-\infty}^{\infty} f(t)\psi\left(\frac{t-b}{a}\right)\mathrm{d}t \qquad (4)$$

where the scale parameter a is a real pozitiv number, and $b \in R$ is the „spatial parameter”. A wavelet function ψ, well localized in both space and frequency such as the Gaussian $\exp\left[-\frac{t^2}{2}\right]$ and its derivatives $\psi^{(m)} = \frac{\mathrm{d}^m}{\mathrm{d}t^m}\left(\exp\left[-\frac{t^2}{2}\right]\right)$ is used.

The meaning of the parameters a and b is quite clear when the analysis wavelet function is the Gaussian, namely $\psi(t) \approx \exp\left[-\frac{(t-\bar{t})^2}{2\sigma^2}\right]$. While b represents the coordinate on which the analysis wavelet function is localized, a is proportional to the dispersion of the Gaussian bell, and consequently, it characterizes the width of the domain where the wavelet function has meaningful values.

By modulus maxima of the wavelet transform, we mean the local maxima of the $\left|T_\psi(b,a)\right|$ function for a given value of *a*. By decreasing the parameter *a,* these maxima result in a wavelet spectrum that reveals a detailed structure of singularities of the initial signal as *b* is scanning the range of values of the coordinate (in the present situation, *t*) [17]. The wavelet analysis for the time series used in this work is shown on Fig.1, where the upper section shows the time series after detrending with the second order polynomial.

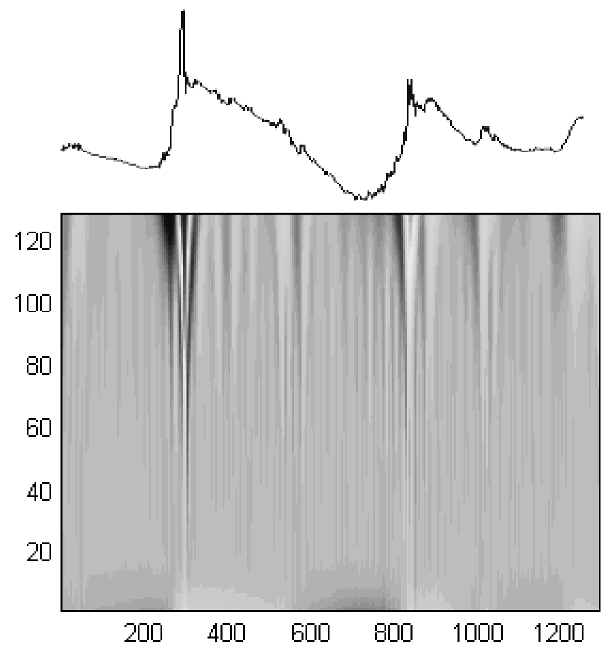


Fig. 1 Wavelet analysis of the detrended financial time series, shown on top of the figure.

WTMM uses the „spatial" partitioning generated by these maxima corresponding to different values of the scale parameter *a*. This allows the definition of an order *q* partition function according to

$$Z(q,a) = \sum_{t_i(a)} \left|T_\psi(t_i(a),a)\right|^q \qquad (5)$$

where $t_i(a)$ are the values of *b* corresponding to the local maxima of $\left|T_\psi(b,a)\right|$, and *q* is a real number including zero.

For fractal signals, the dependence of the partition function on the "dimension of the cell", *a,* for small values of the latter, must be exponential:

$$Z(q,a) \sim a^{\tau(q)}. \qquad (6)$$

The singularity spectrum $f(\alpha)$ of the signal $f(t)$ is obtained [18] using the Legendre transform of the function $\tau(q)$

$$f(\alpha) = \min_{q \in R}(q\alpha - \tau(q)). \tag{7}$$

In many practical situations, as in the present one, $f(t)$ is in the form of a series of discrete values $x(i)$, $i = 1,2,\cdots,N$. In this case, the wavelet transform is

$$T_\psi(n,s) = \frac{1}{s}\sum_{i=1}^{N}\psi\left(\frac{i-n}{s}\right)x(i) \tag{8}$$

and the partition function becomes

$$Z(q,s) = \sum_{l(s)}\left|T_\psi(n_l(s),s)\right|^q \tag{9}$$

where the index l goes over the set of all maxima of $\left|T_\psi(n,s)\right|$ for each value of the scale parameter s and $n_l(s)$ represents the position of any given maximum. For fractal signals the partition function satisfies an equation similar to (6):

$$Z(q,s) \sim s^{\tau(q)} \tag{10}$$

and equation (7) remains unchanged.

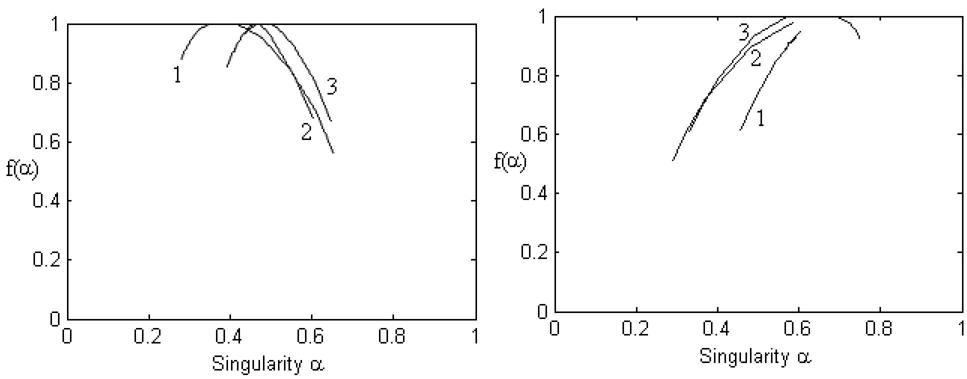
The function $\tau(q)$ is computed as the slope of the graph $\log Z(q,s)$ versus $\log s$ for the straight portion corresponding to small values of s and finally, the singularity spectrum $f(\alpha)$ is given by equation (7).

3. Computational results

The singularity spectrum of the brown noise time series is shown in Fig. 2a and that of the financial time series analysed in this paper is shown in Fig. 2b.

The figures present the singularity spectra for three increasing lengths of the time series. First, we shall discuss the situation for the brown noise (Fig. 2a). The spectrum denoted 1 is obtained for series length of 1024 points, the one denoted 2 for a length of 2048 points and the one denoted 3 for 4096 points. We notice a convergence towards the value D_0=0.5 known to characterise the brown noise. Further increasing of the length has practically no effect on the spectrum.

In the case of the analysed financial time series, we were unable to follow the same steps because its length was restricted to only 1350 points. Consequently, we increased the length of the series by repeating it once and then by adding again the original time-serie. The spectrum denoted 1 in Fig. 2b is obtained for the original time series, the one denoted 2 for the double length time series (2700 points) and the one denoted 3 for the triple length time series (4050 points). The addition of another 1350 points segment does not significantly change the singularity spectrum. In this case, we notice a similar convergence towards a box counting dimension D_0 = 0.65. This is consistent with previously published results [17] and is also proved by the box-counting computation shown in Fig. 3. We stress that the box-counting result is not altered by the lengthening process described.



(a) (b)

Fig. 2 Singularity spectra for brown noise (a) and the analyzed financial time series (b) for different length of the time series

We infer that this artificial process of lengthening of the available time series is really beneficial for the results of a multifractal analysis by the WTMM.

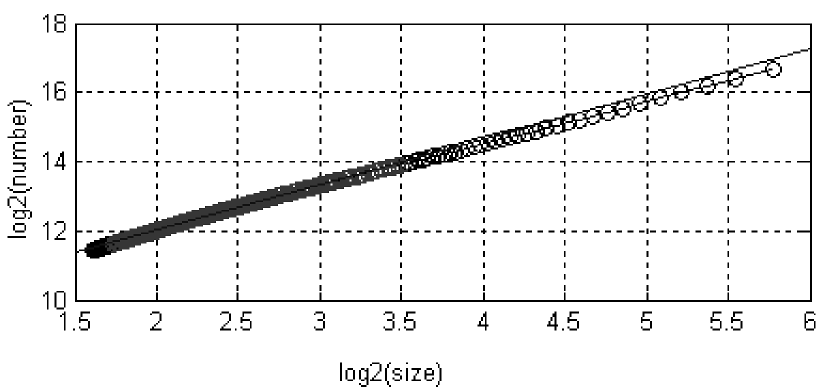


Fig. 3 Computation of the box-counting dimension of the financial time series

4. Conclusions

The analysis of this type is very important because in many practical situations, particularly the present one, the first usually addresed question is whether the data under study is monofractal or multifractal. It should be noticed that both methods mentioned above work satisfactorily well for very long time series. However, the results for short ones (as limited by practical constraints) can be, and indeed are affected by errors leading to a broadening and a translation of the singularity spectrum. It is not pointlike as it should be for monofractal signals. This effect is clearly seen on Fig. 2, because we know from the previous analysis [17] that the signal is monofractal. The result is consistent with a comparative analysis of the accuracy of the two methods which shows that, for short time series, the method based on detrended fluctuation analysis gives more reliable results than the wavelet transform analysis [19].

We find that our multifractal analysis gives reliable results for a time series longer than 4000 points. If the available time series is (much) shorter, considerably improved results are expected via a lenghtening procedure consisting of repeating the available time series. Clearly, a carefully detrended procedure has to be previously applied in order to avoid artificially introduced fluctuations that might alter the singularity spectrum.

REFERENCES

[1] *R. N. Mantegna, H. E. Stanley*, Nature **383** (1996) 587.

[2] *H. E. Stanley, L. A. N. Amaral, X. Gabaix, P. Gopikrishnan, V. Plerou,* Physica A **299** (2001) 1.

[3] *R.N. Mantegna, H.E. Stanley,* An introduction to econophysics: Correlations and Complexity in Finance (fourth edition), Cambridge University Press, Cambridge, 2004.

[4] B. *Mandelbrot*, Scientific American (1999) 70.

[5] *J. W. Kantelhardt, S. A. Zschiegner, A. Bunde, S. Havlin, E. Koscielny-Bunde, and H. E. Stanley*, Physica A **316**, (2002) 87.

[6] *N. Vandewalle, M. Ausloos*, J. Eur. Phys. B **4** (1998) 257.

[7] *N. Scafetta, L. Griffin, B. J*. West, Physica A **328** (2003) 561.

[8] *V. A. Litvin*, Physica A **344** (2004) 178.

[9] *E. W. Montrol and M. F. Schlesinger* in Non-equilibrium Phenomena II. From Stochastics to Hydrodynamics, Ed., J. L. Lebowitz, North-Holland, Amsterdam, 1984.

[10]. A. *Renyi*, Probability Theory, North Holland Amsterdam, 1971.

[11]. *P. Grassberger*, Phys. Lett. A **97**, (1983) 227.

[12]. *E. Ott*, Chaos in Dynamical Systems, Cambridge Univ. Press, New York, 1993.

[13]. C. *–K. Peng, S. V. Buldyrev, S. Havlin, M. Simons, H. E. Stanley, and A. L. Goldberger*, Phys. Rev. E **49,** (1994) 1685.

[14*]. J. Kantelhardt, S.A. Zschiegner, A. Bunde, S. Havlin, E. Koscielny-Bunde and H. E. Stanley*, Physica A **316**, (2002) 87.

[15]. *J. F. Muzy, E. Bacry and A. Arnéodo*, Phys. Rev. Lett. **67,** (1991) 3515.
[16]. *J. F. Muzy, E. Bacry and A. Arnéodo*, Phys. Rev. E **47,** (1993) 875.
[17]. *E. Scarlat, C. Stan and C.P. Cristescu*, Physica A **379**, (2007) 188.
[18]. *S. Mallat, and W. L. Hwang*, IEEE Trans. Inf. Theory **38**, (1992) 617.
[19]. *P. Oswiecimka, J. Kwapien and S. Drozdz*, Phys. Rev. E **74**, (2006) 016103.