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INFLUENCE OF THE PRIMARY SUSPENSION DAMPING ON THE VERTICAL DYNAMIC FORCES AT THE PASSENGER RAILWAY VEHICLES

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În lucrare se analizează influența amortizării suspensiei primare a unui vagon de călători asupra mărimii forțelor dinamice verticale generate la interfața roată-şină. Pentru aceasta s-a adoptat un model complex al sistemului vehicul-cale de rulare care ia în considerare pe de o parte, elasticitatea căii şi a contactului roată-şină, iar pe de alta, vibrațiile vehiculului, inclusiv vibrația de încovoiere a cutiei. Aplicând analiza modală, s-a dat o nouă formă ecuațiilor de mişcare care descriu mişcările simetrice şi antisimetrice ale sistemului vehicul-cale şi modurile lor de excitare. Influența gradului de amortizare a suspensiei primare asupra forțelor dinamice s-a studiat în corelație cu regimul de viteză al vehiculului şi diferiți parametri ai acestuia. S-a arătat că la viteze mari de circulație există un grad de amortizare al suspensiei primare care conduce la valori minime ale forțelor dinamice verticale.

The damping of primary suspension of a passenger railway vehicle influences the magnitude of vertical dynamic forces at the wheel/rail interface. In order to calculate the vertical dynamic forces, a model of the vehicle-track system has been adopted to take into consideration the elasticity of the track and wheel/rail contact, on the one hand, and the vibration of the vehicle, including the carbody bending vibration, on the other hand. Upon applying the modal analysis, a new form has been given to the movement equations that describe the decoupled symmetrical and anti-symmetrical movements of the vehicle/track system and their excitation modes. The influence of the damping ratio of primary suspension upon the magnitude of the vertical dynamic forces has been studied along with the vehicle velocity and various parameters. It has been shown that there is a damping ratio of primary suspension for high velocities, which leads to minimum values of the vertical dynamic forces.

Key words: railway vehicle, vertical dynamic forces, suspension, damping

1. Introduction

While vehicle rolling, the dynamic forces generated at the wheel/rail interface will inevitably trigger the damage in the track and the rolling device, the wear of the rolling surfaces [1] and the rolling noise [2, 3]. It is worthwhile mentioning that the dynamic forces have multiple causes: rolling track

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irregularities [4], irregularities of the rolling surfaces [5], discontinuities of the rail rolling surface (joints, switches, crossings) [6], local defects in the rolling wheel surface (wheel flat) [7]. Among these, the vertical dynamic forces occurring at frequencies up to circa 20 Hz are set off by the vehicle response while passing over the rolling track irregularities. On the contrary, the dynamic forces over 20 Hz come from the structural wheel/rail vibrations [8, 9].

In designing the railway vehicle and the rolling track, the fact that the dynamic forces need to comply with the limits included in the specific standards is taken into consideration [10]. At the construction speeds of the railway vehicles, they are required to have the capacity to pass over the vertical irregularities of the rolling track without generating excessive wheel/rail vertical dynamic forces. In the United Kingdom, the standard GM/TT0088 [11] stipulates the limitation of the vertical forces on wheel at 322 kN, irrespective of the velocity, considering that it is about the totality of the forces generated at the wheel/rail interface (static force on wheel, inertia forces and the forces associated with the dynamic response of the unsuspended masses at the vertical irregularities of the rolling track). In Europe, one of the homologation criteria for the railway vehicles from the dynamic behaviour perspective refers to the fatigue of track [12]. In the document above, the limit values of the vertical wheel/rail forces derived from the addition of static load on wheel and the vertical dynamic forces that occur while vehicle passes over the rolling track irregularities is a velocity factor.

Restricting the dynamic forces at an acceptable level involves, as far as the vehicle is concerned, constructive limits that target the vertical forces on wheel, the suspended and unsuspended massed, the diameter of wheels, the suspension features, etc.

For a passenger vehicle, the damping coefficients that correspond to those two levels of suspension are generally calculated in terms of the vertical comfort criterion [13]. Zhou s.a. [14] mentions, without bringing any argument or pointing out a reference, that the damping ratio of the primary suspension cannot be increased as much as desired, due to the limitations brought about the vertical dynamic wheel/rail forces.

About the latter, this paper studies the influence of the primary suspension damping on the vertical dynamic forces, in correlation with other vehicle parameters.

In order to calculate the vertical dynamic forces, a model of the vehicle-track system has been adopted to pay attention to the elasticity of the rolling track and wheel/rail contact, on one hand, and the vibration of the vehicle, including the carbody bending vibration, on the other hand. The movement equations are processed by applying the technique of modal analysis, in an original way, which allows highlighting the fact that the movement of the vehicle-track system can be decomposed into symmetrical and anti-symmetrical decoupled movements, with their own excitation modes.

The influence of the primary suspension damping upon the vertical dynamic forces is dependent on the velocity, degree of damping of the secondary suspension, the axle mass and the vehicle wheelbase. It is, therefore, shown that the minimizing requirement of the vertical dynamic forces can help to determine the best damping of the primary suspension on this account.

2. The mechanical model and the movement equations

To study the vertical vibrations, a four-axle, two-level suspension railway vehicle, travelling at the constant speed V on a track with random longitudinal irregularities is being looked at.

The size of irregularities against each axle depends on its position. Thus, bearing reference to the track irregularity against the vehicle centre, the defects against the axles are dephased with $2\pi(a_c \pm a_b)/\Lambda$ - opposite the first bogie and with $2\pi(-a_c \pm a_b)/\Lambda$ opposite the second bogie, where Λ is the defect wavelength, $2a_c$ is the vehicle wheelbase and $2a_b$ represents the bogie wheelbase. The defects of the transversal nivelment are overlooked, hence the possibility of exciting the rolling movement is ruled out.

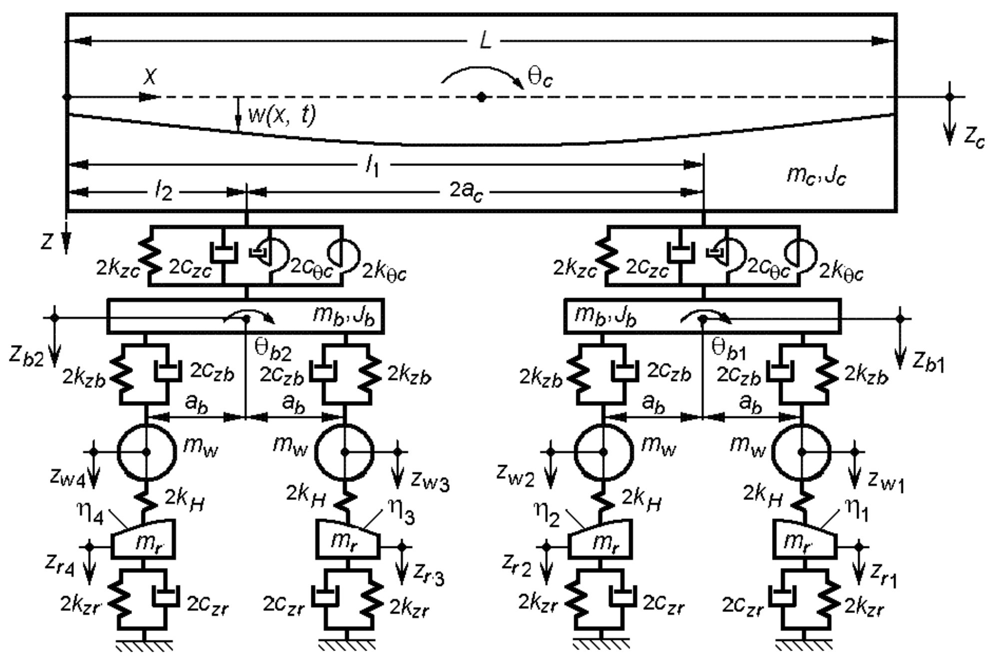

Fig. 1. The mechanical model of the vehicle-track system.

Should we disregard the coupling effects between wheels derived from the propagation of the bending waves in the rails in the frequency range specific to the vertical vibration of the vehicle, an equivalent model with concentrated parameters will be adopted for the track. Opposite each axle, the track is represented by an oscillatory system with a degree of freedom that can move vertically, where the corresponding travelling is z_{rj}, for j = 1÷4. The equivalent track model has the mass m_r, rigidity k_{zr} and the damping coefficient c_{zr}.

The vehicle model includes a body with parameters distributed for the carbody and a system of rigid bodies, namely the axles and the suspended masses of the two bogies.

The carbody of a length L is modelled by an Euler-Bernoulli beam of a constant section and an uniformly distributed mass, with the bending model EI and mass per length unit m. The structural damping of the carbody will be also weighted in, by the damping coefficient μ. The displacement of a beam section in relation to the mobile referential Oxz attached to the rear carbody end is $w(x,t)$, where t is time. The positions of the carbody suspension points on the secondary suspension are given by the distances l_1 and l_2.

To make the analysis simpler a model, has been adopted that allows considering the natural vibration modes only that come from the carbody bending. Also, models based on finite element can be used to study the influence of the complex carbody modes, by observing the construction specific features of a certain vehicle.

The suspended masses of the bogies are considered to be two-degree of freedom rigid bodies, i.e. the bounce movement z_{bi} and pitch θ_{bi}, with i = 1, 2. The mass of a bogie is m_b and its inertia moment $J_b = m_b i_b^2$, where i_b – the gyration radius of the bogie.

The wheelset with mass m_w has only one degree of freedom, which is the vertical movement z_{wj}, with j = 1÷4.

The suspension levels of the vehicle, two per each bogie, are modelled via Kelvin-Voigt systems. The primary suspension has one Kelvin-Voigt system working on the transition motion, and the secondary suspension has two Kelvin-Voigt systems for translation and rotation. The elastic constants are k_{zb}, k_{zc} and $k_{\theta c}$, and the damping ones c_{zb}, c_{zc} and $c_{\theta c}$. It should be mentioned that the elements of the Kelvin-Voigt system that take over the relative angle motion between carbody and bogie take into view the influence of the secondary suspension and of the system of transmitting the longitudinal force between carbody and bogies.

To calculate the vertical dynamic forces, the hypothesis of the linear hertzian contact between wheel and rail is assumed

$$\Delta Q_j = k_H (z_{wj} - z_{rj} - \eta_j), \; j = 1 \text{ to } 4, \tag{1}$$

where η_j, with j = 1÷4, represents the track longitudinal irregularities opposite the axle j, and k_{H} – the rigidity of the wheel-rail contact.

The movement equations are:

- for the carbody bending

$$EI\frac{\partial^4 w(x,t)}{\partial x^4}+\mu I\frac{\partial^5 w(x,t)}{\partial x^4 \partial t}+m\frac{\partial^2 w(x,t)}{\partial t^2}=\sum_{i=1}^{2}\left[F_i\delta(x-l_i)-M_i\frac{\mathrm{d}\delta(x-l_i)}{\mathrm{d}x}\right], \quad (2)$$

where δ(.) is Dirac's delta function, while F_i and M_i represent the force and moment due to the bogie secondary suspension i

$$F_i=-2c_{zc}\left(\frac{\partial w(l_i,t)}{\partial t}-\dot{z}_{bi}\right)-2k_{zc}\left(w(l_i,t)-z_{bi}\right); \quad (3)$$

$$M_i=-2c_{\theta c}\left(\frac{\partial^2 w(l_i,t)}{\partial t \partial x}-\dot{\theta}_{bi}\right)-2k_{\theta c}\left[\frac{\partial w(l_i,t)}{\partial x}-\theta_{bi}\right], \quad (4)$$

which acts from the distance l_i from the carbody end.

Should we assign to the two bogies the indices i = 1 and 2 and considering that they are equipped with the axles j and j+1, where j is 1 or 3, as case may be, then the movement equations of the bogies and of the four axles can write as:

- for bogies bounce

$$\begin{aligned}&m_b\ddot{z}_{bi}+2c_{zb}(2\dot{z}_{bi}-\dot{z}_{wj}-\dot{z}_{w(j+1)})+2k_{zb}(2z_{bi}-z_{wj}-z_{w(j+1)})-\\&+2c_{zc}\left[\dot{z}_{bi}-\frac{\partial w(l_i,t)}{\partial t}\right]-2k_{zc}[z_{bi}-w(l_i,t)]=0;\end{aligned} \quad (5)$$

- for bogies pitch

$$\begin{aligned}&J_b\ddot{\theta}_{bi}+2c_{zb}a_b(2a_b\dot{\theta}_{bi}-\dot{z}_{wj}+\dot{z}_{w(j+1)})+2k_{zb}a_b(2a_b\theta_{bi}-z_{wj}+z_{w(j+1)})+\\&+2c_{\theta c}\left[\dot{\theta}_{bi}-\frac{\partial^2 w(l_i,t)}{\partial x \partial t}\right]+2k_{\theta c}\left[\theta_{bi}-\frac{\partial w(l_i,t)}{\partial x}\right]=0;\end{aligned} \quad (6)$$

- for the vertical movement of axles

$$\begin{aligned}&m_o\ddot{z}_{wj,j+1}+2c_{zb}(\dot{z}_{wj.j+1}\mp a_b\dot{\theta}_{bi}-\dot{z}_{bi})+2k_{zb}(z_{wj,j+1}\mp a_b\theta_{bi}-z_{bi})+\\&+2k_H(z_{wj,j+1}-z_{rj,j+1}-\eta_{j,j+1})=0;\end{aligned} \quad (7)$$

The equation of the rail vertical movement is

$$m_s \ddot{z}_{rj} + 2c_{zr}\dot{z}_{rj} + 2k_{zr}z_{rj} + 2k_H(z_{rj} - z_{wj} + \eta_j) = 0 . \qquad (8)$$

The movement equations with partial derivatives can be turned into equations with ordinary derivatives by applying the method of the modal analysis. For this, the carbody rigid and bending modes are looked at, in the form of

$$w(x,t) = z_c(t) + \left(x - \frac{L}{2}\right)\theta_c(t) + \sum_{n=2}^{\infty} X_n(x)T_n(t) , \qquad (9)$$

where $z_c(t)$ and $\theta_c(t)$ represent the carbody vibration rigid modes, namely the pitch and bounce, $T_n(t)$ is time-dependent function and $X_n(x)$ is the eingenfunction of the bending vibration mode n

$$X_n(x) = \sin\beta_n x + \sinh\beta_n x - \frac{\sin\beta_n L - \sinh\beta_n L}{\cos\beta_n L - \cosh\beta_n L}(\cos\beta_n x + \cosh\beta_n x) ; \qquad (10)$$

with $\beta_n = \sqrt{\omega_n^2 m/(EI)}$ (11)

and $\cos\beta_n L \cosh\beta_n L - 1 = 0$, (12)

where ω_n is the natural pulsation of the vibration mode n.

Upon taking into account the first two natural bending modes only, symmetrical and anti-symmetrical, the vibration of the vehicle-track system is described by a set of 16 coupled equations with ordinary derivatives. In spite of this and after a corect selection of coordinates and an appropriate processing of the of equations, we have its decomposition into two independent sets of eight equations each. The two sets describe the symmetrical and anti-symmetrical movements of the vehicle-track system. As we note below

$$q_1^+ = z_c ; \; q_1^- = \theta_c ; \; q_2^+ = T_2 ; \; q_2^- = T_3 ;$$

$$q_3^+ = \frac{1}{2}(z_{b1} + z_{b2}) ; \; q_3^- = \frac{1}{2}(z_{b1} - z_{b2}) ;$$

$$q_4^+ = \frac{1}{2}(\theta_{b1} - \theta_{b2}) ; \; q_4^- = \frac{1}{2}(\theta_{b1} + \theta_{b2}) ;$$

$$q_5^+ = \frac{1}{4}(z_{o1} + z_{o2} + z_{o3} + z_{o4}) ; \; q_5^- = \frac{1}{4}(z_{o1} + z_{o2} - z_{o3} - z_{o4}) ;$$

$$q_6^+ = \frac{1}{4}(z_{o1} - z_{o2} - z_{o3} + z_{o4})\,;\;\; q_6^- = \frac{1}{4}(z_{o1} - z_{o2} + z_{o3} - z_{o4})\,; \tag{13}$$

$$q_7^+ = \frac{1}{4}(z_{s1} + z_{s2} + z_{s3} + z_{s4})\,;\;\; q_7^- = \frac{1}{4}(z_{s1} + z_{s2} - z_{s3} - z_{s4})\,;$$

$$q_8^+ = \frac{1}{4}(z_{s1} - z_{s2} - z_{s3} + z_{s4})\,;\;\; q_8^- = \frac{1}{4}(z_{s1} - z_{s2} + z_{s3} - z_{s4})\,,$$

the equations of the symmetrical movements (bounce and carbody symmetrical bending, bounce and symmetrical pitch of bogies, the vertical symmetrical movement of axles and rails) are derived,

$$m_c\ddot{q}_1^+ + 4c_{zc}(\dot{q}_1^+ + \varepsilon^+\dot{q}_2^+ - \dot{q}_3^+) + 4k_{zc}(q_1^+ + \varepsilon^+ q_2^+ - q_3^+) = 0\,; \tag{14}$$

$$\begin{aligned} &m_{m2}\ddot{q}_2^+ + c_{m2}\dot{q}_2^+ + k_{m2}q_2^+ + \\ &+ 4c_{zc}\varepsilon^+(\dot{q}_1^+ + \varepsilon^+\dot{q}_2^+ - \dot{q}_3^+) + 4k_{zc}\varepsilon^+(q_1^+ + \varepsilon^+ q_2^+ - q_3^+) + \\ &+ 4c_{\theta c}\lambda^+[\lambda^+\dot{q}_2^+ - \dot{q}_4^+] + 4k_{\theta c}\lambda^+(\lambda^+ q_2^+ - q_4^+) = 0\,; \end{aligned} \tag{15}$$

$$\begin{aligned} &m_b\ddot{q}_3^+ + 4c_{zb}(\dot{q}_3^+ - \dot{q}_5^+) + 4k_{zb}(q_3^+ - q_5^+) + \\ &+ 4c_{zc}(\dot{q}_3^+ - \dot{q}_1^+ - \varepsilon^+\dot{q}_2^+) + 4k_{zc}(q_3^+ - q_1^+ - \varepsilon^+ q_2^+) = 0\,; \end{aligned} \tag{16}$$

$$\begin{aligned} &J_b\ddot{q}_4^+ + 4c_{zb}a_b(a_b\dot{q}_4^+ - \dot{q}_6^+) + 4k_{zb}a_b(a_b q_4^+ - q_6^+) + \\ &+ 2c_{\theta c}(\dot{q}_4^+ - \lambda^+\dot{q}_2^+) + 2k_{\theta c}(q_4^+ - \lambda^+ q_2^+) = 0\,; \end{aligned} \tag{17}$$

$$m_w\ddot{q}_5^+ + 2c_{zb}(\dot{q}_5^+ - \dot{q}_3^+) + 2k_{zb}(q_5^+ - q_3^+) + 2k_H(q_5^+ - q_7^+ - \eta_1^+) = 0\,; \tag{18}$$

$$m_w\ddot{q}_6^+ + 2c_{zb}(\dot{q}_6^+ - a_b\dot{q}_4^+) + 2k_{zb}(q_6^+ - a_b q_4^+) + 2k_H(q_6^+ - q_8^+ - \eta_2^+) = 0\,; \tag{19}$$

$$m_r\ddot{q}_7^+ + 2c_{zr}\dot{q}_7^+ + 2k_{zr}q_7^+ + 2k_H(q_7^+ - q_5^+ + \eta_1^+) = 0\,; \tag{20}$$

$$m_r\ddot{q}_8^+ + 2c_{zr}\dot{q}_8^+ + 2k_{zr}q_8^+ + 2k_H(q_8^+ - q_6^+ + \eta_2^+) = 0 \tag{21}$$

plus the equations of the anti-symmetrical movements (carbody pitch, anti-symmetrical bending, anti-symmetrical bounce and pitch of bogies, vertical anti-symmetrical movement of axles and rails)

$$\begin{aligned} & J_c\ddot{q}_1^- + 4c_{zc}a_c(a_c\dot{q}_1^- + \varepsilon^-\dot{q}_2^- - \dot{q}_3^-) + 4k_{zc}a_c(a_cq_1^- + \varepsilon^-q_2^- - q_3^-) + \\ & + 4c_{\theta c}(\dot{q}_1^- + \lambda^-\dot{q}_2^- - \dot{q}_4^-) + 4k_{\theta c}(q_1^- + \lambda^-q_2^- - q_4^-) = 0\,; \end{aligned} \tag{22}$$

$$\begin{aligned} & m_{m3}\ddot{q}_2^- + c_{m3}\dot{q}_2^- + k_{m3}q_2^- + \\ & + 4c_{zc}\varepsilon^-(\dot{q}_1^- + \varepsilon^-\dot{q}_2^- - \dot{q}_3^-) + 4k_{cz}\varepsilon^-(q_1^- + \varepsilon^-q_2^- - q_3^-) + \\ & + 4c_{\theta c}\lambda^-(\dot{q}_1^- + \lambda^-\dot{q}_2^- - \dot{q}_4^-) + 4k_{\theta c}\lambda^-(q_1^- + \lambda^-q_2^- - q_4^-) = 0\,; \end{aligned} \tag{23}$$

$$\begin{aligned} & m_b\ddot{q}_3^- + 4c_{zb}(\dot{q}_3^- - \dot{q}_5^-) + 4k_{zb}(q_3^- - q_5^-) + \\ & + 4c_{zc}(\dot{q}_3^- - a_c\dot{q}_1^- - \varepsilon^-\dot{q}_2^-) + 4k_{zc}(q_3^- - a_cq_1^- - \varepsilon^-q_2^-) = 0\,; \end{aligned} \tag{24}$$

$$\begin{aligned} & J_b\ddot{q}_4^- + 4c_{zb}a_b(a_b\dot{q}_4^- - \dot{q}_6^-) + 4k_{zb}a_b(a_bq_4^- - q_6^-) + \\ & + 4c_{\theta c}(\dot{q}_4^- - \dot{q}_1^- - \lambda^-\dot{q}_2^-) + 4k_{\theta c}(q_4^- - q_1^- - \lambda^-q_2^-) = 0\,; \end{aligned} \tag{25}$$

$$m_w\ddot{q}_5^- + 2c_{zb}(\dot{q}_5^- - \dot{q}_3^-) + 2k_{zb}(q_5^- - q_3^-) + 2k_H(q_5^- - q_7^- - \eta_1^-) = 0\,; \tag{26}$$

$$m_w\ddot{q}_6^- + 2c_{zb}(\dot{q}_6^- - a_b\dot{q}_4^-) + 2k_{zb}(q_6^- - a_bq_4^-) + 2k_H(q_6^- - q_8^- - \eta_2^-) = 0\,; \tag{27}$$

$$m_r\ddot{q}_7^- + 2c_{zr}\dot{q}_7^- + 2k_{zr}q_7^- + 2k_H(q_7^- - q_5^- + \eta_1^-) = 0\,; \tag{28}$$

$$m_r\ddot{q}_8^- + 2c_{zr}\dot{q}_8^- + 2k_{zr}q_8^- + 2k_H(q_8^- - q_6^- + \eta_2^-) = 0\,, \tag{29}$$

where m_c and $J_c = m_c i_c^2$ are the carbody mass and inertia moment, with i_c – the gyration radius, whereas $m_{m2,3}$, $c_{m2,3}$ and $k_{m2,3}$ are the modal masses, damping constants and rigidities

$$\begin{aligned} & m_{m2,3} = m\int_0^L X_{2,3}^2(x)\mathrm{d}x\,; \\ & c_{m2,3} = \mu I\int_0^L \left(\frac{\mathrm{d}^2X_{2,3}(x)}{\mathrm{d}x^2}\right)^2 \mathrm{d}x\,;\ k_{m2,3} = EI\int_0^L \left(\frac{\mathrm{d}^2X_{2,3}(x)}{\mathrm{d}x^2}\right)^2 \mathrm{d}x\,. \end{aligned} \tag{30}$$

Similarly, the following notations have been used

$$X_2(l_1) = X_2(l_2) = \varepsilon^+; \quad X_3(l_1) = -X_3(l_2) = \varepsilon^-; \tag{31}$$

$$\frac{\mathrm{d}X_2(l_1)}{dx} = -\frac{\mathrm{d}X_2(l_2)}{dx} = \lambda^-; \quad \frac{\mathrm{d}X_3(l_1)}{dx} = \frac{\mathrm{d}X_3(l_2)}{dx} = \lambda^+ . \tag{32}$$

The equations include two excitation modes of the symmetrical movements

$$4\eta_1^+ = \eta_1 + \eta_2 + \eta_3 + \eta_4; \; 4\eta_1^- = \eta_1 + \eta_2 - \eta_3 - \eta_4 \tag{33}$$

and two anti-symmetrical modes of excitation

$$4\eta_2^- = \eta_1 - \eta_2 + \eta_3 - \eta_4; \; 4\eta_2^+ = \eta_1 - \eta_2 - \eta_3 + \eta_4 . \tag{34}$$

In order to facilitate the analysis of the vehicle vibration behaviour, the following damping ratios of the suspension levels are introduced

$$\zeta_{b,c} = \frac{4c_{b,c}}{2\sqrt{4k_{b,c}m_{b,c}}} . \tag{35}$$

In the next section, the movement equations will be used to evaluate the vertical dynamic wheel/rail forces.

3. The vertical dynamic wheel/rail forces

In order to calculate the vertical dynamic forces, the track irregularities are considered as random and stationary. The review literature mentions more calculation ratios of the power spectral densities in the track irregularities. The ORE recommended form is shown below [15]

$$S(\Omega) = \frac{A\Omega_c^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)} , \tag{36}$$

where Ω is the wave number, $\Omega_c = 0{,}8246$ rad/m, $\Omega_r = 0{,}0206$ rad/m, and $A = 4{,}032 \cdot 10^{-7}$ rad m or $A = 1{,}080 \cdot 10^{-6}$ rad m, depending on the track quality.

Since the track irregularities become an excitation factor for a vehicle travelling at speed V, then the power spectral density of the track irregularities must be expressed reported to the angular frequency $\omega = V\Omega$ as seen in the general relation

$$G(\omega) = \frac{S(\omega / V)}{V} . \tag{37}$$

From equations (36) and (37) we have

$$G(\omega) = \frac{A\Omega_c^2 V^3}{[\omega^2 + (V\Omega_c)^2][\omega^2 + (V\Omega_r)^2]}. \tag{38}$$

Starting from the frequency response factors of dynamic forces and the power spectral density of track irregularities, the power spectral density of the dynamic forces can be calculated

$$G_{Qj}(\omega) = G(\omega)\left|\overline{H}_{Qj}(\omega)\right|^2, \text{ for } j = 1 \text{ to } 4. \tag{39}$$

where $\overline{H}_{Qj}(\omega) = k_H \left[\overline{H}_{wj}(\omega) - \overline{H}_{rj}(\omega) - \overline{H}_{\eta j}(\omega)\right],$ (40)

with $\overline{H}_{wj}(\omega)$ and $\overline{H}_{rj}(\omega)$ the frequency responses of the wheel and rail j,

$$\overline{H}_{\eta j}(\omega) = \exp(\mathrm{i}\omega(a_c \pm a_b)/V), \text{ for } j = 1 \text{ and } 2; \tag{41}$$

$$\overline{H}_{\eta j}(\omega) = \exp(\mathrm{i}\omega(-a_c \pm a_b)/V), \text{ for } j = 3 \text{ and } 4. \tag{42}$$

Next, the root mean square of the vertical dynamic wheel/rail forces is determined on basis of the above

$$\sigma_{Qj} = \sqrt{\frac{1}{\pi}\int_0^{\infty} G_{Qj}(\omega)\mathrm{d}\omega}, \; j = 1 \text{ to } 4. \tag{43}$$

4. Numerical application

This chapter presents the results of the numerical simulations concerning the influence of the primary suspension damping upon the vertical dynamic forces obtained from the model and method above. Starting from this criterion of minimizing the vertical dynamic forces, the best damping of primary suspension is derived, in relation to various parameters of vehicle.

The parameters of the model in Table 1 correspond to a passenger railway vehicle.

The vertical dynamic forces that occur at frequencies up to 20 Hz are induced by the response of the vehicle carbody, bogies and of other masses while passing over the track irregularities. As a consequence, the track irregularities wavelengths that contribute to the excitation of the vehicle natural frequencies will be taken into account, namely between 3 and 100 m.

Table 1

Parameters of the vehicle-track system

Carbody mass	m_c = 34320 kg
Bending module	$EI = 3.2 \cdot 10^9$ Nm^2
Carbody length	L = 26.4 m
Carbody wheelbase	$2a_c$ = 19 m
Carbody gyration radius	i_c = 7.6 m
Carbody model damping ratio	$\zeta_{m2,3}$ = 0.015
Vertical rigidity of secondary suspension	$4k_{zc}$ = 2.4 MN/m
Carbody-bogie angle rigidity	$2k_{\theta c}$ = 1.6 MNm
Vertical damping of secondary suspension	$4c_{zc}$ = 68.88 kNs/m
Carbody-bogie angle damping	$2c_{\theta c}$ = 2.87 kNm
Bogie mass	m_b = 3200 kg
Bogie gyration radius	i_b = 0.8 m
Bogie wheelbase	$2a_b$ = 2.56 m
Vertical rigidity of primary suspension	$4k_{zb}$ = 4.4 MN/m
Vertical damping of primary suspension	$4c_{zb}$ = 52.21 kNs/m
Axle mass	m_w = 1686 kg
Track mass	m_r = 180 kg
Track vertical rigidity	$2k_{zr}$ = 170 MN/m
Track vertical damping	$2c_{zr}$ = 52 kNs/m
Rigidity of wheel/rail contact	$2k_H$ = 3000 MN/m

In Fig. 2, we see the influence of the damping ratio of primary suspension on the vertical dynamic wheel-rail forces in those four axles of vehicle, for velocities between 100 and 300 km/h. The charts prove that the magnitude of the vertical dynamic forces depend on the position of axles inside the vehicle, irrespective of the speed behaviour of vehicle, where the highest value of dynamic forces is recorded for the rear axle of the second bogie. Should the reference velocity is 250 km/h and the suspension damping ratios match the reference parameters in table 1, ζ_b = 0.22 and ζ_c = 0.12, we will have the following values: ΔQ_1 = 1553 N, ΔQ_2= 1740 N, ΔQ_3 = 1664 N and ΔQ_4 = 1860 N. Similarly, it is obvious that the ΔQ forces increase along with velocity. In reference to this observation and to the influence of the damping of primary suspension upon the vertical dynamic forces, it needs to be shown that the dynamic forces depend only a little on ζ_b velocities under 200 km/h.

At higher velocities, a minimum value of the vertical dynamic forces is present for a certain value of the damping ratio of primary suspension, a damping hereinafter called the best damping. For instance, V = 250 km/h of the rear end axle of vehicle corresponds to the damping ratio ζ_b = 0.09, and at V = 300 km/h, the best damping is obtained for ζ_b = 0.07.

As already shown, the rear end axle of the vehicle presents higher dynamic forces than the other three, a reason for including below the results of the numerical simulations for this case exclusively.

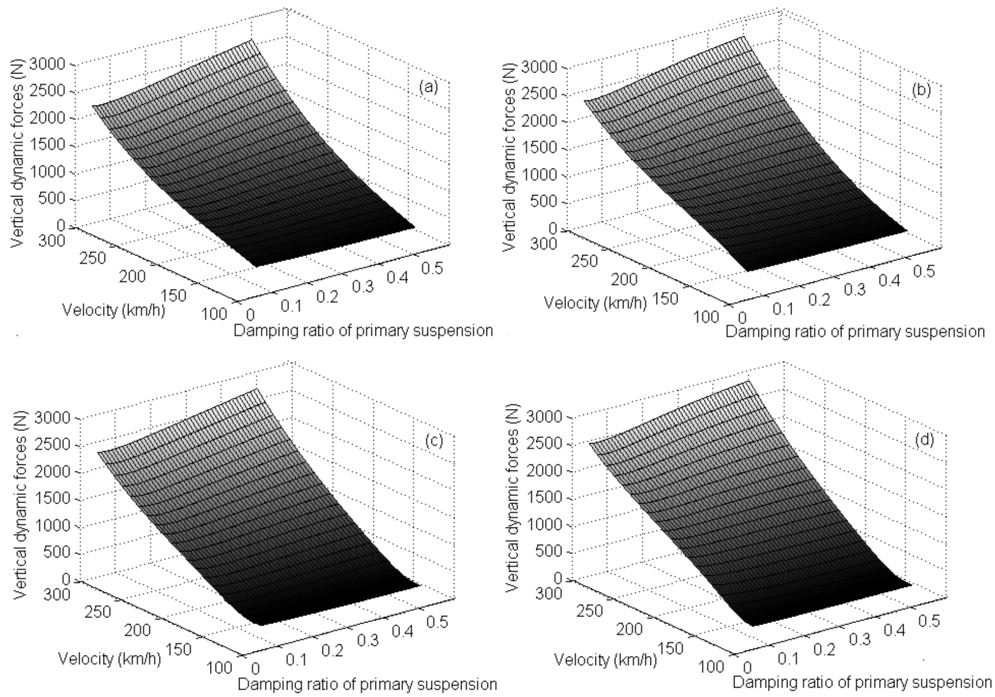

Fig. 2. Influence of velocity upon the vertical dynamic forces:
(a) ΔQ_1; (b) ΔQ_2; (c) ΔQ_3; (d) ΔQ_4.

Based on the diagram in Fig. 3, an analysis of the influence of the damping ratio of secondary suspension upon the magnitude of vertical dynamic wheel/rail forces was un dertaken. The premise is that the vehicle travels at velocity $V = 250$ km/h. A first general observation is related to the fact that a high value of ζ_c triggers the decrease of the dynamic wheel/rail forces ΔQ. It is evident that the damping ratio of the primary suspension leading to the minimizing of dynamic forces is not significantly influenced by value of ζ_c. Thus, if ζ_c is 0.1, then the best damping of primary suspension is 0.1; if the damping ratio ζ_c is 0.3 for the secondary suspension, the best damping of primary suspension is 0.09.

The vertical dynamic wheel/rail forces calculated for three values of the axle mass, namely 1400, 1600 and 1800 kg, at velocity 250 km/h, are shown in figure 4. The other parameters of the vehicle are considered as reference in table 1. While the axle mass increases, as well as the vertical dynamic forces ΔQ, the damping ratio of primary suspension for which the dynamic forces are minimum decreases. These downturns are not significant though: for the values of axle mass considered here, the best damping will take the value of around 0.1. Thus, the minimizing of the vertical dynamic load ΔQ_4 is done for $\zeta_b = 0.11$ if $m_w = 1400$ kg, for $\zeta_b = 0.10$ if $m_w = 1600$ kg and $\zeta_b = 0.09$ if the axle mass is 1800 kg.

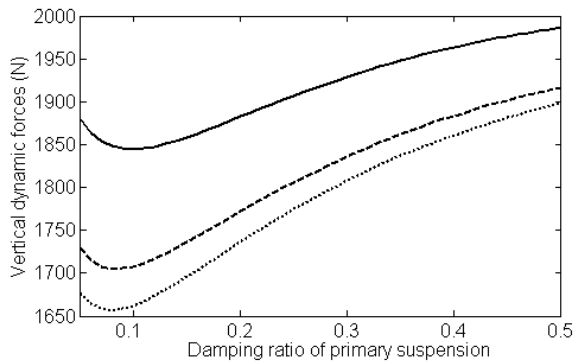

Fig. 3. Influence of damping ratio of secondary suspension upon the vertical dynamic forces: ——, $\zeta_c = 0.1$; – – –, $\zeta_c = 0.2$; · · · ·, $\zeta_c = 0.3$.

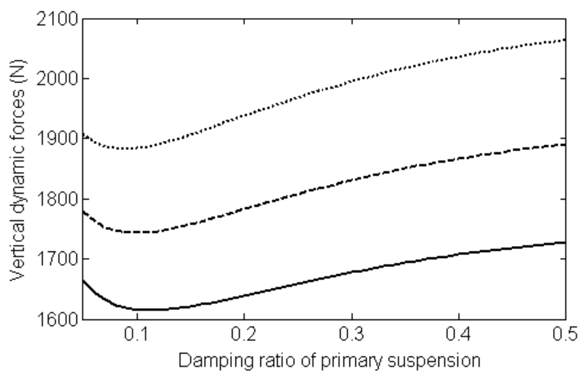

Fig. 4. Influence of axle mass upon the vertical dynamic forces: ——, $m_o = 1400$ kg; – – –, $m_o = 1600$ kg; · · · ·, $m_o = 1800$ kg.

The change in the bogie wheelbase will have contrary effects upon the magnitude of dynamic forces, in dependence with the damping ratio of primary suspension, as seen in Fig. 5. To analyse it, the reference value of bogie wheelbase $2a_b = 2.56$ m has been considered, plus two more values $2a_b = 2.3$ m and $2a_b = 2.8$ m, respectively.

It can be ascertained that a decrease in the wheelbase versus the reference value will result into a decrease of the dynamic forces, should the damping ratio of primary suspension is lower than 0.38. Moreover, the reduction of the wheelbase will lead to an increase of the damping ratio of primary suspension, for which the values of vertical dynamic forces are minimum, from 0.09 to 0.11.

We will thus have $\Delta Q_{4\min} = 1760$ N for $\zeta_b = 0.11$ and $\Delta Q_{4\min} = 1802$ N for $\zeta_b = 0.09$. The increase in the wheelbase to 2.8 m will result into higher vertical dynamic forces if the damping ratio of primary suspension is lower than 0.26, versus the reference value, i.e. for $2a_b = 2.56$ m. It should be mentioned that the best damping that leads to the minimization of the vertical dynamic forces does not change for this modification of the bogie wheelbase – and we are talking about the same damping ratio $\zeta_b = 0.09$.

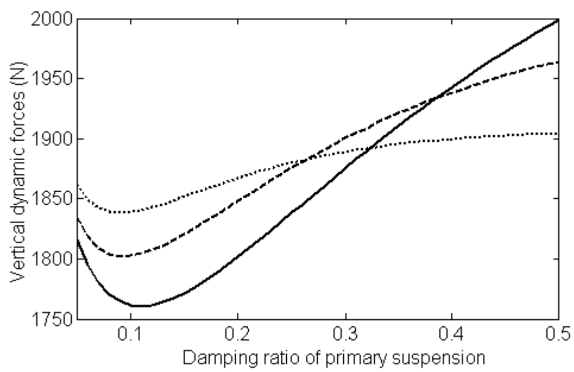

Fig. 5. Influence of bogie wheelbase upon vertical dynamic forces:
——, $2a_b = 2.3$ m; − − −, $2a_b = 2.56$ m; · · · ·, $2a_b = 2.8$ m.

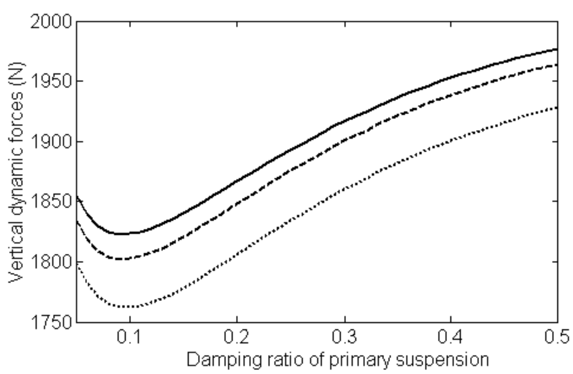

Fig. 6. Influence of vehicle wheelbase upon vertical dynamic forces:
——, $2a_c = 17$ m; − − −, $2a_c = 19$ m; · · · ·, $2a_c = 21$ m.

Finally, another issue to be examined refers to the influence of the vehicle wheelbase, in relation to the damping ratio of primary suspension upon the vertical dynamic forces. Hence, the same reference velocity $V = 250$ km/h will be considered, versus the reference value $2a_c = 19$ m, along with other two values of the carbody wheelbase, namely 17 and 21 m (see Fig. 6). A first observation would be that an increase in the carbody wheelbase results into a decrease of the vertical dynamic wheel/rail forces, irrespective of the damping ratio of primary suspension. Should we refer to the minimum values of the vertical dynamic forces and weigh in the extreme cases analysed above ($2a_c$ = 17 m and $2a_c$ = 21 m), we see a dimininuation of ΔQ_4 by 61 N. The decrease or increase of the wheelbase compared to the value of reference will not bring a change of the best damping of primary suspension, as this has the value of ζ_b = 0.09 for all three cases under discussion.

5. Conclusions

The vertical dynamic forces that occur while a railway vehicle travels over the rolling track irregularities are marked off by the frequency range spreading up to circa 20 Hz. To limit them, construction-related measures for both the rolling track and the vehicle will be taken. Building a track of an almost perfect geometry or maintaining it at its initial designing parameters is difficult, hence a great attention should be paid to designing the railway vehicle. The limitation of the dynamic forces generated at the wheel/rail interface presumes the adoption of a series of measures concerning construction parameters for the vehicle, which includes the characteristics of damping of the primary suspension.

In order to analyse the influence of the damping ratio of primary suspension upon dynamic forces, a complex model of the vehicle-track system has been taken into account. The vehicle model encloses a body with parameters distributed for the carbody and a system of rigid bodies, namely the axles and the suspended masses of the two bogies; for the track, an equivalent model with concentrated parameters has been assumed. Upon applying the modal analysis for processing the movement equations, the decoupled symmetrical and anti-symmetrical movements of the vehicle/rail systems and their excitation modes have been pointed out.

The examination of the influence of velocity behaviour upon the size of the vertical forces has indicated that the highest values are recorded by the rear end vehicle axle, no matter the velocity. Plus, it has been shown that there is a minimum value of the dynamic forces for a certain value of the damping ratio of primary suspension at velocities higher than 200 km/h. On this account, this damping value can be determined as the best, in terms of lowering the vertical dynamic wheel/rail forces.

The influence of the damping ratio of primary suspension upon the magnitude of the vertical dynamic forces has been studied along with various parameters of the vehicle, where for each case the value of damping ratio leading to minimum values of the dynamic forces has been set apart.

It has been clearly shown that the lowering of the dynamic forces can be obtained from increasing the damping ratio of secondary suspension, by diminishing the axle mass or increasing the vehicle wheelbase – these results are independent from the damping ratio of primary suspension. Likewise, the vertical dynamic forces can be decreased by reducing the bogie wheelbase, but it is only for a certain range of values in the damping ratio of primary suspension.

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