UNSTEADY STATE NUMERICAL SIMULATION OF RECIPROCATING COMPRESSOR IN THE PRESENCE OF THE SUCTION AND DISCHARGE PROCESS

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This work presents a simplified and relevant mathematical model of universal use intended for the estimation of the performances of the reciprocating compressors pistons in unsteady state where the real thermodynamic cycle of compression of gas in the cylinder is considered. While, the performances under the effect of the operating conditions, the thermodynamic conditions of the fluid at the entry of the compressor are strongly affected by the design of the orifices of the suction and discharge valves, and the time of opening and closing of these valves, as well as, their unsteady movement influences the flow rate of the flow of gas at the inlet and the outlet of the compressor. However, this movement requires a suitable severe mathematical model. This model is based on the conservation equations of mass and the first principle of thermodynamics was developed, moreover, a kinematic analysis of crank-connecting rod system is held to account in this study whose objective is to simulate the temporal variations of mass flow rate at the exit of the compressor, the state of the refrigerant in the cylinder and the movements of the valves. The state equation of a real gas is used, also, the heat transfer between the solid parts and its contact with the flow of the fluid is considered. The profiles of the results obtained are similar to the results found in the literature from profiles point of view.

Keywords: reciprocating compressor, refrigeration, fluid, valve, simulation, modeling, flow.

1. Introduction

The compressor is one of the five essential parts of a vapor compression refrigeration system and its role is to pass the refrigerant to superheated vapor from a low pressure (LP) level to a high pressure level (HP). It is also the most

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complex and critical component in the refrigerated installations. In a compressor, it has always existed two types of valves, one for the suction of the gas leaving the evaporator and the other for the delivery of the gas through the condenser. These plates acting as the joints between the various zones of pressure in the compressor and they are opened when the differential pressures between the zones are reached and make it possible for gases to circulate in the regions of high pressure to low pressure during a cycle of compression.

These plates have several geometrical shapes and have some of the most complicated operating characteristics in a refrigeration system. They are usually composed of seat, buffer and a thin plate or annular disk. The variation of the pressure force applied on the valve plate and on the seat can lead to oscillations and displacement of this valve. The different geometries of plates, ports and seats and also the differential pressure between upstream and downstream of the plate respectively affect the effective surface of the flow and the effective surface of the force acting on the plate. In addition, the delay of time of the opening and the closing of the valves caused by the downward or upward flow directly affects the flow rate and the compressor efficiency. For this purpose, the improvement of the performances of the hermetic compressors requires a better understanding of the physical phenomena during the operation of compressor such as:

- Various heat transfers which can be existed in the compressor;
- The resistance of the flow in the canals of gas and the friction losses between the piston rings and the cylinder wall;
- Distribution of the temperature in the compressor elements;
- Various forces acting on the plates of the valves;
- The kinematic movement of crank-connecting rod system;
- The re-expansion of volume of gas in the cylinder;

As a result, the compressor is the seat of simultaneous heat exchange with the refrigerant fluid and the ambient. These transfers result, among other things, from the dissipation of energy due to the friction between the different moving pieces.

In these last years, many progress was made and we found in the literature an infinity of theoretical and experimental work such as: L. Boswirth [8], Si-Yoing Sun [25], G.A. Longo [6], E.H. Ng [3], M.N. Srinivas [18], S. Lee [26] and Craig R. Bradshaw [1],… etc, the principal results obtained show the different pressures variations in the suction, discharge and cylinder plenums chambers, also the displacements of the valves plates.

For this purpose, the objective of this current work is to complete the knowledge recorded on the mathematical modeling of hermetic piston compressors in dynamic regime, also, to develop a mathematical model that includes the thermodynamic formulation and the different heat exchanges producing inside the compressor, adding to that a mathematical analysis of kinematic of crank-connecting rod system and the displacements of the plates of the valves, whose purpose to simulate the performance of the compressor.

2. Kinematic modeling of the piston

The piston displacement is given in terms of lengths of the connecting rod, of the crank, and the rotation angle (see Fig. 1). The use of Pythagoras method, the expressions of displacement, Z_{cyl} and speed, V_p are [21,28]:

$$Z_{cyl}(t) = R_{crank}(1 - cos(\theta)) + L_{rod}\left\{1 - \sqrt{1 - \left(\frac{R_{crank}}{L_{rod}} sin(\theta)\right)^2}\right\} \quad (1)$$

$$V_p(t) = \frac{dZ_{cyl}(t)}{dt} = R_{crank}\omega\left[sin(\theta) + \frac{R_{crank}}{2L_{rod}} cos(\theta)\right] \quad (2)$$

The temporary derivative of the rotation angle is $(d\theta = \omega dt)$ and the angular velocity is $(\omega = 2\pi N/60)$. The expressions of temporal volume in the cylinder, V_{cyl} and its temporary variation are given as follows [9,20]:

$$V_{cyl}(t) = V_{nox} + \pi R_{cyl}^2 Z_{cyl}(t) \quad (3)$$

$$dV_{cyl}(t)/dt = \pi R_{cyl}^2 \frac{dz_{cyl}(t)}{dt} = \pi R_{cyl}^2 V_p(t) \quad (4)$$

The dead volume, V_{nox} is defined as follows: [7]:

$$V_{nox} = \boldsymbol{V_{nox}} 2A_{cyl}\eta_v R_{crank} \quad (5)$$

The mass of fluid enclosed in this volume can be calculated for a pre-project according to Boswirth. [16] as follows [7] (see Fig. 2 and 3):

$$m_0 = \frac{P_d V_{nox}}{R_{spec}T_2} \quad (6)$$

The temporal variation of the pressure between the entry and the exit of a valve is given by [14]:

$$\Delta P_{12}(t) = P_1 - P_2(t) = P_1 - \frac{m(t)R_{spec}T_2}{V_2(t)} \quad (1\text{: entry}, 2\text{: exit}) \quad (7)$$

3. Mathematical model of the gas in the cylinder

The main purpose of the thermodynamic model of the gas in dynamics regime is to provide a system of equations describing the different temporal changes in the state of the gas in the cylinder. Therefore, it is convenient to use the formulation of the first principle of thermodynamics which can be written for our case as follows (see Fig. 1.) [1,28]:

$$\frac{dQ}{d\theta} + h_s\frac{dm_s}{d\theta} = \frac{dW}{d\theta} + h_d\frac{dm_d}{d\theta} + \frac{d(m_c u_c)}{d\theta} \quad (8)$$

The definitions of the work, the enthalpy and the mass of the gas in the cylinder are given as follows [13,22]:

$$\frac{dW}{d\theta} = P_c \frac{dV_c}{d\theta} \tag{9}$$

$$h = u + PV \tag{10}$$

$$m_c = \frac{V_c}{v_c} \tag{11}$$

The angular variation of the total mass of the gas in the cylinder is [19, 25]:

$$\frac{dm_c}{d\theta} = \frac{dm_s}{d\theta} - \frac{dm_d}{d\theta} \tag{12}$$

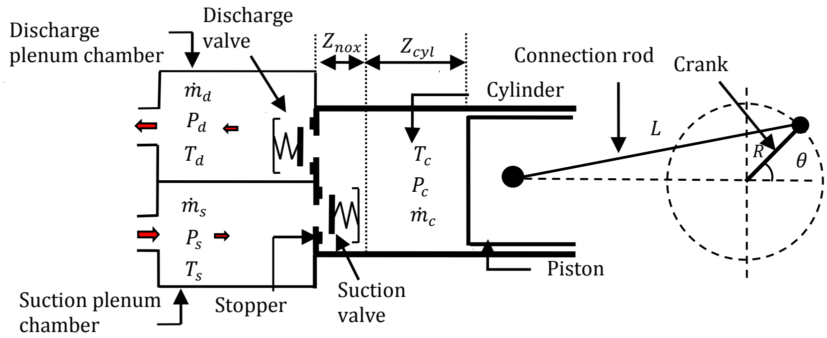

Fig. 1. Compressor model showing the various system elements.

In order to make the overall mathematical a simple model for manipulating, we adopt the following simplifying assumptions:

- The mass quantities aspirated or discharged through the clearances between the piston and the cylinder walls are negligible;
- The mass flow rate of the gas which enters or leaves the cylinder is controlled by the opening of the valves and the pressures difference;
- The pulsations of gas in the plenums chambers of the aspiration and discharge are not considered in this study;
- The pressures in the plenums chambers of aspiration and discharge are assumed constant;

Taking into account equation (10), the derivation of the quantity $(m_c u_c)$ with respect to the angle of rotation is:

$$\frac{d(m_c u_c)}{d\theta} = m_c \frac{dh_c}{d\theta} + h_c \frac{dm_c}{d\theta} - P_c \frac{dV_c}{d\theta} - V_c \frac{dP_c}{d\theta} \tag{13}$$

The introduction of this equation into equation (8) and after some rearrangements we can express the angular variation of the enthalpy of the gas as follows [25,27]:

$$\frac{dh_c}{d\theta} = \frac{1}{m_c}\left[\frac{dQ}{d\theta} + (h_s - h_c)\frac{dm_s}{d\theta} - (h_d - h_c)\frac{dm_d}{d\theta} + V_c \frac{dP_c}{d\theta}\right] \tag{14}$$

The total derivations of enthalpy and pressure are given as follows [25]:

$$\frac{dh}{d\theta} = \frac{\partial h}{\partial T}\bigg|_{\rho}\frac{dT}{d\theta} + \frac{\partial h}{\partial \rho}\bigg|_{T}\frac{d\rho}{d\theta} \tag{15}$$

$$\frac{dP}{d\theta} = \frac{\partial P}{\partial T}\bigg|_{\rho}\frac{dT}{d\theta} + \frac{\partial P}{\partial \rho}\bigg|_{T}\frac{d\rho}{d\theta} \tag{16}$$

The introduction of these last two equations in equation (14), and following the several manipulations, leads to write the expression of the angular variation of the fluid temperature in the cylinder as follows [4,25]:

$$\frac{dT_c}{d\theta} = \frac{\frac{1}{m_c}\left\{\frac{dQ}{d\theta} + (h_s - h_c)\frac{dm_s}{d\theta} - (h_d - h_c)\frac{dm_d}{d\theta}\right\} + \left\{v_c\left(\frac{\partial P_c}{\partial \rho_c}\right)_T - \left(\frac{\partial h_c}{\partial \rho_c}\right)_T\right\}\frac{d\rho_c}{d\theta}}{\left[\left(\frac{\partial h_c}{\partial T_c}\right)_\rho - v_c\left(\frac{\partial P_c}{\partial T_c}\right)_\rho\right]} \tag{17}$$

The angular variation of the mass in the cylinder can be formulated as follows [25, 27]:

$$\frac{dm_c}{d\theta} = \frac{d(\rho_c V_c)}{d\theta} = \rho_c\frac{dV_c}{d\theta} + V_c\frac{d\rho_c}{d\theta} \tag{18}$$

The total angular variation of the density of the gas is [18, 27]:

$$\frac{d\rho_c}{d\theta} = \frac{1}{V_c}\left[\frac{dm_c}{d\theta} - \rho_c\frac{dV_c}{d\theta}\right] \tag{19}$$

4. Heat transfer between the gas in the cylinder and the wall

Available correlations on heat transfer in a piston cylinder are generally developed for internal combustion engines. In fact, the thermal exchange in the cylinder is periodic and will take place between the walls and the superheated vapor of the refrigerant resulting from the considerable variation of the temperature of the vapor with respecting to the temperature of the wall and it is given during early part of the compression by the equation below [10,22]:

$$\frac{dQ}{dt} = \alpha(t)A_x(t)\big(T_w(\mathrm{t}) - T_c(t)\big) \tag{20}$$

Where: $A_x(t) = A_{pis} + A_{cl} + A_w(t)$ (21a)

$$A_w(t) = \pi D_c Z_{cyl}(t) \tag{21b}$$

During the latter part of the compression, the heat transferred is negative [20]. The heat transfer coefficient is calculated using correlation of Woschni. [5,12] and it is expressed as follows:

$$\alpha(W/m^2K) = 3.26D_c(m)^{-0.2}P_c(kPa)^{0.8}T_c(K)^{-0.53}w(m/s)^{0.8} \tag{22}$$

The correlations of the temperatures of the cylinder wall, T_w, the cylinder head, T_{wc} and of the gas in the suction plenum, T_{sr} are cited in the work of Y. Wu et al. [29].

5. Dynamic model of fluid flow through the valve

During the movement of the piston, the volume of the fluid in the cylinder V_c varies and produces a pressure difference ΔP_{12} between the suction plenum chamber V_1 and the volume in the cylinder V_2 (see Figures 2 and 3). This variation in pressure produces a force exerted on the plate, F_{pl}, and a variation of the jet speed of the fluid, W_2. On the other hand, the plate of the valve has a mass, m_v, a displacement X, and speed $\dot{X}$. Thus, we assume that the fluid is incompressible and non-viscous and taking into account the work done by Bhoswirth. [15,17], the expression of the pressure difference between the suction plenum chamber $P_1(= P_s)$ and the pressure in the cylinder $(P_2 = P_c)$, all along a streamline, S is defined as follows [15,16]:

$$\frac{\Delta \mathrm{P}_{12}(t)}{\rho} = \frac{W_2^2(t)}{2} + J\left(\dot{W}_2(t)X(t) + W_2(t)\dot{X}(t)\right) + \frac{A_p C_p W_2(t)\dot{X}(t)}{2LC_D X(t)} \quad (23)$$

According to Bhoswirth. [15] and for the case where the valve has a constant orifice, A_p, the parameter J becomes (see Fig.4):

$$J = LC_D\left(h_{seat} + \kappa D_{port}\right)/A_p \quad (24)$$

A typical value for the correction coefficient is (κ = 1.4) (Bhoswirth. [15]).

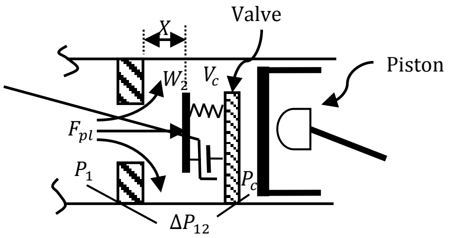

Fig. 2. Schema presents the dynamic of the valve and grandeurs.

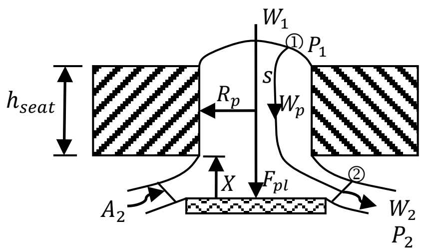

Fig .3. Flow through a valve and its housing.

The force exerted on the valve plate, F_{pl}, is expressed as follows [15,16]:

$$F_{pl}(t) = A_p C_p \Delta P_{12}(t) = \frac{1}{2}\rho A_p C_p W_2^2(t) \quad (25)$$

The expression of mass flow rate as a function of the jet section is:

$$\dot{m}(t) = \frac{dm}{dt} = \rho A_2(t)W_2(t) = \rho LC_D X(t)W_2(t) \quad (26)$$

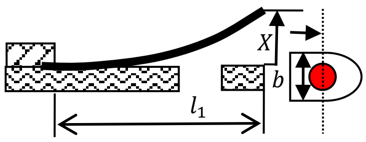

Fig.4. Diagram of illustration of a valve in movement with the geometrical parameters.

6. Valve plate displacement equation

The dynamic of valve plate can be characterized as an oscillatory system of a mass coupled to a spring and damper. The equation of motion of this valve is defined as follows [11,18]:

$$m_v\ddot{X}(t) + B(t)\dot{X}(t) + K_v X(t) = F_{pl}(t) \tag{27}$$

According to Bhoswirth. [15], the damping coefficient for a flexible plaque is given by the following expression [2,8]:

$$B(t) = \begin{cases} \dot{X}(t) < 0 \quad c_v + \kappa_d \left(\dfrac{b}{X(t)}\right)^3 \mu_{gaz} l_1 + \dfrac{C_{stic}}{X(t)^3} \\ \dot{X}(t) > 0 \qquad\qquad c_v + \dfrac{C_{stic}}{X(t)^3} \end{cases} \tag{28}$$

The valve speed, $\dot{X}$, is set normal to the positive valve seat is shown in Fig. 5. The damping coefficient is presented in the works of H. Bukac et al. [8] and Roland Aigner et al. [23]. Therefore, the motion equation of the plate is a second order differential equation and coupled.

7. Global mathematical model of the compressor

The global mathematical model that describes the different physical phenomena existing during each process (expansion, suction, compression, discharge) consists of differential equations of first orders coupled except the equation of movement of the valve which is of second order. The different grandeurs to predict in these equations are presented by the vector of unknowns$\left(T_c, P_c, \rho_c, m_c, W_{2,i}, X_i, \dot{X}_i\right)$ where$(i = s, d)$. Indeed, to make the equation of movement of the valve in first order we make the following transformations: $x_1(t) = X(t)$, $x_2(t) = dx_1(t)/dt = \dot{X}(t)$. From where, we deduce that the acceleration of the plaque is: $\ddot{X}\ (t) = \frac{d\dot{X}(t)}{dt} = \frac{dx_2(t)}{dt} = \dot{x}_2(t)$.

The related relation between time t, angular velocity ω, and rotation angle θ, with respect to a physical quantity ϕ is given as follows by: $\frac{d\phi}{dt} = \frac{d\phi}{d\theta}\frac{d\theta}{dt} = \omega\frac{d\phi}{d\theta}$. Taking into account the equations (17), (18), (19), (23) and (27), we can express the new system of equations adopted under the conditions of the formulation of the Runge Kutta method as follows [7,11]

$$\{f_1, f_2, f_3, f_4, f_5, f_6\} = \left\{\frac{dx_{1,i}(t)}{d\theta}, \frac{dx_{2,i}(t)}{d\theta}, \frac{dW_{2,i}(t)}{d\theta}, \frac{dm_c(t)}{d\theta}, \frac{dT_c}{d\theta}, \frac{d\rho_c}{d\theta}\right\} \tag{29}$$

Where:

$$f_1 = x_{2,i}(t) \tag{30.a}$$

$$f_2 = \begin{cases} x_{2,i}(t) < 0 & \frac{1}{2}\frac{A_{P,i}C_P\rho W_{2,i}^2(t)}{m_{v,i}} - \frac{c_v x_{2,i}(t)}{m_{v,i}} - \frac{\kappa_d \mu_{gaz} lb^3}{m_{v,i}}\frac{x_{2,i}(t)}{x_{1,i}^3(t)} \\ & -\frac{C_{stic}}{m_{v,i}}\frac{x_{2,i}(t)}{x_{1,i}^3(t)} - \frac{\kappa_v}{m_{v,i}}x_{1,i}(t) \\ x_{2,i}(t) > 0 & \frac{1}{2}\frac{A_{P,i}C_P\rho_c W_{2,i}^2(t)}{m_{v,i}} - \frac{c_v x_{2,i}(t)}{m_{v,i}} - \frac{C_{stic}}{m_{v,i}}\frac{x_{2,i}(t)}{x_{1,i}^3(t)} - \frac{\kappa_v}{m_{v,i}}x_{1,i}(t) \end{cases} \tag{30.b}$$

$$f_3 = \frac{P_1}{J\rho_c}\frac{1}{x_{1,i}(t)} - \frac{R_{sp}}{J\rho_c V_c(t)}\frac{m_c(t)T_c}{x_{1,i}(t)} - \frac{1}{2J}\frac{W_{2,i}^2(t)}{x_{1,i}(t)} - \frac{W_{2,i}(t)x_{2,i}(t)}{x_{1,i}(t)} - \frac{1}{2}\frac{A_{P,i}C_P}{JLC_D}\frac{W_{2,i}(t)x_{2,i}(t)}{x_{1,i}^2(t)} \tag{30.c}$$

$$f_4 = \rho_c L C_D x_{1,i}(t) W_{2,i}(t) \tag{30.d}$$

$$f_5 = \frac{\left\{\frac{1}{m_c}\left[\frac{dQ}{d\theta} + (h_s - h_c)\frac{dm_s}{d\theta} - (h_d - h_c)\frac{dm_d}{d\theta}\right] + \left[v_c\left(\frac{\partial P_c}{\partial \rho}\right)_T - \left(\frac{\partial h_c}{\partial \rho_c}\right)_T\right]\frac{d\rho_c}{d\theta}\right\}}{\left[\left(\frac{\partial h_c}{\partial T_c}\right)_\rho - v_c\left(\frac{\partial P_c}{\partial T_c}\right)_\rho\right]} \tag{30.e}$$

$$f_6 = \frac{1}{V_c}\left[\frac{dm_c}{d\theta} - \rho_c\frac{dV_c}{d\theta}\right] \tag{30.f}$$

While, the temporal variation of the volume of gases in the cylinder $(dV_c/d\theta)$ is given by equation (4).

8. The phases of operations and the conditions of transitions

During a cycle of compression of the vapors, it exists different principal phases of operation which can be distinguished by the position or displacement from the valves plate and they are defined by equations in the table. 1. As the phases of the suction and discharge are subdivided in two phases: a phase where the valve plate is really in movement, $(X_{s,d} \neq 0)$ and the other when the movement of the valve plate is limited in its stroke by the seat of the valve $(X_{s,d} = 0.)$ and by the stopper $(X_{s,d} = X_{s,d,max})$.

Taking into account the results of Habing. [24], when a mobile body impact on a fixed wall it will rebound and reverse its direction of movement with a speed which is generally lower at the speed before the impact. The coefficients of restitution are defined as:

- For the stopper : $e_{res} = 0.2 \mp 0.1$;
- For the seat : $e_{res} = 0.3 \mp 0.1$.

And the speed of the valve plate can be expressed as follows [24]:

$$\frac{dX(t^+)}{dt} = -e_{res}\frac{dX(t^-)}{dt} \tag{30}$$

Consequently, the equation describing the dynamics of valve plate is only necessary in the global model when the valves plates are really in movement.

Table. 1

Conditions for transitions [4].

<table>
<tr><td colspan="3">Phase of operations</td><td>Suction valve</td><td>Conditions</td><td>Discharge valve</td></tr>
<tr><td rowspan="3">I</td><td rowspan="3">Aspiration</td><td rowspan="2">a</td><td>Opened, moving
$\frac{dm_c}{d\theta} = \frac{dm_s}{d\theta}, X_s$
$\frac{dX_s}{d\theta}, \frac{dX_s^2}{d\theta}$</td><td>$0 \leq X_s(t) \leq X_{s,max}$
$P_s \geq P_c(t)$</td><td rowspan="2">closed</td></tr>
<tr><td colspan="2">$(P_s - P_c)(CA)_{eff,s} < \left(X_{s,max} + X_{s,0}\right)K_{s,raid}$</td></tr>
<tr><td>b</td><td>Maximum opening
$\frac{dm_c}{d\theta} = \frac{dm_s}{d\theta}$
$X_s = X_{s,MAX}$
$\frac{dX_s}{d\theta} = \frac{dX_s^2}{d\theta} = 0$</td><td>$X_s(t) = X_{s,max}$
$P_s > P_c(t)$</td><td>closed</td></tr>
<tr><td rowspan="3">II</td><td rowspan="3">Discharge</td><td>a</td><td>closed</td><td>$0 \leq X_d(t) \leq X_{d,max}$
$P_c(t) \geq P_d$</td><td>Opened, moving
$\frac{dm_c}{d\theta} = -\frac{dm_d}{d\theta} X_d, \frac{dX_d}{d\theta}$
$\frac{dX_d^2}{d\theta}$</td></tr>
<tr><td rowspan="2">b</td><td>closed</td><td>$X_d(t) = X_{d,max}$
$P_c(t) > P_d$</td><td rowspan="2">Maximum opening
$\frac{dm_c}{d\theta} = -\frac{dm_d}{d\theta}$
$X_d = X_{d,MAX}$
$\frac{dX_d}{d\theta} = \frac{dX_d^2}{d\theta} = 0$</td></tr>
<tr><td colspan="2">$(P_s - P_c)(CA)_{eff,d} < \left(X_{d,max} + X_{d0}\right)K_{d,raid}$</td></tr>
<tr><td>III</td><td colspan="2">Process of compression or expansion</td><td>closed</td><td>$\frac{dm_c}{d\theta} = 0$
$X_d(t) = X_s(t) = 0$</td><td>closed</td></tr>
</table>

9. Results and discussions

In this study, we have adopted the dimensions of the compressor, the conditions of fluid at inlet and outlet, and the different parameters and coefficients for the numerical simulation dynamic regime conditions, the values mentioned in the table. 2.

The angle of rotation is assumed to be zero at TDC and the both plenums chambers pressures of the suction and discharge respectively are assumed to be constant, whereas, the initial point at the beginning of the expansion of the gas is assumed equal to the end of the gas discharge point. In the calculation program, the different partials derivatives such as : $\left(\frac{\partial P}{\partial \rho}\right)_T$, $\left(\frac{\partial h}{\partial \rho}\right)_T$, $\left(\frac{\partial h}{\partial T}\right)_\rho$ and $\left(\frac{\partial P}{\partial T}\right)_\rho$ and the speed of sound are calculated using the subroutines of Refprop V6 [30]. In addition, we have chosen the method of Runge Kutta of order 4 for the resolution of system of equations (equations. 30). The numerical simulation results are presented by profiles showing the different variations of the thermodynamic and physical grandeurs of the gas in the cylinder as a function of the crank angle rotation.

Table. 2

Compressor dimensioning and operating conditions.

$R_{cyl} = 10.15\ mm$	Radius of cylinder	*N=1500*	Number of rotation per minute
$R_{crank} = 11.51\ mm$	Length of crank shaft	$R_{port} = 4.2\ mm$	Radius of valve port
$L_{rod} = 49.50\ mm$	Length of connected rod	$R_{seal} = 4.22\ mm$	Radius of seal ring
$n_{nox} = 3.8\%$	Clearance volume ratio	$h_{seat} = 3.6\ mm$	Height of seat
$P_{ev} = 0.349.10^6\ Pa$	Suction pressure	$C_d = 1.$	Discharge coefficient
$T_{ev} = 15\ C°$	Suction temperature	$C_P = 1.$	Pressure coefficient
$\rho_{ev} = 16.283\ Kg/m^3$	Suction density	$m_v = 0.1\ g$	Mass of valve
$P_d = 2.364\ MPa$	Discharge Pressure	$k_v = 600.5\ N/m$	Stiffness of valve
$T_d = 96.93\ C°$	Discharge temperature	$k_d = 15.$	Non dimensional empirical value
$\rho_d = 106.05\ Kg/m^3$	Discharge Density	$l_1 = 8.1\ mm$	Effective valve plate length
$m_0 = 2.21910^{-5}\ Kg$	The initial mass in the dead volume	$b_1 = 10\ mm$	Width of valve plate
$R134a$	refrigerant	$X_{s,max} = 1.5\ mm$ $X_{d,max} = 3\ mm$	Maximum displacements of the plates of the valves

The profile of angular variations of the pressure of the refrigerant in the cylinder is presented by the figures (5). We notice that the pressure decreases progressively with the increase in the angle of rotation for the case of expansion process until the opening of the suction valve ($\theta = 49.65°, P_c = 0.3207\ MPa$). Beyond this angle, the process of aspiration begins and the profile of the pressure has an oscillation of maximum amplitude of order ($\Delta P_c = 0.043\ MPa$) and deadened before the closing of this valve ($\theta = 190°, P_c = 0.3505\ MPa$).

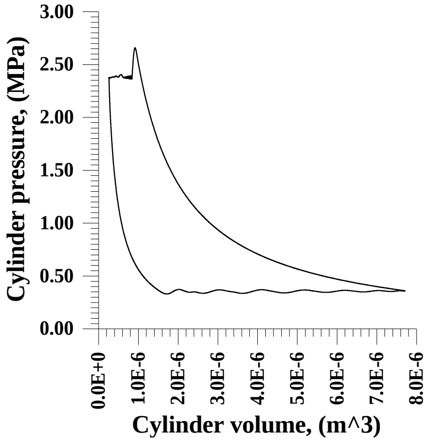

Fig. 5. Compression cycle in the PV diagram.

This oscillation is due to the vibration of the plate of the valve as shown in Figure (10). In addition, the delay of the opening and the closing of the valve plate are from where with the rigidity of the plate and the force of adherence (oil viscosity). After having that the suction valve is closed, the process of compression begins and the profile of the pressure increases parabolically with the angle of rotation until the opening of the valve of discharge ($\theta = 330.8°, P_c = 2.65\,MPa$). Therefore, the process of expansion will take place after this last angle and we observe that the profile of the pressure decreases suddenly by $P_c = 2.65\,MPa$ to $P_c = 2.35\,MPa$ and after this reduction the profile presents oscillations of maximum amplitude of order ($\Delta P_c = 0.0452\,MPa$) and the same deadened before the closing of valve ($\theta = 362°, P_c = 2.35\,MPa$).

The figure (6) presents the different angular variations of the temperatures, of gas in the cylinder T_c, of the cylinder wall T_w , of cylinder head T_{wc} and in the suction chamber plenum chamber T_{sr}. The three last temperatures are calculated using the data correlations mentioned in the Ref. [29]. We note that both cylinder head and suction plenum chamber temperatures remain constant with increasing angle, so the head temperature is higher than that of suction plenum. On the other hand, the temporal cycle of the temperature of the cylinder wall T_w follows only the similar configuration of the refrigerant temperature curve in the cylinder T_c, and consequently, it varied periodically around a mean value. While the profile of the gas temperature in the cylinder is similar to the pressure profile because the pressure of superheated vapor increases with temperature and density.

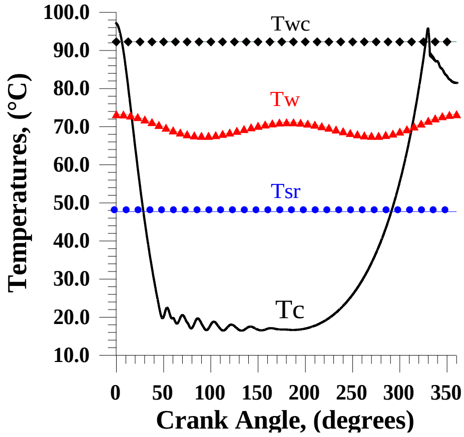

Fig. 6. Temperatures behavior during the compression cycle.

The angular variation of the amount of heat exchanged between the gas in the cylinder and the wall of the cylinder is shown in Figure (7). It is noted that this quantity gradually decreases during the compression and aspiration processes; in this case the cylinder receives the heat given off by the fluid. As can be seen, heat transfer rate during the discharge process is much higher than that in the suction process. This occurs because the piston is very close to the cylinder head and, as a consequence, high levels of velocity are present in the flow along the small clearance left between the piston and the cylinder head as the gas is directed towards the discharge valve. After the discharge valve opens, there is a peak of heat flux associated with gas compression and increase of flow velocity along the cylinder clearance. In the discharge process the exhaust of hot gas causes the curve to decrease. If the heat exhaust is ignored in the theoretical computation, the curve becomes horizontal [25]

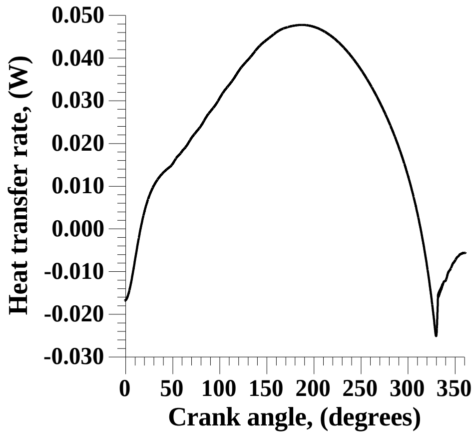

Fig. 7. Heat transfer flow rate between refrigerant and cylinder wall during the compression cycle.

The angular variation of the mass of gas contained in the cylinder is presented by the figure (8). We see that this mass is constant during the process of expansion and equal to the mass contained in the dead volume and also for the case of a process of discharge and this mass admits a maximum value. During the process of aspiration it increases with the angle of rotation in a way parabolic and sinusoidal is that is due to the vibration of the plate of the valve. On the other hand, for a process of discharge it falls gradually with the increase in the rotation angle.

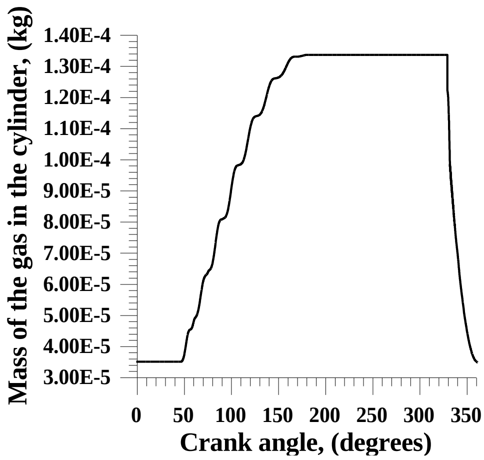

Fig. 8. Mass contained in the cylinder during compression cycle.

The profiles of the angular variations of the speeds of the jet and the sound respectively of the gas through the two ports of the suction and discharge valves are presented by the figure (9). We see that the profile of the velocity of the jet is sinusoidal during the aspiration but for the case of discharge this velocity decreases suddenly and we find that the gas flow is always subcritical.

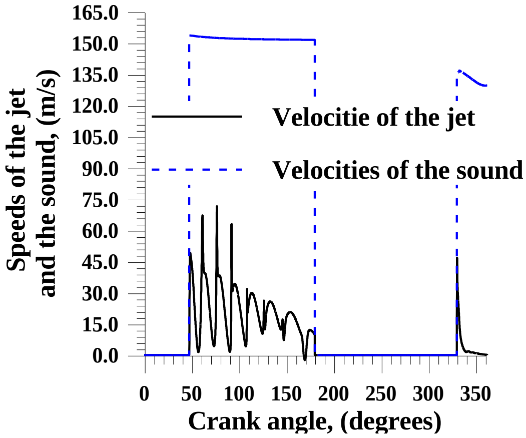

Fig. 9. Angular variations of the speeds of the jet and the sound respectively of the gas through the suction and discharge valves during compression cycle.

In the same way, the profile of the angular variation of the displacements of the valves's plates are presented in the figure (10).We observe that this profile is sinusoidal during the aspiration and the amplitude is amortized with the increase of the angle and that the plate under the conditions indicated in table (1) does not reach the stopper $\left(X_{s,max}\right)$ and on the other hand, for the discharge process, the plate always coincides with the stopper$\left(X = X_{d,max}\right)$.

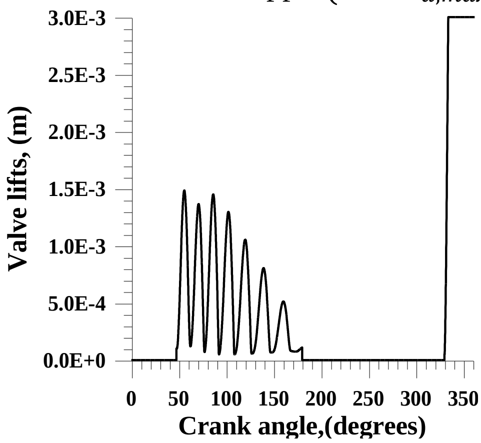

Fig. 10. Valve lifts-crank angle diagram.

While, the profile presents the angular variation of these plates is given by the figure (11). In the same way, this profile is sinusoidal during the suction and the amplitude is amortized with the increase of the angle, and on the other hand decreases gradually during the discharge.

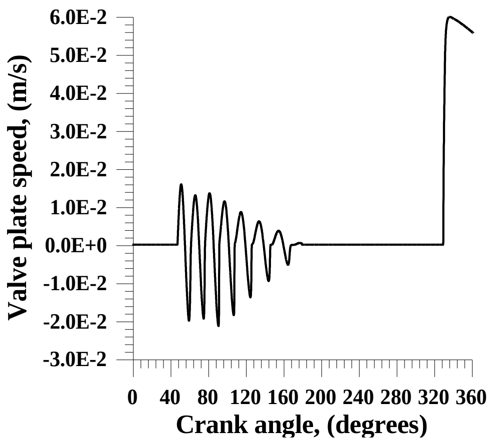

Fig. 11. Valve plate speed-crank angle diagram.

10. Model validation

To validate the numerical method discussed above, a single cylinder reciprocating compressor is modeled and the numerical results have been compared with available data values of authors Kasper H. F. et al. [32]. These data is for a cylinder volume of 20 cm^3, the working refrigerant is **propane**. The geometric dimensions, the physical parameters and the operating conditions are cited in the Ref. [32 (tab. 7 and 8)]. The comparison between the results obtained from this model and presented by Kasper H. F. et al. [32] concerning only to the processes of expansion and suction.

The variation of cylinder pressure against crank angle is shown in Fig.12. We notice that the both pressures profiles of this model and the 2-D planar CFD/FSI model of Kasper H. F. et al. [32] (is set up in ANSYS Fluent for 142 cells mesh sizes) have a difference in the two processes, this difference is mainly due to state of the fluid. In the Kasper H. F. model, the gas is assumed as an ideal gas, the density is calculated using the ideal gas law. Thus, the heat transfer between the refrigerant in the cylinder and the cylinder wall is negligible, when the valve impacts the stopper or the seat it does not rebound and the motion of the mass-plat valve system is undamped. In addition, the compression and the expansion of the gas are assumed to be isentropic (for more details see Refs. [25,26]).

The variation of the valve of suction against crank angle is shown in Fig.13. Also, we notice the existence of a difference between the both displacement profiles of the suction valve, and this difference is due to the reasons mentioned before, because, the profile data presented by the author [32] are calculated by using the code KV-DYN [32,33] and which has the same assumptions on the fluid. While, the opening and closing angles of valve are almost similar.

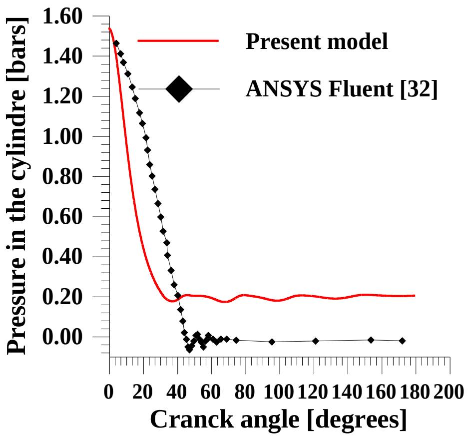

Fig. 12. Pressure in the cylinder-crank angle diagram

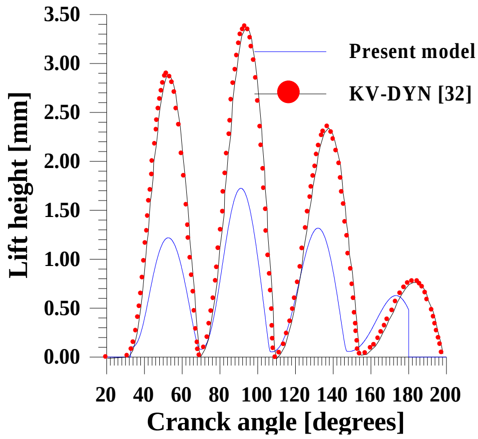

Fig. 13. Valve plate displacement-crank angle diagram

Conclusion

In this article we have briefly presented several mathematical models reflecting the different physical phenomena existing in a hermetic compressor piston. A mathematical model in dynamic régime based on the first principle of thermodynamics has been applied in order to evaluate the thermodynamic properties of the fluid in the cylinder where the fluid is considered real. Thus, two other models of the kinematics of system connecting rod-crank and the valve plate movement. The mass flow model adopted in this study is based on the universal expression of the flows through the convergent and divergent nozzles where the critical condition of the flow is considered. In this study the heat transfer between the fluid in the cylinder and the wall is considered. All these models make it possible to construct a global dynamic model of the flow of a refrigerant fluid in a hermetic piston compressor.

The results of the numerical simulation are presented by profiles. The analysis presented proves to be correct and reliable, and the method can reflect the practical conditions of operation. Thus, the profiles of the results obtained are similar to the results found in the literature profile point of view. The results from the present model, the 2-D CFD/FSI model, and KV-DYN are compared. Differences between the models and the reliability of each individual model are discussed. The present work can be useful on the one hand for the numerical simulation of refrigeration installations in dynamic regime, because the pressures and the temperatures respectively of aspiration and discharge of the fluid and the mass flow rate which circulates in the refrigerant circuit are very important parameters to be introduced with the aim of calculating the powers of the various components of the installation (evaporator, condenser, ...) after knowing the mass flow. On the other hand, it allows compressor constrictors to choose in a precise manner the technical characteristics of the plate of the valve.

Nomenclature

A_{cyl}	Section of the cylinder [m²];
A_x	Total heat exchange area between the cylinder walls and the refrigerant contained in this cylinder [m²];
A_{pis}	The lateral surface of the piston $\left(=\frac{\pi}{4}D_c^2\right)$ [m²];
A_{cl}	The exchange surface in the clearance volume$\left(=4\frac{V_{nox}}{D_c}\right)$ [m²];
A_w	The exchange surface in the cylinder [m²];
A_p	The valve port section [m²];
B	The damping coefficient [/];
b	The width of the valve [m];
C_D	The discharge coefficient for an orifice [/];
C_P	The pressure coefficient [/];
c_v	The damping coefficient of the valve plate [/];
Cc_{stic}	The damping coefficient [/];
D_c	The diameter of the cylinder [m];
D_{port}	The port diameter of the valve [m];
e_{res}	Coefficient of restitution [/];
F_{pl}	Pressure force applied to the valve plate [N];
F_{stic}	Damping force [N];
h	Specific enthalpy [J/kg.K];
h_{seat}	The seat height of the valve [m];
J	Parameter depending on the geometry only [/];
K	Thermal conductivity [W/m².K];
K_v	Stiffness constant of the valve [N/m];
κ	Coefficient of correction takes into; account the acceleration of the mass at the exit port of the valve $(= 1.4)$ [/];
L_{rod}	The crank length [m];
L	The circumference of the valve [m];
l_1	The length of the plate [m];
m	The mass [kg];
$\dot{m}$	Mass flow rate [kg/s];
M	Molar mass [kg/kmol];
m_0	The initial mass in the cylinder [kg];
m_v	The equivalent mass of the valve plate [kg];
N	The number of crankshaft system turns [revolutions/sec];
Nu	Number of Nulselt [/];
Q	The quantity of heat exchanged [W/m²K];
P	Pressure [Pa];
P_M	The work exchange between the gas and valve [J];
R_{crank}	The length of the connecting rod [m];
R_{cyl}	The radius of the cylinder [m²];
R_{spec}	The specific gas constant [N.m/kmol.K];
Re	Number of Reynolds [/];
T	Temperature [K];
T_c	Temperature of the refrigerant in the cylinder [K];
T_w	Temperatures of the cylinder wall [K];
T_{wc}	Cylinder bottom temperature [K];
T_{sr}	Temperature of the gas in the suction chamber [K];
T_h	Lubricating temperature [K];
u	Internal energy [J/kg.K];
v	Specific volume [kg/m³];
V_p	The volume in the cylinder [m³];
V_{nox}	The clearance volume [m³];
W	The work [W];
W_2	The jet velocity of the gas [m/s];
$\dot{W}_2$	The acceleration of gases [m/s²];
X	The displacement of the plate of the valve [m];
$\dot{X}$	The speed of the valve plate [m/s];
$\ddot{X}$	Acceleration of the valve plate [m/s²];
Z_{cyl}	Piston displacement [m];
t	The time [s];

Numbers

ε	The compression ratio [/];
ρ	The density [kg/m^3];
μ	Dynamic viscosity [Pa-s];
θ	The rotation angle of crank connecting rod system [radian];
ω	The angular speed of crank connecting rod system [rad/s];
η_v	Volumetric efficiency [/];
α	The heat transfer coefficient [W/m^2.K];
κ	Correction Factor for a Circular Chanel [/];

Index

s	Admission;
d	Discharge;
c	Cylinder;
ent	Entrance;
$sort$	Output;
ev	Evaporation;
$seat$	Seat;
$stic$	Stiction

R E F E R E N C E S

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