# AN INTUITIVE METHOD TO AUTOMATICALLY DETECT THE COMMON AND NOT COMMON FREQUENCIES FOR TWO OR MORE TIME-VARYING SIGNALS

Cezar DOCA[1], Constantin PĂUNOIU[2]

*Sampling a time-varying signal and his spectral analysis are, both, subjected to theoretically compelling, such as Shannon's theorem and the objectively limiting of the frequency's resolution. After obtaining the signal's (Fourier) spectrum, this is processed and interpreted usually by a scientist who, presumably, has sufficient prior information about the monitored signal to conclude, for example, on the significant frequencies. Obviously, processing and interpretation of individual spectra are routine tasks that can be automated by suitable software (PC application). The problems complicate if we need to compare two or more spectra corresponding to different signals and/or phenomena. In the above context, this paper presents an intuitive method for automatic identification of the common and not common frequencies for two or more congruent spectra. The method is illustrated by numerical simulations, and by the results obtained in the analysis of the noise from some experimental measured signals.*

**Keywords:** Signals' analysis; frequencies spectrum; automation process

## 1. Introduction

It is known that sampling a time-varying signal and his spectral analysis are both subjected to theoretically compelling, such as Shannon's theorem and the objectively limiting of the frequency's resolution [1] – [5]. After obtaining the signal's (Fourier) spectrum, this is processed and interpreted usually by a scientist who, presumably, has sufficient prior information about the monitored signal to conclude, for example, on the significant frequencies. In this case, a quickly discernment criterion is the magnitude of spectral lines, but also the change of the associated phases. Thus, it is accepted that *the more important frequency in signal has the greater magnitude from the spectral lines*.

Obviously, processing and interpretation of individual spectra are routine tasks that can be automated by transferring the magnitudes comparison to suitable software (PC application). The problems complicate if we need to compare two or more spectra corresponding to different signals and/or phenomena. In the above

[1] PhD Senior Researcher, Institute for Nuclear Research Piteşti, Romania, e-mail: cezar.doca@nuclear.ro

[2] PhD Eng., Senior Researcher, Institute for Nuclear Research Piteşti, Romania, e-mail: constantin.paunoiu@nuclear.ro

context, the next paragraphs present an intuitive method for automatic identification of the common and not common frequencies for two or more congruent spectra.

## 2. Two sentences

Let $s(t)$ be a time-varying signal. We can accept that the signal's *frequencies spectrum* is a collection $S(\Delta\nu) = \{(\nu_i, A(\nu_i)) | i = 0,...,N\}$ of $N$ doublets $(\nu, A(\nu))$ where $A(\nu)$ is the magnitude of the *spectral line* having as abscise the frequency $\nu$. The value $\Delta\nu = \nu_i - \nu_{i-1} = const.$ is the spectrum's *resolution*. We name two spectra $S_1(\Delta\nu_1)$ and $S_2(\Delta\nu_2)$ as *congruent* if they have the same resolution $\Delta\nu_1 = \Delta\nu_2 = \Delta\nu$. We also name the spectral lines having the same abscise $\nu_k$, i,e. $(\nu_k, A_1(\nu_k))$ and $(\nu_k, A_2(\nu_k))$, as *correspondent*.

Let $s_1(t)$ and $s_2(t)$ be two time-varying signals as in Fig. 1 and Fig. 2 (numerical simulations in Microsoft Office Excel with *sampling time* $t_s$, *sampling frequency* $\nu_s$, and *number of samples* $N_s$).

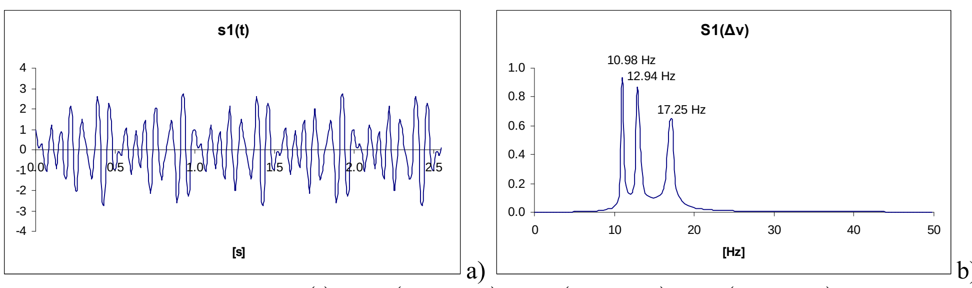



Fig. 1. *Signal* $s_1(t) = \sin(2\pi \cdot 11 \cdot t) + \cos(2\pi \cdot 13 \cdot t) - \sin(2\pi \cdot 17 \cdot t)$

*a) sampling:* $t_s = 2.55s; \nu_s = 100Hz; N_s = 256$ *; b) spectrum:* $\Delta\nu = 0.392157Hz; N = 128$

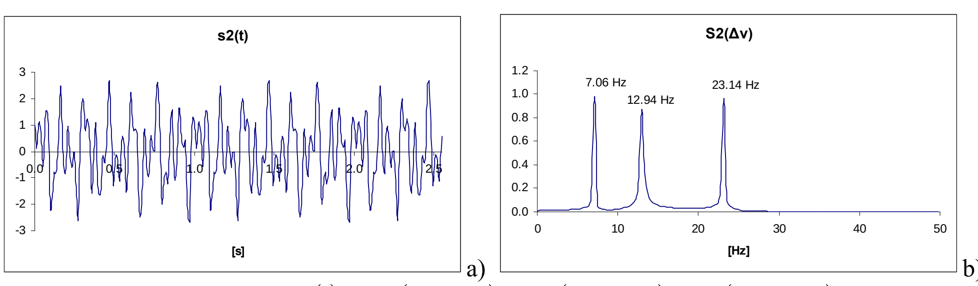
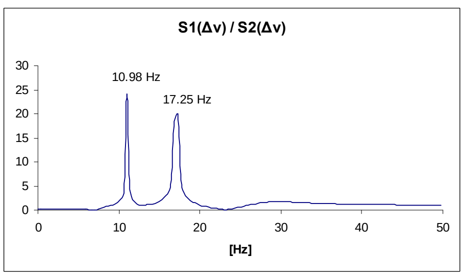



Fig. 2. *Signal* $s_2(t) = \sin(2\pi \cdot 7 \cdot t) + \cos(2\pi \cdot 13 \cdot t) - \sin(2\pi \cdot 23 \cdot t)$

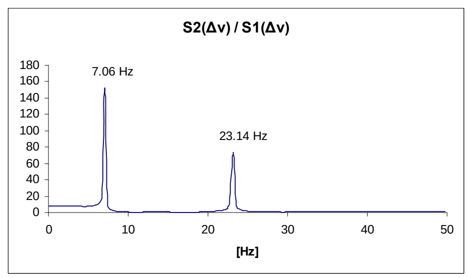
*a) sampling:* $t_s = 2.55s; \nu_s = 100Hz; N_s = 256$ *; b) spectrum:* $\Delta\nu = 0.392157\,Hz; N = 128$

If $S_1(\Delta\nu_1)$ and $S_2(\Delta\nu_2)$ are congruent, then the next two sentences are true:

1. *The ratio of the correspondent spectral lines' magnitudes emphasizes especially the not common frequencies, significantly existing in the numerator spectrum*; indeed, briefly, the value of the ratio $A_1(\nu_k)/A_2(\nu_k)$ amplifies when the value $A_1(\nu_k)$ increases (i.e. the frequency $\nu_k$ is significant in the numerator spectrum) and when the value $A_2(\nu_k)$ decreases (i.e. the frequency $\nu_k$ is insignificant in the denominator spectrum) (Fig. 3).



a)



b)

Fig. 3. *Not common frequencies:*
*a) ratio* $S_1(\Delta\nu)/S_2(\Delta\nu)$*; b) ratio* $S_2(\Delta\nu)/S_1(\Delta\nu)$

2. *The product of the correspondent spectral lines' magnitudes emphasizes especially the common frequencies, significantly existing in the both spectra*; indeed, briefly, the value of the product $A_1(\nu_k)\cdot A_2(\nu_k)$ amplifies when the both values $A_1(\nu_k)$ and $A_2(\nu_k)$ increase (i.e. the frequency $\nu_k$ is significant in the both spectra) (Fig. 4).

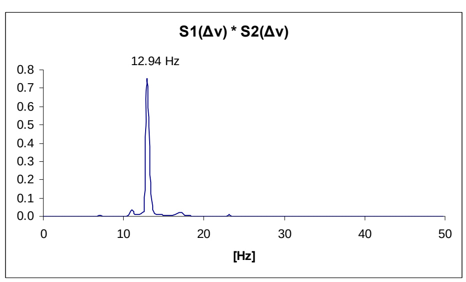


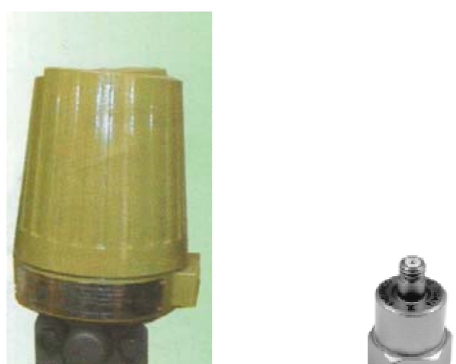

Fig. 4. *Common frequency: product* $S_1(\Delta\nu)\cdot S_2(\Delta\nu)$

### 3. Applications for the noise of some experimental measured signals

The two above sentences were applied in the analysis of the noise from some experimental measured signals for two absolute pressure transducers installed at the ends of a pipe through which water circulates (Tebeică I. ş.a., 2011), [6].

**Testing conditions**: *pipe's parameters*: $L = 10m$, $EI_z = 1.8 \times 10^5 Nm^2$, $\rho A = 7.333 kg/m$; *mass of water filling the pipe*: $m = 5.281 kg/m$ (Fig. 5); *measurement instruments*: two absolute pressure transducers FEPA Barlad FE1GM $(0-14)bar/(4-20)mA$, precision 0.5%, installed at the upstream and downstream pipe's ends (Fig. 6); two ENDEVCO *i*-TEDS accelerometers installed in $L/2$ and $L/4$ points; *data acquisition hardware*: Bruel & Kjaer Rack 3560C; *data processing software (PC application)*: Bruel & Kjaer PULSE LabShop Versions 12.5.1 (Fig. 7).

Fig. 5. *Pipe's end with pressure connection*



a) b)

Fig. 6. Instrumentation: a) FEPA Barlad FE1GM absolute pressure transducer b) ENDEVCO i-TEDS Accelerometer

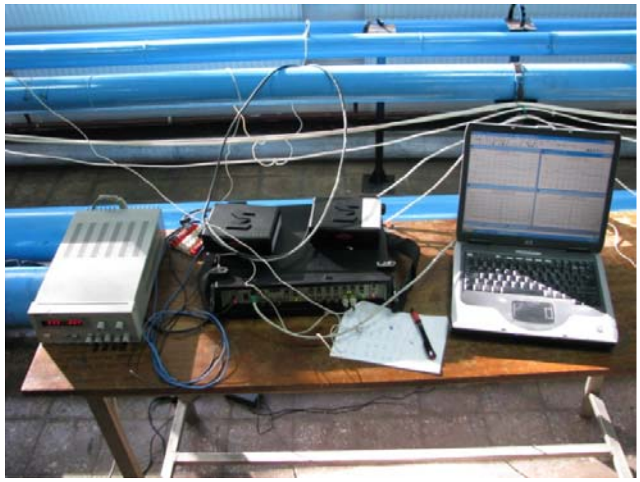


Fig. 7. Bruel & Kjaer measuring instrumentation

**Experimental results**: are presented as graphics in Fig. 8 – Fig. 19.

- In the case of the pipe filled with water, pressurized, but without flow (background signals)

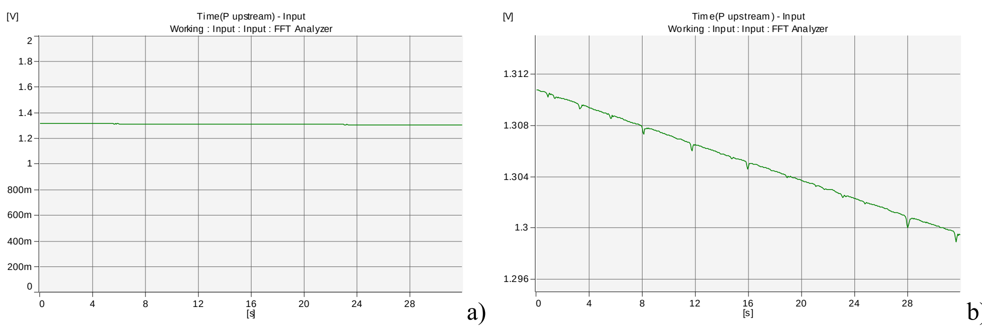



Fig. 8. Background (no flow): pressure transducer installed at the pipe's **upstream** end
a) time-varying signal; b) time-varying signal (zoom)

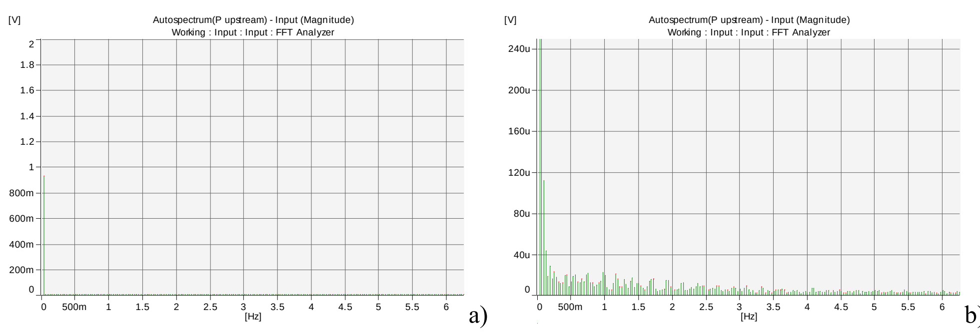



Fig. 9. Background (no flow): pressure transducer installed at the pipe's **upstream** end
a) frequencies spectrum; b) frequencies spectrum (zoom)

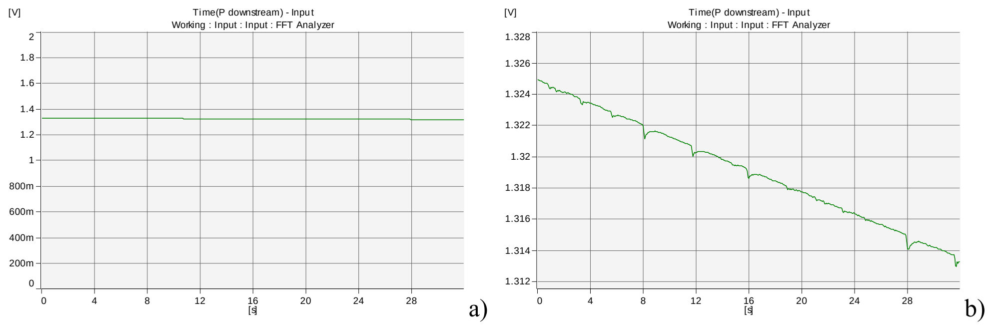



Fig. 10. Background (no flow): pressure transducer installed at the pipe's **downstream** end
a) time-varying signal; b) time-varying signal (zoom)

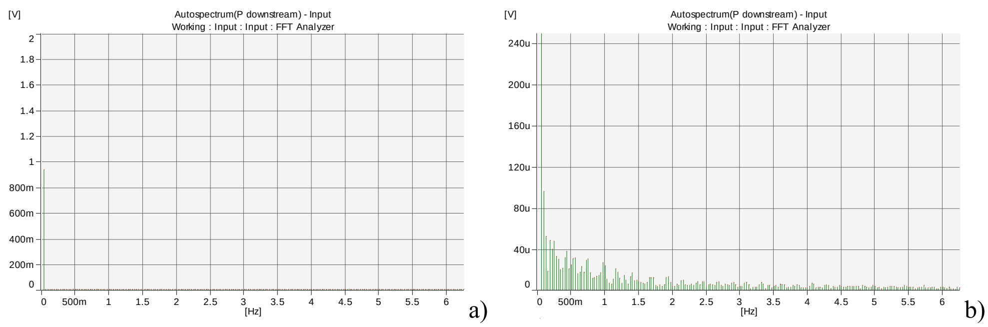



Fig. 11. Background (no flow): pressure transducer installed at the pipe's **downstream** end
a) frequencies spectrum; b) frequencies spectrum (zoom)

All measured signals were FFT analysed using the Bruel & Kjaer PULSE software (PC application) with: *sampling time* $t_s = 32s$; *sampling frequency* $\nu_s = 16Hz$; *number of samples* $N_s = 512$; *span frequency* $\nu_{\max} = 6.25Hz$; *number of spectral lines* $N = 200$; *spectral resolution* $\Delta\nu = 0.03125Hz$;.

- Repeating the same above measurements in the case of the pipe filled with water and flow at $70m^3/h$ (flow induced vibrations)

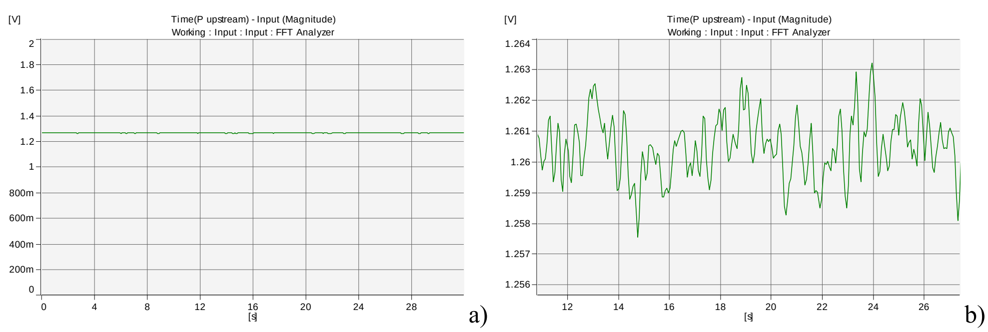



Fig. 12. Flow induced vibrations: pressure transducer installed at the pipe's **upstream** end
a) time-varying signal; b) time-varying signal (zoom)

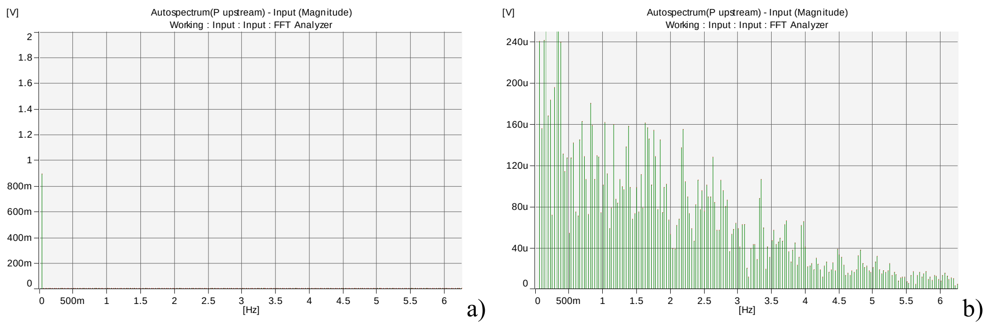



Fig. 13. Flow induced vibrations: pressure transducer installed at the pipe's **upstream** end a) frequencies spectrum; b) frequencies spectrum (zoom)

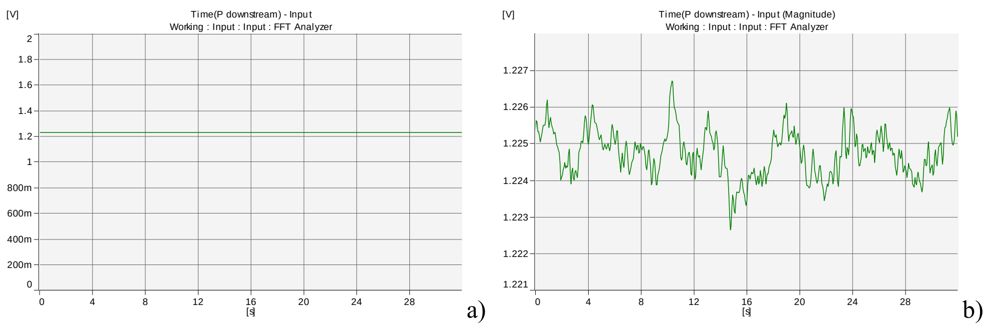



Fig. 14. Flow induced vibrations: pressure transducer installed at the pipe's **downstream** end a) time-varying signal; b) time-varying signal (zoom)

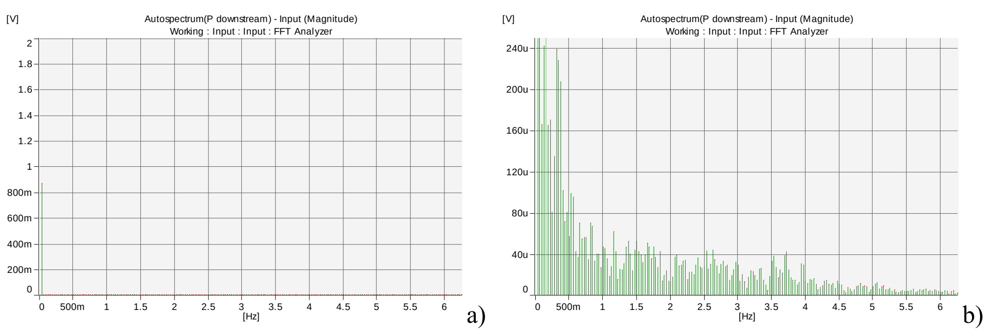
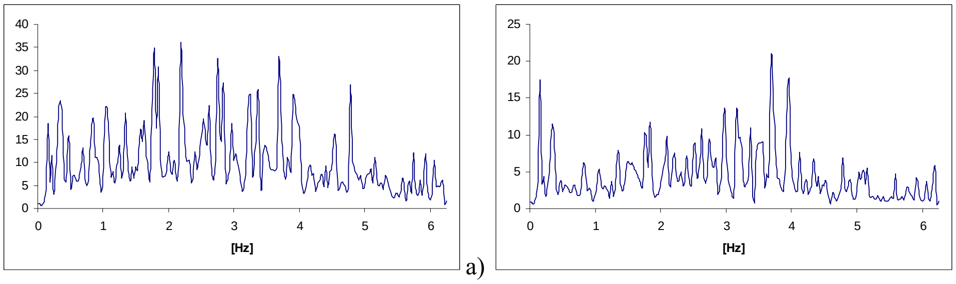



Fig. 15. Flow induced vibrations: pressure transducer installed at the pipe's **downstream** end a) frequencies spectrum; b) frequencies spectrum (zoom)

All these above numerical experimental data were copied in a Microsoft Office Excel Worksheet and, applying the two above sentences on the experimental congruent spectra, we obtained:

- The ratio between the magnitudes of the correspondent flow-induced-vibrations spectral lines (as numerator) and background spectral lines (as denominator) emphasized the significant vibrations frequencies from the both absolute pressure transducers' signals (Fig. 16)




a) b)

Fig. 16. Not common frequencies: ratios **vibrations** / **background**; emphasized vibrations frequencies for: a) upstream pressure transducer; b) downstream pressure transducer

- The product between the magnitudes of the correspondent emphasized spectral lines (Fig. 16) accentuated the common vibrations frequencies for the both absolute pressure transducers' signals (Fig. 17)

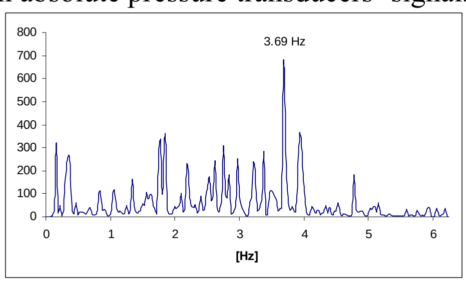
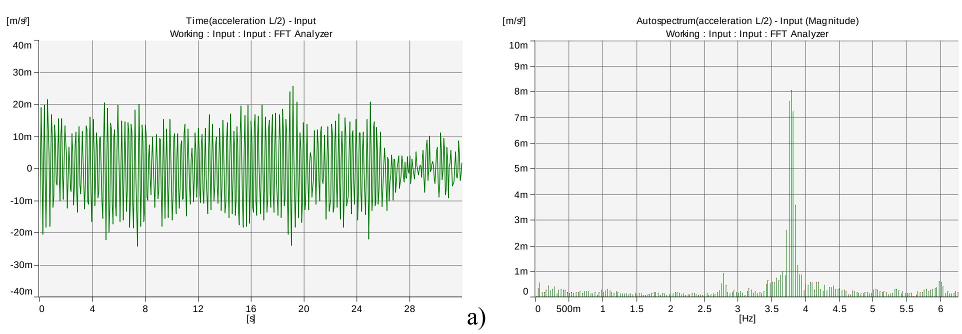



Fig. 17. Common frequencies: product **upstream*** **downstream**; emphasized vibrations frequencies; the most significant common frequency: $\nu = 3.69\,Hz\,(\Delta\nu = 0.03125\,Hz)$

The common frequency $\nu = 3.69\,Hz$ is very near-by the measured first significant frequency of oscillation $\nu = 3.76\,Hz$ in the case of the water-filled pipe's bending vibrations (Fig. 18 and Fig. 19).




a) b)

Fig. 18. Water-filled pipe's bending vibrations: accelerometers in the point $L/2$
a) flow induced vibrations signal; b) significant frequency $\nu = 3.76\,Hz\,(\Delta\nu = 0.03125\,Hz)$

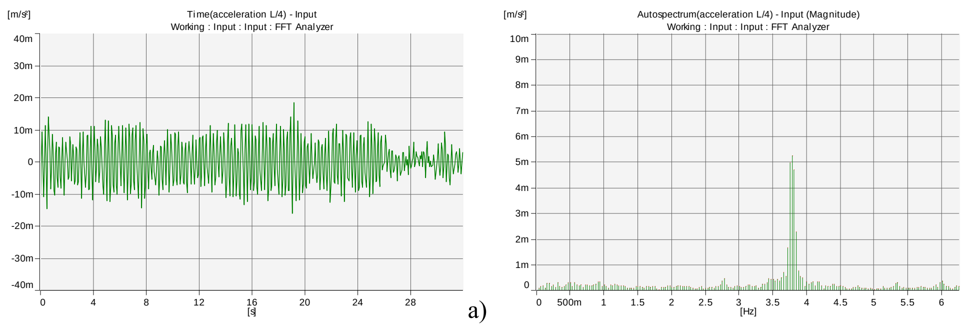



Fig. 19. Water-filled pipe's bending vibrations: accelerometer in the point $L/4$
a) flow induced vibrations signal; b) significant frequency $\nu = 3.76 Hz (\Delta\nu = 0.03125 Hz)$

We note that, starting from the general equation of bending vibrations of a beam (Buzdugan Gh. ş.a., 1979), [7]:

$$\frac{\partial^4 w(x,t)}{\partial x^4} + \frac{\rho A}{EI_z} \cdot \frac{\partial^2 w(x,t)}{\partial t^2} = \frac{p(x,t)}{EI_z}$$

in the case $p(x,t) = 0$ (i.e. free vibrations), for a pipe with $L = 10m$, $EI_z = 1.8 \times 10^5 Nm^2$, $\rho A = 7.333 kg/m$, filled with water $m_w = 5.281 kg/m$, and having at the ends two identical general suspensions (experimental situation from [6]) described by the equations (Chen S.S., 1972) [8]:

$$\begin{cases} -EI_z(x_*) \cdot \left.\dfrac{d^2 w(x)}{dx^2}\right|_{x=x_*} = -c \cdot \left.\dfrac{dw(x)}{dx}\right|_{x=x_*} \\ -EI_z(x_*) \cdot \left.\dfrac{d^3 w(x)}{dx^3}\right|_{x=x_*} = k \cdot w(x_*) \end{cases}$$

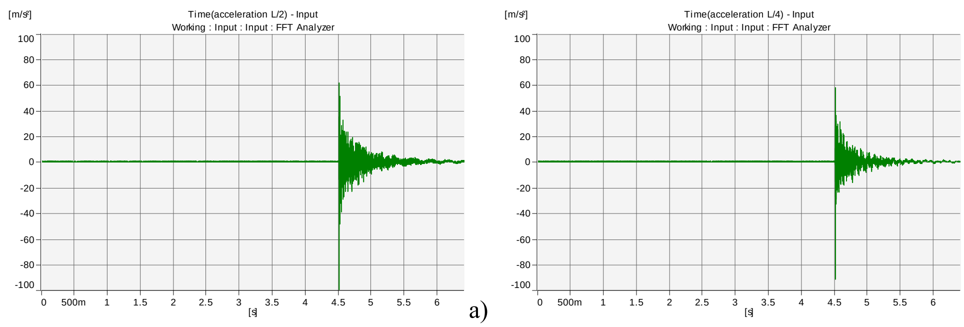
the first three free vibrations frequencies calculated as:

$$f_i = \frac{1}{2\pi}\left(\frac{\beta_i}{L}\right)^2 \sqrt{\frac{EI}{\rho A + m_w}}; \quad i = 1,2,3$$

are: $f_1 = 3.76 Hz$ for $\beta_1 = 4.441$, $f_2 = 8.34 Hz$ for $\beta_2 = 6.609$, and $f_3 = 17.17 Hz$ for $\beta_3 = 9.484$, if the elastic constants are $c \approx 2 \times 10^7 Nm$ and $k \approx 4 \times 10^7 N/m$.

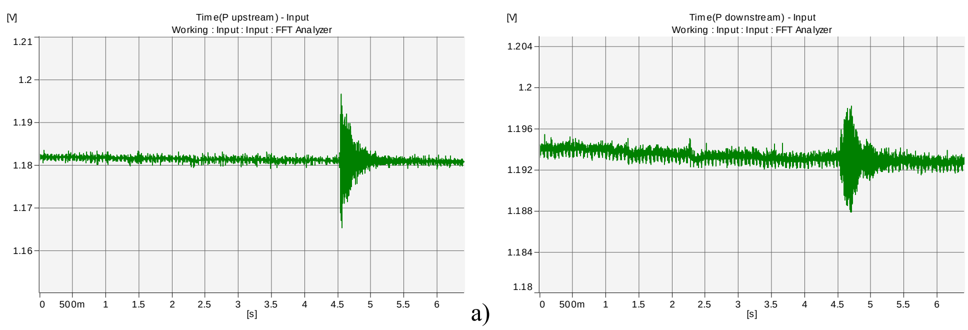
## 4. Monitoring the pipe's bending vibrations using absolute pressure transducers?

In this moment we don't have enough information to undoubtedly correlate the two values $\nu = 3.69\,Hz$ and $\nu = 3.76\,Hz$ . Anyway, the next pictures show four simultaneously measured signals for the bending vibrations of the water-filled pipe subjected to a mechanical shock.



a) b)

Fig. 20. Experimental: water-filled pipe's bending vibrations at **shock**
a) accelerometer's signal in the point $L/2$ ; b) accelerometer's signal in the point $L/4$



a) b)

Fig. 21– Experimental: signals for the **shock** "felt" by the absolute pressure transducers (zoom)
a) installed at the pipe's **upstream** end; b) installed at the pipe's **downstream** end

In the graphical results from Fig. 20 and Fig. 21 we can see that the both absolute pressure transducers "felt" the mechanical shock.

Having, in the case of shock tests, the *sampling frequency* $\nu_s = 2kHz$ , we also can see that there is a detectable delay $\left(\Delta t = t_p - t_a = 0.02s\right)$ between the shock time-moment for the accelerometers $\left(t_a = 4.506s\right)$ and the shock time-moment for the absolute pressure transducers $\left(t_p = 4.526s\right)$.

More precise data, more justified questions! We are preparing new needed tests.

## 5. Conclusions

The paper started with four unconventional definitions for *frequencies spectrum*, *spectral line*, *congruent spectra* and *correspondent spectral lines*.

Then the paper presented an intuitive method for the detection of common and not common frequencies from two or more congruent spectra, based on the mathematical demonstration of the next two sentences:

1. *The ratio of the correspondent spectral lines' magnitudes emphasizes especially the not common frequencies, significantly existing in the numerator spectrum*, and
2. *The product of the correspondent spectral lines' magnitudes emphasizes especially the common frequencies, significantly existing in the both spectra*.

The method was illustrated by numerical simulations made using the Microsoft Office Excel mathematical tools, and by the results obtained in the analysis of the noise existing in the experimental measured signals of two absolute pressure transducers installed at the ends of a pipe through which water circulates.

For the experimental cases, we don't have (yet) enough information to undoubtedly correlate the significant value $\nu = 3.69Hz$ emphasized in the signals' noise, and the measured value $\nu = 3.76Hz$ characteristic for the flow induced vibrations of the testing pipe, both obtained with the resolution $\Delta\nu = 0.03125Hz$ .

Forcing the limits:

$\nu_1 = (3.69 + 0.03)Hz = 3.72Hz$ (for absolute pressure transducers' signals);

$\nu_2 = (3.76 - 0.03)Hz = 3.73Hz$ (for accelerometers' signals);

$\nu_2 - \nu_1 = (3.73 - 3.72)Hz = 0.01Hz$ .

We need (and are preparing) new tests.

Sure enough, the method presented in this paper can be easy implemented in any automated vibrations monitoring task (i.e. using a PC application), with an adequately degree of reliability.

Sometimes, more important than *at what we look* is *what we see*.

## REFERENCES

[1]. *Bernd Girod*, Signals and Systems, Copyright © 2001 by John Wiley & Sons Ltd.

[2]. Steven W. Smith, *The Scientist and Engineer's Guide to Digital Signal Processing*, Second Edition, Copyright © 1997-1999 by California Technical Publishing

[3]. *E.S. Gopi*, Algorithm Collections for Digital Signal Processing Applications Using Matlab®, All Rights Reserved © 2007 Springer

[4]. *Robert M. Gray* and *Lee D. Davisson*, An Introduction to Statistical Signal Processing, Copyright © 2004 by Cambridge University Press.
[5]. *D.S.G. Pollock*, A Handbook of Time-Series Analysis, Signal Processing and Dynamics, Copyright © 1999 by ACADEMIC PRESS
[6]. *I. Tebeică, C. Doca, G. Dună, N. Anghel, E. Pantilie and C. Tudor*, "Measurements and analyses of the noise from absolute pressure transducers' signals", Internal Report 9288/2011, SCN Piteşti (in Romanian)
[7]. *Gh. Buzdugan, L. Fetcu, M. Radeş*, "Mechanical Vibrations", Ed.Did.Ped., Bucharest, 1979 (in Romanian)
[8]. *S. S. Chen,* "Parallel-flow-induced vibration of fuel rods", Nuclear Engineering and Design, Vol.18 (1972)