VALIDATION METHODS OF INTRINSIC AERODYNAMIC FORCE FORMULATIONS

Alexandru FILIP[1]

Exista doua metode de evaluare experimentala a fortelor aerodinamice exercitate de un curent asupra unui corp scufundat in fluid: metoda extrinseca si cea intrinseca. Formualarea intrinseca a fortelor aerodinamice este obtinuta prin observarea si evaluarea schimbarilor din fluid in urma interactiunii fluid-corp. Articolul se concentreaza asupra curgerilor incompresibile plane si prezinta trei metodologii de validare pentru ecuatia"flux", aceasta fiind una din formularile intrinseci cele mai uzitate.

There are two methods of evaluating experimentally the aerodynamic loads exerted by the flow on a submerged body: extrinsic and intrinsic. The intrinsic formulations of the aerodynamic forces are obtained by observing and evaluating the changes in the fluid due to the fluid-body interaction. The article concentrates on the two dimensional incompressible flows and presents three validation methodologies for the "flux" equation which is one of the most used intrinsic methods.

Keywords: experimental aerodynamics, intrinsic methods, particle image velocimetry

1. Introduction

There are two methods of evaluating experimentally the loads exerted by the flow on a submerged body: extrinsic and intrinsic. The extrinsic methods are easier to formulate and apply but they have the disadvantage of not being capable of examining local effects and the body-fluid interaction. The intrinsic methods however require a more complex experimental set-up and also have the constraints of the difficulty to measure the pressure field without influencing the real flow. This difficulty was surmounted by the development of the Particle Image Velocimetry technique [1][2][3] and the theoretical formalism developed by Noca [4][5]. This study aims at detailing the mathematical formulation of the "flux" equation and to describe the methods of validation. The derivation starts from writing the momentum equation in it's integral form for a general state of the fluid, compressible and viscid, emphasizing the creation of several "flows". Furthermore, a simplification is made, considering an incompressible flow. The article will also emphasize on the limitations of each method and will recommend future directions of research.

[1] PhD Student, University POLITEHNICA of Bucharest, Romania, e-mail: alexandru.filip@gmail.com

2. Intrinsic formulations of the aerodynamic forces

The intrinsic formulations of the aerodynamic forces acting on a submerged body in fluid are obtained by observing and evaluating the changes in the fluid due to the fluid-body interaction. In opposition, extrinsic formulations observe and evaluate the changes in the body behavior due to the same fluid-body interaction. All the intrinsic formulations are based on the momentum equation over a control volume which leads to the equation below:

$$\frac{F}{\rho}=-\frac{d}{dt}\int_{V(t)}\boldsymbol{u}dV+\oint_{S(t)}\boldsymbol{n}\cdot\left(-\frac{p}{\rho}\boldsymbol{I}-(\boldsymbol{u}-\boldsymbol{u_s})\boldsymbol{u}+\boldsymbol{T}\right)dS-\oint_{S_b(t)}\boldsymbol{n}\cdot(\boldsymbol{u}-\boldsymbol{u_s})\boldsymbol{u}dS \tag{1}$$

$$\boldsymbol{T}=\lambda(\nabla\cdot\boldsymbol{u})\boldsymbol{I}+\mu(\nabla\boldsymbol{u}+\nabla\boldsymbol{u}^{\mathrm{T}}) \tag{2}$$

where $\boldsymbol{n}$ is the unit vector normal to the surface $\boldsymbol{S(t)}$ as shown in Fig. 1, $\boldsymbol{u}$ is the flow velocity, $\boldsymbol{u}_s$ is the body wall velocity, p is the pressure, $\boldsymbol{I}$ is the unit tensor, $\boldsymbol{T}$ is the viscous stress tensor and μ is the coefficient of dynamic viscosity.

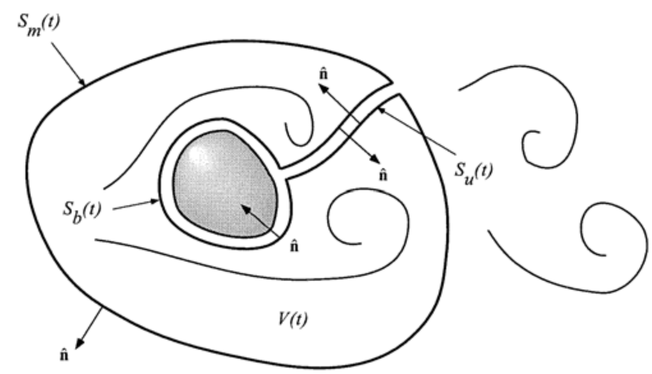

Fig. 1. Control volume analysis

For an incompressible flow ($\nabla\cdot\boldsymbol{u}=\boldsymbol{0}$) the tensor equation becomes:

$$\boldsymbol{T}=\mu(\nabla\boldsymbol{u}+\nabla\boldsymbol{u}^{\mathrm{T}}) \tag{3}$$

Equation (1) requires the knowledge of the pressure field and the velocity field. While the velocity field can be evaluated experimentally using the Particle Image Velocimetry method, the pressure field is the unknown. For this, different formulations where derived with different constraints [4].

The “impulse equation”:

$$\frac{F}{\rho} = -\frac{1}{N-1}\frac{d}{dt}\int_{V(t)} \boldsymbol{x} \times \omega dV + \frac{d}{dt}\oint_{S(t)} \boldsymbol{n} \cdot \boldsymbol{\gamma_i} dS + \tag{4}$$

$$+\frac{1}{N-1}\frac{d}{dt}\oint_{S_b(t)} \boldsymbol{x} \times (\boldsymbol{n} \times \boldsymbol{u}) dS - \oint_{S_b(t)} \boldsymbol{n} \cdot (\boldsymbol{u} - \boldsymbol{u_s}) \boldsymbol{u} dS$$

$$\gamma_i = \frac{1}{2}u^2 I - \boldsymbol{uu} - \frac{1}{N-1}(\boldsymbol{u} - \boldsymbol{u_s})(\boldsymbol{x} \times \boldsymbol{\omega}) + \frac{1}{N-1}\boldsymbol{\omega}(\boldsymbol{x} \times \boldsymbol{u}) + \tag{5}$$

$$+\frac{1}{N-1}[\boldsymbol{x} \cdot (\nabla \cdot \boldsymbol{T})\boldsymbol{I} - \boldsymbol{x}(\nabla \cdot \boldsymbol{T})] + \boldsymbol{T}$$

is a very general formulation and it is valid for rotational and viscous flows; the body does not have to be rigid or solid, the control volume is arbitrary as long as it contains the body. N is the space dimension. The downside of this formulation comes from the fact that the volume integrals involve not only the velocity field but also derivatives of this velocity field, namely the vorticity, ω. The Particle Image Velocimetry method provides an under-resolved set of data in the vicinity of the body. This is because, the experiment set-up is based on the flow velocity but the velocity decreases near the body and the data is not good enough in this vicinity.

Another formulation is the ”momentum” equation:

$$\frac{F}{\rho} = -\frac{d}{dt}\int_{V(t)} \boldsymbol{u} dV + \oint_{S(t)} \boldsymbol{n} \cdot \boldsymbol{\gamma}_P dS - \oint_{S_b(t)} \boldsymbol{n} \cdot (\boldsymbol{u} - \boldsymbol{u_s}) \boldsymbol{u} dS \tag{6}$$

where

$$\boldsymbol{\gamma}_P = \frac{1}{2}u^2 I + (\boldsymbol{u}_S - \boldsymbol{u})\boldsymbol{u} - \frac{1}{N-1}\boldsymbol{u}(\boldsymbol{x} \times \boldsymbol{\omega}) + \frac{1}{N-1}\boldsymbol{\omega}(\boldsymbol{x} \times \boldsymbol{u}) + \tag{7}$$

$$+\frac{1}{N-1}[\boldsymbol{x} \cdot (\nabla \cdot \boldsymbol{T})\boldsymbol{I} - \boldsymbol{x}(\nabla \cdot \boldsymbol{T})] + \boldsymbol{T} - \frac{1}{N-1}[(\boldsymbol{x} \cdot \frac{\partial \boldsymbol{u}}{\partial t})\boldsymbol{I} - \boldsymbol{x}\frac{\partial \boldsymbol{u}}{\partial t}]$$

This is also a very general equation. The difference is that the volume integral assumes only the knowledge of the velocity field which should lead to lower errors than the volume integral in equation (4).

In order to eliminate volume integrals, the “flux” equation was derived but with the constraint of incompressible flows:

$$\frac{F}{\rho} = \oint_{S(t)} n \cdot \gamma dS - \oint_{S_b(t)} \boldsymbol{n} \cdot (u - u_S) u dS - \frac{d}{dt}\oint_{S_b(t)} \boldsymbol{n} \cdot (\boldsymbol{u}x) dS \tag{8}$$

Where

$$\boldsymbol{\gamma}_P = \frac{1}{2}u^2 I - \boldsymbol{uu} - \frac{1}{N-1}\boldsymbol{u(x}\times\boldsymbol{\omega}) + \frac{1}{N-1}\boldsymbol{\omega(x}\times\boldsymbol{u}) + \\ + \frac{1}{N-1}[\boldsymbol{x}\cdot(\nabla\cdot\boldsymbol{T})\boldsymbol{I} - \boldsymbol{x}(\nabla\cdot\boldsymbol{T})] + \boldsymbol{T} - \frac{1}{N-1}[(\boldsymbol{x}\cdot\frac{\partial\boldsymbol{u}}{\partial t})\boldsymbol{I} - \boldsymbol{x}\frac{\partial\boldsymbol{u}}{\partial t} + (N-1)\frac{\partial u}{\partial t}x] \tag{9}$$

3. Two dimensional flux equation formulation

From this point forward we will focus on the incompressible two dimensional flows (N=2). In this chapter we will explicit the flux equation terms into more detail. We will note the term equation as follows:

$$\frac{F}{\rho} = \left(\oint_{S(t)} n\cdot\boldsymbol{\gamma} dS\right)_I - \left(\oint_{S_b(t)} \boldsymbol{n}\cdot(u-u_S)udS\right)_{II} - \left(\frac{d}{dt}\oint_{S_b(t)} \boldsymbol{n}\cdot(\boldsymbol{u}x)dS\right)_{III} \tag{10}$$

$$\boldsymbol{\gamma}_P = \left(\frac{1}{2}u^2 I\right)_A - (uu)_B - \left(\frac{1}{N-1}\boldsymbol{u(x}\times\boldsymbol{\omega})\right)_C + \left(\frac{1}{N-1}\boldsymbol{\omega(x}\times\boldsymbol{u})\right)_D + \\ + \frac{1}{N-1}[(\boldsymbol{x}\cdot(\nabla\cdot\boldsymbol{T})\boldsymbol{I})_E - (\boldsymbol{x}(\nabla\cdot\boldsymbol{T}))_F] + (\boldsymbol{T})_G - \\ - \frac{1}{N-1}[\left((\boldsymbol{x}\cdot\frac{\partial\boldsymbol{u}}{\partial t})\boldsymbol{I}\right)_H - \left(\boldsymbol{x}\frac{\partial\boldsymbol{u}}{\partial t}\right)_I + \left((N-1)\frac{\partial u}{\partial t}x\right)_J] \tag{11}$$

Thus:

$$A = \frac{1}{2}|u|^2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}\frac{1}{2}(u^2+v^2) & 0\\ 0 & \frac{1}{2}(u^2+v^2)\end{bmatrix} \tag{12}$$

$$B = \begin{pmatrix}u\\ v\end{pmatrix}\begin{pmatrix}u & v\end{pmatrix} = \begin{bmatrix}u^2 & uv\\ uv & v^2\end{bmatrix} \tag{13}$$

$$C = \begin{pmatrix}u\\ v\\ 0\end{pmatrix}\left[\begin{pmatrix}x\\ y\\ z\end{pmatrix}\times\begin{pmatrix}w_x\\ w_y\\ w_z\end{pmatrix}\right]^T = \begin{pmatrix}u\\ v\\ 0\end{pmatrix}\begin{pmatrix}yw_z & -xw_z & 0\end{pmatrix} = \\ = \begin{bmatrix}uyw_z & -uxw_z\\ vyw_z & -vxw_z\end{bmatrix} \tag{14}$$

$$D=\begin{pmatrix}0\\0\\w_z\end{pmatrix}\left[\begin{pmatrix}x\\y\\0\end{pmatrix}\times\begin{pmatrix}u\\v\\0\end{pmatrix}\right]^T=\begin{pmatrix}0\\0\\w_z\end{pmatrix}\begin{bmatrix}0 & 0 & xv-yu\end{bmatrix}=0 \tag{15}$$

$$E=\begin{pmatrix}x & y\end{pmatrix}\cdot\left(\nabla\cdot\mu\left(\nabla\begin{pmatrix}u\\v\end{pmatrix}+\left(\nabla\begin{pmatrix}u\\v\end{pmatrix}\right)^T\right)\right)\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}= \tag{16}$$

$$=\mu\begin{bmatrix}2x\frac{\partial^2u}{\partial x^2}+x\frac{\partial^2u}{\partial y^2}+x\frac{\partial^2v}{\partial x\partial y}+2y\frac{\partial^2v}{\partial y^2}+y\frac{\partial^2v}{\partial x^2}+y\frac{\partial^2u}{\partial x\partial y} & 0\\ 0 & E_{1,1}\end{bmatrix}$$

$$F=\begin{pmatrix}x\\y\end{pmatrix}\left(\nabla\cdot\mu\left(\nabla\begin{pmatrix}u\\v\end{pmatrix}+\left(\nabla\begin{pmatrix}u\\v\end{pmatrix}\right)^T\right)\right)= \tag{17}$$

$$=\mu\begin{bmatrix}2x\frac{\partial^2u}{\partial x^2}+x\frac{\partial^2u}{\partial y^2}+x\frac{\partial^2v}{\partial x\partial y} & 2x\frac{\partial^2v}{\partial y^2}+x\frac{\partial^2v}{\partial x^2}+x\frac{\partial^2u}{\partial x\partial y}\\ 2y\frac{\partial^2u}{\partial x^2}+y\frac{\partial^2u}{\partial y^2}+y\frac{\partial^2v}{\partial x\partial y} & 2y\frac{\partial^2v}{\partial y^2}+y\frac{\partial^2v}{\partial x^2}+y\frac{\partial^2u}{\partial x\partial y}\end{bmatrix}$$

$$G=\nabla\cdot\mu\left(\nabla\begin{pmatrix}u\\v\end{pmatrix}+\left(\nabla\begin{pmatrix}u\\v\end{pmatrix}\right)^T\right)=\mu\begin{bmatrix}2\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x} & 2\frac{\partial v}{\partial y}\end{bmatrix} \tag{18}$$

$$H=\begin{pmatrix}x & y\end{pmatrix}\cdot\frac{\partial}{\partial t}\begin{pmatrix}u\\v\end{pmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}x\frac{\partial u}{\partial t}+y\frac{\partial v}{\partial t} & 0\\ 0 & x\frac{\partial u}{\partial t}+y\frac{\partial v}{\partial t}\end{bmatrix} \tag{19}$$

$$I=\begin{pmatrix}x\\y\end{pmatrix}\frac{\partial\begin{pmatrix}u & v\end{pmatrix}}{\partial t}=\begin{bmatrix}x\frac{\partial u}{\partial t} & x\frac{\partial v}{\partial t}\\ y\frac{\partial u}{\partial t} & y\frac{\partial v}{\partial t}\end{bmatrix} \tag{20}$$

$$J = \frac{\partial \begin{pmatrix} u \\ v \end{pmatrix}}{\partial t} \begin{pmatrix} x & y \end{pmatrix} = \begin{bmatrix} x\frac{\partial u}{\partial t} & y\frac{\partial u}{\partial t} \\ x\frac{\partial v}{\partial t} & y\frac{\partial v}{\partial t} \end{bmatrix} \tag{21}$$

Thus, the control volume surface integral becomes:

$$\left(\oint_{S(t)} n \cdot \gamma dS\right)_I = \oint_{S(t)} n \cdot (A - B - C + D + E - F + G - H + I - J) dS \tag{22}$$

The second term of equation (10) becomes:

$$\left(\oint_{S_b(t)} \boldsymbol{n} \cdot (u - u_s) u dS\right)_{II} = \oint_{S_b(t)} n \cdot \left(\begin{pmatrix} u \\ v \end{pmatrix} - \begin{pmatrix} u_s \\ v_s \end{pmatrix}\right)\begin{pmatrix} u & v \end{pmatrix} dS = \tag{23}$$

$$= \oint_{S_b(t)} n \cdot \begin{bmatrix} u^2 - u_s u & uv - u_s v \\ vu - v_s u & v^2 - v_s v \end{bmatrix} dS$$

The third term becomes:

$$\left(\frac{d}{dt}\oint_{S_b(t)} \boldsymbol{n} \cdot (\boldsymbol{u} x) dS\right)_{III} = \frac{d}{dt}\oint_{S_b(t)} n \cdot \begin{pmatrix} u \\ v \end{pmatrix}\begin{pmatrix} x & y \end{pmatrix} dS = \tag{24}$$

$$= \frac{d}{dt}\oint_{S_b(t)} n \cdot \begin{bmatrix} ux & uy \\ vx & vy \end{bmatrix} dS$$

4. Validation of the two dimensional flux equation using analytical methods

In order to validate the equation analytically, we need to use two dimensional incompressible flows for which we have analytical formulations of the force and velocity fields.

The first validation example is the potential flow fixed vortex. The conditions for this case are: steady flow, inviscid, incompressible and irrotational.

This leads to equation (10) to become:

$$\frac{F}{\rho} = \left(\oint_{S(t)} n \cdot \gamma dS\right)_I = \oint_{S_b(t)} \left(\begin{bmatrix} \frac{1}{2}\left(u^2 + v^2\right) & 0 \\ 0 & \frac{1}{2}\left(u^2 + v^2\right) \end{bmatrix} - \begin{bmatrix} u^2 & uv \\ uv & v^2 \end{bmatrix} \right) dS \tag{25}$$

For this case we know the equation for the velocity field [6]:

$$\begin{pmatrix} u \\ v \end{pmatrix} = \frac{\Gamma}{2\pi r}\begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} \tag{26}$$

Γ is the vortex circulation and θ is the angle between the x axis and the position vector r.

Thus (25) further becomes:

$$\frac{F}{\rho} = \oint_{C_b(t)} \left(\begin{bmatrix} \frac{1}{2}|U|^2 & 0 \\ 0 & \frac{1}{2}|U|^2 \end{bmatrix} - \left(\frac{\Gamma}{2\pi r}\right)^2 \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix} \right) r d\theta = \tag{27}$$

$$= \left(\frac{\Gamma}{2\pi r}\right)^2 r \int_0^{2\pi} \left(\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} - \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix} \right) d\theta =$$

$$= \left(\frac{\Gamma}{2\pi r}\right)^2 r \left(\begin{bmatrix} \pi & 0 \\ 0 & \pi \end{bmatrix} - \begin{bmatrix} \pi & 0 \\ 0 & \pi \end{bmatrix} \right) = 0$$

The result above is in line with the result using the Kutta-Joukowski theorem [7]:

$$\frac{F}{\rho} = \Gamma \times U_\infty = 0 \tag{28}$$

This case can be validated also using different other contour integrals and also using numeric integration.

More complex potential flow cases can be used such as moving vortex or combination of vorteces.

5. Validation of the two dimensional flux equation using CFD

In order to validate the formulation for even more complex flow conditions. For this, the process is as follows. We use the CFD code in order to obtain the velocity field, along with the vorticity field along a chosen contour around a wing profile. Using an extrapolation technique, we export these data to a Matlab application which introduces them in the flux equation in order to compute the force exerted on the body.

On the other hand we use the CFD code to directly calculate the loads on the body and compare the results to the ones previously obtained.

This validation was performed by Marelli [8] for a steady flow, viscous, incompressible and rotational, using a NACA0015 airfoil using the procedure already described.

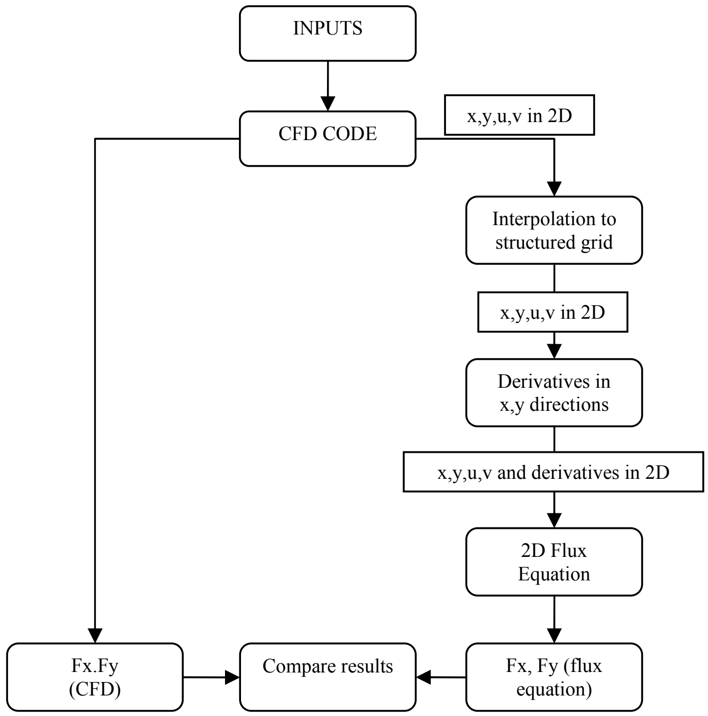

Fig. 2. Validation procedure using CFD code

This method is very sensitive to the interpolation method use to extract data from an unstructured grid to a structures one. Different computations can be performed also for various contours.

6. Experimental validation method

So far, the methods presented validate the flux equation against other theoretical results obtained either analytically or numerically. In addition, the validation domain was limited due to analytic or numeric constraints. For these reasons but also because these formulations were developed in order to be used experimentally, a powerful validation method is the experiment in itself [9][10][11].

The procedure consists in setting-up an experiment in which we can impose various degrees of liberty so that the flow could be steady, unsteady or quasi-steady. Also the flow should be in the limitations of incompressibility so that the flux equation applies. Naturally, the flow will be rotational and viscous.

Special attention has to be paid to ensuring and checking that the flow is as close to the two-dimensional hypothesis. Corrections can be applied to the results in order to allow for model, by CFD codes simulations.

Once the experimental design is performed, two different experimental measurements should be performed, for the same experimental conditions. One type of measurements will rely on sensor readings regarding strain gauges, loads applied by the flow on the whole body (wing) and will be used to calculate the global aerodynamic forces. This method is also named the extrinsic method. These results will have to be corrected in order to obtain the 2D flow case results.

The second type of measurement will be based on the Particle Image Velocimetry which will extract the velocity and its derivatives field around the body at a particular section. These results will be used in the flux equation and the section loads will be computed. These sectional loads will then be integrated over the entire wing span, considering a 2D case.

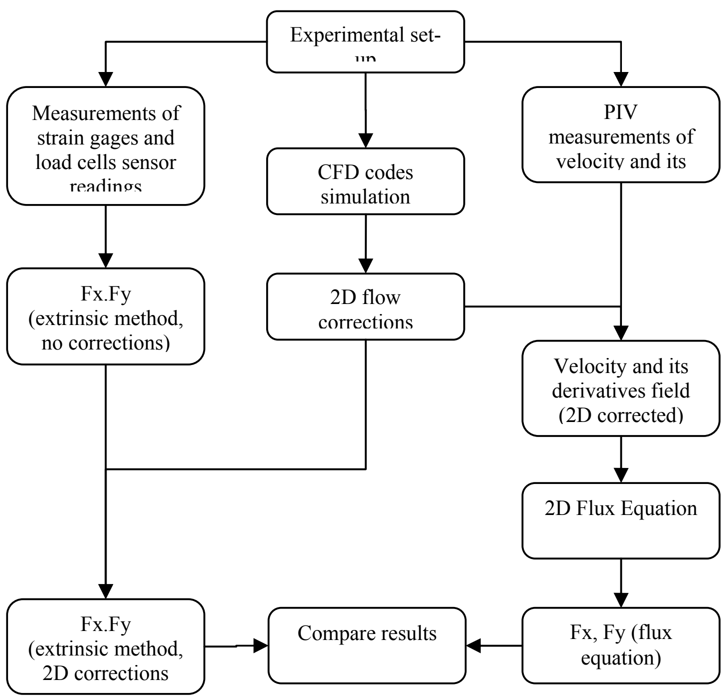

Fig. 3. Validation procedure using the experimental method

7. Conclusions

The article described several methods of validation for the flux equation that evaluates the loads applied by a flowing incompressible fluid on a submerged body. The general assumptions was that of a two dimensional flow.

The conclusion is that the formulation can be easily validated through the analytical methods but with big constraints of the flow conditions domain. A more broad validation is expected by using the CFD codes on its own or even better in symbiosis with the experimental method. The recommendation is to design experiments that will allow the flow domain conditions to be as varied as possible.

As future work, two directions of research can be identified. One is to further validate the two dimensional formulations through the experimental set-up, and the latter is to develop the three dimensional formulation and validate this as well. This also implies a significantly more complex experimental set-up and procedure due to the difficulties of the three dimensional PIV method.

Acknowledgements

This study is financed by the European Union through the SOPHRD ID 5159 project.

R E F E R E N C E S

[1] *M. Raffel, C. Willert, S. Wereley, and J. Kompenhans*, Particle Image Velocimetry, Springer, Berlin/Heidelberg, second edition, 2007

[2] *H. Zare-Behtash, N. Gongora, C. Lada, D. Kounadis and K. Kontis,* Application of pressure-sensitive paints in high-speed flows, Shock Waves, Part VII, 497-502, 2009

[3] *J. W. Gregory, K. Asai, M. Kameda, T. Liu and J. P. Sullivan,* A review of pressure-sensitive paint for high-speed and unsteady aerodynamics, Proceedings of the I Mech E Part G Journal of Aerospace Engineering, 222, number 2, 249-290, 2008

[4] *F. Noca, D. Shields, D. Jeon*, A Comparison of methods for evaluating time-dependent fluid dynamic forces on bodies, using only velocity fields and their derivatives, Journal of Fluids and Structures, vol. 13, 1999, pp. 551-578

[5] *B.T. Tan, M.C. Thompson, K. Hourigan,* Evaluating fluid forces on bluff bodies using partial velocity data, Journal of fluids and structures, 20, 5-25, 2005

[6] *J. Katz and A. Plotkin*, Low-Speed Aerodynamics, Cambridge Press, second edition, 2001

[7] *E. Carafoli, V.N. Constantinescu*, Dinamica fluidelor incompresibile (Dynamics of incompressible flows), Editura Academiei, Bucureşti, 1981

[8] *M. Marelli*, Determining loads over a 3D HAWT blade using Particle Image Velocimetry, MSc. Thesis, Faculty of Aerospace Engineering, Delft University of Technology, 2010

[9] *B. van Oudheusden, F. Scarano, F. Roosenboom, E. Casimiri and L. Souverein,* Evaluation of integral forces and pressure fields from planar velocimetry data for incompressible flows, Experiments in Fluids, 43, 153–162, 2007

[10] *C. Klein, R. H. Engler, U. Henne and W. E. Sachs*, Application of pressure-sensitive paint for determination of the pressure field and calculation of the forces and moments of models in a wind tunnel, Experiments in Fluids, 39, number 2, 475-483, 2005

[11] *D. Ragni, A. Ashok, B. W. van Oudheusden and F .Scarano,* Surface pressure and aerodynamic loads determination of a transonic airfoil based on particle image velocimetry, Measurement Science and Technology, 20, number 7, 2009