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MATHEMATICAL MODELLING OF PRODUCT REPRESENTATIONS

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Product modelling has been of continuous interest. Mathematical modelling and adequate terminology are important instruments for efficient operational developments in product design and manufacturing. The present paper contributes to a mathematical modelling on product representation, by introduction a conceptual frame, implicit constraints and reference elements type of component, relation, structure, characteristic, cardinality, hypostasis, as well as by proofing outline model and case study. The achievement has a high degree of generality, as a basis for the future models.

Keywords: mathematical modelling, product, component, relation, structure, characteristic, hypostasis.

1. Introduction

An advanced product model is of a high interest among researchers.

The relations between parts ”within a whole”, subject of mereology, the theory of parthood [1], is of scientific and applicative interest in product modelling, for knowledge, design, manufacturing, etc.

As part of cohesive product-related information, an operational system has been developed to include annotations in CAD design environment, where the feature is considered a specific geometric element [2].

In knowledge representation, it is not possible to enumerate all properties, but only a subset of these, so that ”identity based on property matching is under-determined. One solution is to have *some* properties count as those necessary for identity, namely an explicit *theory of identity criteria*.” [3].

Maximizing the product modularity is possible during product architecture design, based on interactions of different components. A binary rooted tree representation and a novel mathematical programming model are developed to build up the product structure for an optimal modularity [4].

“A product is more than a tangible thing”: satisfies consumer needs, but also reveals an abstract value. From both these perceptions, five product levels can be denoted: core product, associated with main function; generic product, representing all the qualities about material, design, etc.; expected product, i.e.,

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about all aspects to get from product; augmented product, referring to features as fashion brand, warranty, etc., making it a distinct one; potential product, that is about future possible developments because of technology progress [5].

Languages and reasoning methods are at the core of knowledge representation. There have been developed many representation languages for the effective description of different kind of information [6].

Product variety is “the range of product models” being produced in order to satisfy the market evolution. It is underlined that products built in modular system can be relative easily modified [7].

2. Objective and research method

The objective of the present research is to develop a mathematical modelling on product representations with a high degree of generality, as a basis for the future models.

The present research has been approached with regard to certain reference achievements: selection and introducing of proper concepts, entities, and relations, as well as proofing case studies.

Let *P* product be a *real* product or a *generic* product in a broad sense.

In numerous cases, a *generic* product is associated with a product *family*.

Product representations are referring to product *components*, *relations*, and other *defining entities*.

The product *representations* may be type of analytical relation, graph, etc.

3. Components, relations and structure

Let $X \neq \emptyset$ be a not empty and finite set and let n be its cardinal, i.e.,

$$X = \{x_i;\ i = 1, 2, \dots, n\} \tag{1}$$

The elements x_i are called *components.*

On the set of components, X, let Λ be a finite family of relations, i.e.,

$$\Lambda = (R_k)_{k \in J},\ J = \{1, 2, \dots, m\},\ R_k \subseteq X \times X,\ \forall k \in J \tag{2}$$

We shall denote as usual

$$x_i R_k x_j \Leftrightarrow (x_i, x_j) \in R_k, \forall i, j = 1, 2, \dots, n, \forall k \in J \tag{3}$$

It is to emphasize that the relations R_k are general, and the usual properties they may have are reflexivity, symmetry, antisymmetry, transitivity, as well as (fuzzy) generalizations such as similarity (a fuzzy relation which is reflexive, symmetric and transitive), fuzzy ordering, etc. The set Λ may also include functional and geometric – defining relations.

If X and Λ are as above, then the couple (X, Λ) is called a *structure*, S, i.e.

$$S = (X, \Lambda) \tag{4}$$

4. Characteristics, product, transformations and hypostases

Let (X, Λ) be a structure as above.

In order to satisfy all the necessary requirements, during the development process of the product, a set of *characteristics* - shape, size, relative position, electrical conductivity, corrosion resistance, hardness, etc., as the case -, must be attributed to the components.

Let K be a not empty and finite set,

$$K = \{C_h;\ h = 1, 2, \ldots, s\} \tag{5}$$

An element C_h of K called a *characteristic*.

The characteristics are associated to the components through a map F ,

$$F: X \rightarrow 2^K \tag{6}$$

in the sense that every *component* has a set of *characteristics* (as usual, 2^K denotes the power set of K.

Finally, a P product is defined as the tuple

$$P = (X, \Lambda, K, F) \tag{7}$$

Based on the above approach, a P product is defined by its structure and its characteristics.

It is important to mention that not all the elements of X are necessarily involved in the P product. Let $x_i \in X$ and if:

$$\begin{aligned} &F(x_i) = \emptyset\,, \\ &(x_i, \mathrm{x}) \notin R_k,\ \forall x \in X,\ \forall k \in J, \\ &(\mathrm{x}, x_i) \notin R_k,\ \forall x \in X,\ \forall k \in J \end{aligned} \tag{8}$$

then the *component* x_i is not in fact a part of P. Such a component is called a *potential component.* All the other components will be called *actual components.* We shall denote by $X(a)$ the set of all actual components and by $X(p)$ the set of all potential components of the given product. In this way, one can add components to the set X without changing the basic function of the product, just by adding potential components.

If:

$$F(x) = \emptyset\,,\ \forall x \in X,$$
$$R_k = \emptyset\,,\ \forall k \in J \tag{9}$$

then P is called the *null product* for every fixed set of components X.

One can add components to the null product and still get the null product.

A specific property of the P product is its *cardinality*, (n, m, r), i.e. the number of components, the number of the relations involved and the number of its characteristics.

If two P_1 and P_2 products have the same set of components, i.e.

$$P_1 = (X, \Lambda_1, K_1, F_1),\ P_2 = (X, \Lambda_2, K_2, F_2) \tag{10}$$

we say that P_2 is a *transformation* or *hypostasis* of P_1; we denote this fact by

$$P_2 = U(P_1) \text{ or } P_1 \to P_2 \tag{11}$$

Obviously, " $\to$ " is a transitive relation on the set of products.

As a result of arbitrary (different) successive transformations, $\{U_\alpha,\ \alpha = 1, 2, \dots, p\}$, the product P may be in corresponding *hypostases*, $P, U_1(P), U_2\big(U_1(P)\big), \dots, U_p(U_{p-1}\big(\dots(U_1(P)\dots)\big)$.

We shall call *the orbit starting from P*, defined by the transformations $\{U_\alpha$, $\alpha = 1, 2, \dots, p\}$, the set

$$\{P, U_1(P), U_2\big(U_1(P)\big), \dots, U_p(U_{p-1}\big(\dots(U_1(P)\dots)\big)\} \tag{12}$$

where P product could be the *initial real*, *generic*, etc. product.

Certain product *hypostases* are specific in the product development process, as *concept*, *prototype*, *version*, etc.

5. Outline model and case study

In order to strengthening the concepts and entities which have been introduced, an outline model and a case study are unrolled.

5.1. Outline model

Let X be the set of components,

$$X = \{x, y, z, v\} \quad (13)$$

R_1 - the relation defined on X (Fig. 1) by

$$R_1 = \{(x, x), (y, y), (x, y), (y, z)\} \quad (14)$$

and

$$\Lambda_1 = \{R_1\} \quad (15)$$

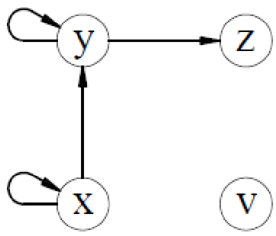


Fig. 1. Graph of R_1 relation

Consider the set of characteristics

$$K_1 = \{C_1, \ C_2, \ C_3\} \quad (16)$$

and define the map F_1,

$$F_1: X \rightarrow 2^{K_1}, \ F_1(x) = \{C_2, \ C_3\},$$
$$F_1(y) = \{C_1, \ C_3\}, \ F_1(z) = \{C_1, \ C_2\}, \ F_1(v) = \emptyset \quad (17)$$

It results the product

$$P_1 = (X, \ \Lambda_1, \ K_1, \ F_1) \quad (18)$$

One can observe that $X(a) = \{x, y, z\}$ is the set of actual components, while $X(p) = \{v\}$ is the set of potential components.

We now define a new P_2 product.

Let

$$\Lambda_2 = \{\rho_1, \rho_2\} \quad (19)$$

where ρ_1 and ρ_2 are the relations on X defined (Fig. 2, a, b) by:

$$\rho_1 = \{(x, x), (y, y), (z, z), (x, y), (y, z), (x, z)\}, \quad (20)$$

$$\rho_2 = \{(x,v),(y,v),(z,v)\}$$

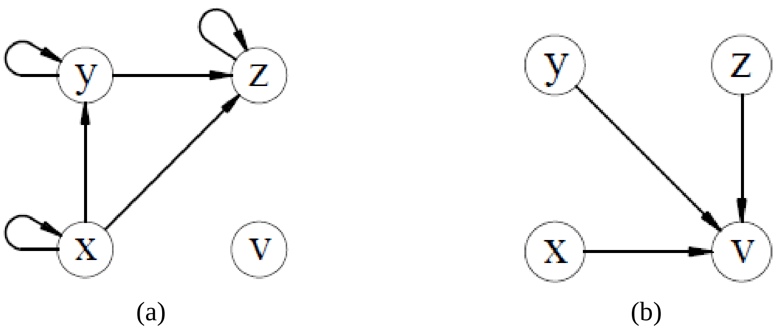


Fig. 2. Graphs of ρ_1 relation (a) and ρ_2 relation (b)

One can observe that ρ_1 is the reflexive and transitive closure of R_1 on the subset $\{x, y, z\}$.

The new set of characteristics is

$$K_2 = \{\mathrm{C}_1,\ C_2,\ C_3,\ C_4\} \tag{21}$$

and define the map F_2,

$$F_2\colon X \ \to\ 2^{K_2},\ F_2(x) = \{C_2,\ C_3\},$$
$$F_2(y) = \{C_1,\ C_3\}, F_2(z) = \{C_1,\ C_2\}, F_2(v) = \{C_4\} \tag{22}$$

The product

$$P_2 = (X,\ \Lambda_2,\ K_2,\ F_2) \tag{23}$$

is a transformation (hypostasis) of P_1, i.e.

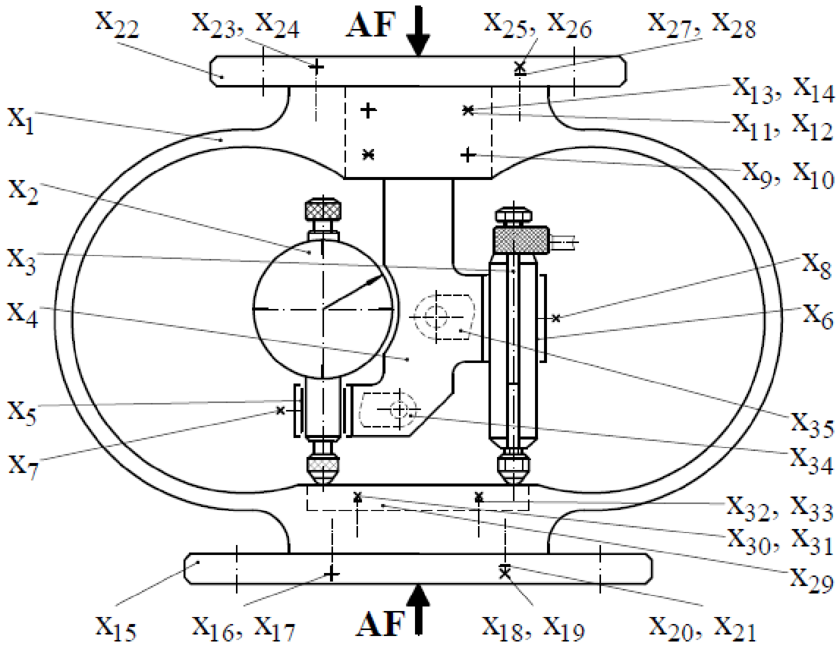
$$P_2 = U(P_1) \tag{24}$$

5.2. Case study

The case study is concerning on a *product* type of dynamometer with ellipsoidal elastic element and dial gauge or/and inductive transducer.

The considered *product* is part of an apparatus for the control of elastic elements rigidity characteristic [8].

The product *components* and general features are as presented in Fig. 3.



x_i	Type	x_i	Type
x_1	Elastic element	x_9, x_{10}; x_{16}, x_{17}; x_{23}, x_{24}	Centring bolt
x_2	Dial gauge	x_{11}, x_{12}; x_{18}, x_{19}; x_{25}, x_{26}	Fixing screw
x_3	Inductive transducer	x_{13}, x_{14}; x_{20}, x_{21}; x_{27}, x_{28}	Safety washer
x_4	Support	x_{29}	Intermediary plate
x_5, x_6	Collet	x_{30}, x_{31}	Fixing screw
x_7, x_8	Fixing screw	x_{32}, x_{33}	Safety washer
x_{15}; x_{22}	Lower plate; Upper plate	x_{34}; x_{35}	Fixing mechanism

Fig. 3. Product *components* and general features

So, the X set of the considered product components is

$$X = \{x_i;\ i = 1, 2, \ldots, 35\} \tag{25}$$

The relations and the characteristics associated to the product components are defined based on specific elements of the functional process, constructive links, technological requirements, etc. Some of these are further reflected.

x_{15} and x_{22} components are contributing to transferring the acting force, AF, to x_1, generating its elastic deformation; x_2 or/and x_3, x_4 and x_{29} are contributing to capturing and transformation of the x_1 deformation into force units. As consequences: x_4, x_{15}, x_{22} and x_{29} behave as rigid extensions of x_1; there are well defined relative contacts, positions and fixing associated to components as x_1, x_2 and x_4, x_1, x_3 and x_4, x_1 and x_{15}, etc.

A detailed work is unrolled in order to prescribe the necessary qualitative and quantitative *characteristics* to all product *components*. For instance, the proper fits are associated to specific couples, as interference fit to each of couple x_9 - x_1, x_{10} - x_1, x_{16} - x_1, etc., clearance fit to x_9 - x_4, x_{10} - x_4, x_{16} - x_{15}, etc., and threaded fit to x_7 - x_4, x_8 - x_4, x_{11} - x_1, etc. The total number of the considered product *characteristics* is of order of hundreds, among which some are according to specific norms (type of standard, subassembly documentation, etc.). Thus, only a part of product characteristics are explicit presented.

On the above grounding, the P_1, P_2 and P_3 products are as follows.

P_1 product

The Λ_1 family of *relations* is defined by set (26), taking into consideration the significance of relations based on certain elements associated to functional process and geometric contact as presented in Table 1.

Table 1

Significance of the relations $R_1, R_2, \ldots, R_{11}$

R_k	Functional process	Geometric contact
R_1	Transfer of deformation	Point
R_2	Positioning	Cylindrical
R_3	Positioning	Cylindrical
R_4	Fixing	Helicoidal
R_5	Fixing	Linear
R_6	Transfer of deformation	Plane
R_7	Positioning	Cylindrical
R_8	Positioning	Cylindrical
R_9	Fixing	Plane
R_{10}	Fixing	Helicoidal
R_{11}	Fixing	Plane

$$\Lambda_1 = \left\{ \begin{array}{l} x_2 R_1 x_1,\ x_3 R_1 x_1; \\ x_5 R_2 x_4,\ x_6 R_2 x_4;\ x_5 R_3 x_2,\ x_6 R_3 x_3; \\ x_7 R_4 x_4,\ x_8 R_4 x_4;\ x_7 R_5 x_5,\ x_8 R_5 x_6; \\ x_4 R_6 x_1; \\ x_9 R_7 x_1,\ x_{10} R_7 x_1;\ x_9 R_8 x_4,\ x_{10} R_8 x_4; \\ x_{11} R_9 x_4,\ x_{12} R_9 x_4;\ x_{11} R_{10} x_1,\ x_{12} R_{10} x_1; \\ x_{13} R_{11} x_4,\ x_{14} R_{11} x_4;\ x_{13} R_{11} x_{11},\ x_{14} R_{11} x_{12}; \\ x_{15} R_6 x_1; \\ x_{16} R_7 x_1,\ x_{17} R_7 x_1;\ x_{16} R_8 x_{15},\ x_{17} R_8 x_{15}; \\ x_{18} R_9 x_{15},\ x_{19} R_9 x_{15};\ x_{18} R_{10} x_1,\ x_{19} R_{10} x_1; \\ x_{20} R_{11} x_{15},\ x_{21} R_{11} x_{15};\ x_{20} R_{11} x_{18},\ x_{21} R_{11} x_{19}; \\ x_{22} R_6 x_1; \\ x_{23} R_7 x_1,\ x_{24} R_7 x_1;\ x_{23} R_8 x_{22},\ x_{24} R_8 x_{22}; \\ x_{25} R_9 x_{22},\ x_{26} R_9 x_{22};\ x_{25} R_{10} x_1,\ x_{26} R_{10} x_1; \\ x_{27} R_{11} x_{22},\ x_{28} R_{11} x_{22};\ x_{28} R_{11} x_{25},\ x_{28} R_{11} x_{26} \end{array} \right\} \quad (26)$$

In accord with the above, the P_1 product *structure* is $S_1 = (X, \Lambda_1)$.

The K_1 family of *characteristics* is developed. A part of its elements is presented by rel. (27) and Table 2.

$$K_1 = \{C_1, C_2, \ldots, C_{48}, \ldots, C_{N1.w}, C_{N2.w}, \ldots, C_{N8.w}, w = 1, 2, \ldots\} \quad (27)$$

Table 2

The characteristics $C_1, C_2, \ldots, C_{48}, C_{N1.w}, C_{N2.w}, \ldots, C_{N8.w}$

Significance	Numerical values
E – the Young's modulus, in Pa; H1 – hardness in HRC; H2 – hardness in HB; r – radius, t – thickness, l – length, h – height, D – interior diameter and d – exterior diameter, in mm; c x 45° - chamfer, with c in mm; Ra_0 – general roughness and Ra – roughness of contact surface, in μm; T1 – tolerance to parallelism, T2 – tolerance to perpendicularity and T3 – tolerance to coaxiality, in mm; $C_{N\upsilon.w}$ – characteristic according to the specific norm $N\upsilon$, $\upsilon = 1, 2, ..., w = 1, 2, \ldots$	C_1: E = 210*10^9, C_2: H1 = 35 ± 5, C_3: H1 = 30 ± 4, C_4: H2 = 215 ± 5; C_5: r = 40, C_6: t = 3, C_7: t = 0.5, C_8: t = 0.8, C_9: l = 170, C_{10}: l = 60, C_{11}: l = 68, C_{12}: l = 34, C_{13}: l = 14, C_{14}: l = 26, C_{15}: l = 108, C_{16}: l = 96, C_{17}: h = 108, C_{18}: h = 22, C_{19}: h = 16, C_{20}: h = 80, C_{21}: h = 21, C_{22}: h = 20, C_{23}: h = 12, C_{24}: h = 10, C_{25}: h = 45, C_{26}: h = 8; C_{27}: D = $Ø\,4^{-0.011}_{-0.023}$ (R7), C_{28}: D = $Ø\,4^{+0.012}_{0}$(H7), C_{29}: d = $Ø\,4^{0}_{-0.008}$ (h6), C_{30}: D = $Ø\,9^{+0,07}_{0}$(H10), C_{31}: d = $Ø\,9^{0}_{-0.07}$ (h10), C_{32}: D = $Ø\,9.6^{+0,07}_{0}$(H10), C_{33}: d = $9.6^{0}_{-0.07}$ (h10), C_{34}: D = $Ø\,8^{-0.013}_{-0.028}$(R7), C_{35}: D = $Ø\,8^{+0.015}_{0}$(H7), C_{36}: d = $Ø\,8^{0}_{-0.009}$ (h6); C_{37}: r = 8, C_{38}: r = 18, C_{39}: r = 3, C_{40}: 0.5 x 45°, C_{41}: 1.5 x 45°; C_{42}: Ra_0 = 3.2, C_{43}: Ra = 1.6, C_{44}: Ra = 0.4; C_{45}: T1 = 0.008, C_{46}: T1 = 0.02, C_{47}: T2 = 0.02, C_{48}: T3 = 0.04; $C_{N1.w}$, $C_{N2.w}$,…, $C_{N8.w}$, w = 1, 2, …

Through a specific map F_1, characteristics from K_1 are prescribed to the considered $x_1, x_2, ..., x_{28}$ *components* (28):

$$F_1 : X \to 2^{K_1},$$
$$F_1(x_1) = \{C_1, C_2, C_5, C_6, C_9, C_{10}, C_{17}, C_{18}, C_{19}, C_{27}, C_{34}, C_{37}, C_{40}, C_{42}, C_{43}, C_{44}, C_{45}, ...\},$$
$$F_1(x_2) = \{C_{N1.w}, w = 1, 2, \ldots\}, F_1(x_3) = \{C_{N2.w}, w = 1, 2, \ldots\},$$
$$F_1(x_4) = \{C_4, C_{11}, ..., C_{14}, C_{20}, ..., C_{23}, C_{28}, C_{30}, C_{32}, C_{38}, C_{39}, C_{40}, C_{42}, C_{43}, C_{47}, ...\},$$
$$F_1(x_5) = \{C_3, C_7, C_{24}, C_{33}, C_{42}, C_{43}, C_{48}, ...\}, F_1(x_6) = \{C_3, C_8, C_{25}, C_{33}, C_{42}, C_{43}, C_{48}, ...\},$$
$$F_1(x_7) = \{C_{N3.w}, w = 1, 2, \ldots, a\}, F_1(x_8) = \{C_{N3.w}, w = a + 1, a + 2, \ldots\},$$
$$F_1(x_9) = F_1(x_{10}) = \{C_3, C_{29}, C_{42}, C_{43}, C_{N4.w}, w = 1, 2, \ldots, b\},$$
$$F_1(x_{11}) = F_1(x_{12}) = \{C_{N5.w}, w = 1, 2, \ldots, c\}, F_1(x_{13}) = F_1(x_{14}) = \{C_{N6.w}, w = 1, 2, \ldots, e\},$$
$$F_1(x_{15}) = \{C_2, C_{15}, C_{26}, C_{35}, C_{41}, C_{42}, C_{43}, C_{46}, ...\},$$
$$F_1(x_{22}) = \{C_3, C_{16}, C_{26}, C_{35}, C_{41}, C_{42}, C_{43}, C_{46}, ...\},$$
$$F_1(x_{16}) = F_1(x_{17}) = F_1(x_{23}) = F_1(x_{24}) = \{C_3, C_{36}, C_{42}, C_{43}, C_{N4.w}, w = b + 1, b + 2, \ldots\},$$
$$F_1(x_{18}) = F_1(x_{19}) = F_1(x_{25}) = F_1(x_{26}) = \{C_{N7.w}, w = 1, 2, \ldots\},$$
$$F_1(x_{20}) = F_1(x_{21}) = F_1(x_{27}) = F_1(x_{28}) = \{C_{N8.w}, w = 1, 2, \ldots\} \quad (28)$$

Thus, the P_1 product is defined as

$$P_1 = (X, \Lambda_1, K_1, F_1) \quad (29)$$

P_1 product is a transformation (hypostasis) of the generic P product, i.e.

$$P_1 = U_1(P) \quad (30)$$

It is to note that $X(a) = \{x_i;\ i = 1, 2, \ldots, 28\}$ is the set of actual components, and $X(p) = \{x_i;\ i = 29, 30, \ldots 35\}$ is the set of potential components.

P_2 product

Let Λ_2 be defined by enlargement Λ_1 with the relations upon $x_{29}, x_{30}, \ldots, x_{33}$ components as presented by (31), taking into consideration the significance of relations presented in Table 1.

$$\Lambda_2 = \Lambda_1 \cup \left\{ \begin{array}{l} x_{29}\ R_6\ x_1;\ x_{30}\ R_9\ x_{29},\ x_{31}\ R_9\ x_{29};\ x_{30}\ R_{10}\ x_1,\ x_{31}\ R_{10}\ x_1; \\ x_{32}\ R_{11}\ x_{29},\ x_{33}\ R_{11}\ x_{29};\ x_{32}\ R_{11}\ x_{30},\ x_{33}\ R_{11}\ x_{31} \end{array} \right\} \qquad (31)$$

So, the P_2 product *structure* is $S_2 = (X, \Lambda_2)$.

Similarly, the K_2 family of *characteristics* is developed, by enlargement of K_1 with the characteristics prescribed to $x_{29}, x_{30}, \ldots, x_{33}$ components. A part of K_2 elements are presented by rel. (32) and Tables 2 and 3.

$$K_2 = \{C_1, C_2, \ldots, C_{52}, \ldots, C_{N1.w}, C_{N2.w}, \ldots, C_{N8.w},\ w = 1, 2, \ldots\} \qquad (32)$$

Table 3

The characteristics C_{49}, C_{50}, C_{51}, C_{52}

Significance	**Numerical values**
See Table 2	C_{49}: l = 50, C_{50}: h = 9, C_{51}: 1 x 45°, C_{52}: T1 = 0.005

Through a specific map F_2, characteristics from K_2 are prescribed to the considered $x_1, x_2, \ldots, x_{33}$ *components* (33).

$$\begin{array}{c} F_2 : X \rightarrow 2^{K_2},\ F_2(x_i) = F_1(x_i),\ i = 1, 2, \ldots, 28, \\ F_2(x_{29}) = \{C_2, C_{42}, C_{43}, C_{44}, C_{49}, C_{50}, C_{51}, C_{52}, \ldots\}, \\ F_2(x_{30}) = F_2(x_{31}) = \{C_{N5.w},\ w = c+1, c+2, \ldots,\}, \\ F_2(x_{32}) = F_2(x_{33}) = \{C_{N6.w},\ w = e+1, e+2, \ldots,\} \end{array} \qquad (33)$$

Thus, the P_2 product is defined as

$$P_2 = (X, \Lambda_2, K_2, F_2) \qquad (34)$$

P_2 product is a transformation (hypostasis) of P_1, i.e.

$$P_2 = U(P_1) \Leftrightarrow P_2 = U_2(U_1(P)) \qquad (35)$$

P_3 product

The new P_3 product is developed by replacing the $x_4, x_7, x_8, \ldots, x_{14}$ components with an extension of x_1 component, and by adding the x_{34}, x_{35} fixing mechanisms (Fig. 3).

Let Λ_3 be defined with respect to Λ_2, by subtracting the relations upon x_4, x_7, x_8, …, x_{14}, and adding the new relations required by x_1, x_{34}, x_{35} components, as presented by (36), taking into consideration the significance of the new relations presented in Table 4.

$$\Lambda_3 = \Lambda_2 - \left\{ \begin{matrix} x_i\ R_k\ x_j;\ i \text{ or } j = \\ 4, 7, 8, \ldots, 14 \end{matrix} \right\} \cup \left\{ \begin{matrix} x_{34}\ R_{12}\ x_1,\ \ x_{34}\ R_{13}\ x_1; \\ x_{34}\ R_{14}\ x_5;\ \ x_{35}\ R_{12}\ x_1, \\ x_{35}\ R_{13}\ x_1;\ \ x_{35}\ R_{14}\ x_5 \end{matrix} \right\} \tag{36}$$

Table 4

Significance of the relations R_{12}, R_{13}, R_{14}

R_k	Functional process	Geometric contact
R_{12}	Positioning	Plane - cylindrical
R_{13}	Fixing	Helicoidal
R_{14}	Fixing	Linear

So, the P_3 product *structure* is $S_3 = (X, \Lambda_3)$.

Similarly, the K_3 family of *characteristics* is developed with respect to K_2, by subtracting the characteristics prescribed to x_4, x_7, x_8, …, x_{14} components, and adding the required characteristic to x_1, x_{34}, x_{35} components. A part of K_3 elements are presented by rel. (37) and Tables 2, 3 and 5.

$$\begin{aligned} K_2 = \{&C_1, C_2, C_3, C_5, \ldots, C_{10}, C_{15}, C_{16}, C_{19}, C_{24}, \ldots, C_{27}, C_{30}, \ldots, C_{52}, \ldots; \\ &C_{N1.w}, C_{N2.w}, C_{N4.w}, w = 1, 2, \ldots; C_{N5.w}, w = c + 1, c + 2, \ldots; \\ &C_{N6.w}, w = e + 1, e + 2, \ldots; C_{N7.w}, C_{N8.w}, w = 1, 2, \ldots\} \end{aligned} \tag{37}$$

Table 5

The characteristics $C_{N9.w}$ and $C_{N10.w}$

Significance	**Numerical values**
See Table 2	$C_{N9.w}$, w = 1, 2, …; $C_{N10.w}$, w = 1, 2, …

Through a specific map F_3, *characteristics* from K_3 are prescribed to the considered x_1, x_2, ..., x_{33} *components* (38).

$$\begin{gathered} F_3 : X \rightarrow 2^{K_3},\ F_3(x_1) = F_2(x_1) \cup \{C_{30}, C_{32}, C_{38}, C_{39}, C_{47}\}, \\ F_3(x_i) = F_2(x_i),\ \text{i} = 2, 3, 5, 6, 15, 16, \ldots, 33, \\ F_3(x_{34}) = \{C_{N9.w}, w = 1, 2, \ldots\},\ F_3(x_{35}) = \{C_{N10.w}, w = 1, 2, \ldots\} \end{gathered} \tag{38}$$

Thus, the P_3 product is defined as

$$P_3 = (X, \Lambda_3, K_3, F_3) \tag{39}$$

P_3 product is a transformation (hypostasis) of P_2, i.e.

$$P_3 = U(P_2) \Leftrightarrow P_3 = U_3(U_2(U_1(P))) \tag{40}$$

6. Conclusions

A mathematical modelling on product representations has been developed. This includes a conceptual frame and reference elements type of component, relation, structure, characteristic, cardinality, hypostasis.

The unrolled outline model and case study exemplify the main elements of the mathematical modelling which has been accomplished.

The achieved mathematical modelling is a basis for the future developments.

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