REVERSIBILITY AND IRREVERSIBILITY IN PHYSICAL SYSTEMS

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Se propune o abordare sistematică a reversibilităţii şi ireversibilităţii temporale în sistemele fizice. Din acest punct de vedere sunt apoi analizate, pe baza unui formalism simplu, unele mărimi şi legi de evoluţie din diferite domenii ale fizicii (mecanică clasică şi cuantică, electrodinamică, etc.).

This paper deals with a systematic approach to the study of temporal reversibility and irreversibility in physical systems. Using a simple formalism, several physical quantities and kinetic laws are then checked in some domains (classical and quantum mechanics, electrodynamics, et al.) through this point of view.

Keywords: reversibility, irreversibility, time - reversal operator, symmetrical and anti-symmetrical quantity

1. Introduction

There are many papers linked to the problem of reversibility and irreversibility in physical systems [2, 4, 6]. Generally this problem is connected to the behavior of a kinetic equation to the time - reversal operation $t \to -t$. A reversible evolution is proved by the invariance of a kinetic equation in respect with the time reversal operation.

One considers that mechanical kinetics are reversible, while the statistical evolutions are irreversible, although some irreversible features are involved in the last case, too [2, 5].

Another distinction may be amphasized between classical and quantum systems. In quantum systems we must distinguish between a deterministic - reversible behavior (described by the Schrödinger equation) and a probabilistic - irreversible kinetics. The quantum irreversibility is due to the measuring process which converts into a statistical mixture of states, the initial coherent state. A number of theories and models have been developed to prove several selection rules which forbids a coherent superposition of some macroscopically states [1, 3]. The main purpose of these "dechoerence models" is to explain the passing from a microscopic reversible kinetics to a macroscopic irreversible behavior [3, 7].

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This paper proposes a systematic path to establish the feature of a kinetic law, based on a time-reversal operator. Some particular situations are than investigated.

2. The Time - Reversal Operator in Classical Physics

Let it $A = A(q, p, t)$ a physical quantity depending only by the spatial coordonates (q), generalized moment a (p) and time (t). We shall define now the time - reversal operator $\hat{R}$, by

$$\hat{R} \equiv \{q \to q' = q, p \to p' = -p, t \to t' = -t\}. \tag{1}$$

Some properties of this operator are obviously accordingly (1):

$$\begin{aligned} &\hat{R}C = C \quad (C = ct.) \\ &\hat{R}(CA) = C\hat{R}A \\ &\hat{R}(A_1 + A_2) = \hat{R}A_1 + \hat{R}A_2 \\ &\hat{R}(A_1 A_2) = \hat{R}A_1 \hat{R}A_2 . \end{aligned} \tag{2}$$

The $\hat{R}$ operator is therefore a linear one. The eigenvalue equation for $\hat{R}$ is then

$$\hat{R}A = aA .$$

Obviously, by applying once more the $\hat{R}$ operator we return to the initial state.

$$\hat{R}^2 A = \hat{R}(aA) = a\hat{R}A = a^2 A = A ,$$

therefore $a = \pm 1$. If $\hat{R}A = A$, we shall denote A as a symmetrical quantity; if $\hat{R}A = -A$, we shall call A an antisymmetrical quantity (by reference to the time reversal operator).

More generally, let it be now;

$$A = A(A_1, A_2, ..., A_n, t)$$

a physical law, with $A_1, A_2, ..., A_n$ physical quantities depending on q, p and *t*. If

$$\hat{R}A = A'\left(\hat{R}A_1, \hat{R}A_2, ..., \hat{R}A_n, t\right) = A$$

we shall call *A* a symmetrical function. If $\hat{R}A = -A$, the *A* is antisimetrical.

For example, in the classical mechanics, according to the time uniformity and to the space isotropy, the Lagrangian $\mathsf{L}(p, q)$ and the Hamiltonian $\mathsf{H}(p, q)$ must be symmetrical. It follows that the generalized force $F_i = -\frac{\partial L}{\partial q_i}$ is symmetrical. In particular, the symmetry of the Lorentz force $\vec{F} = e(\vec{v} \times \vec{B})$ implies $\hat{R}\vec{B} = -\vec{B}$, therefore the induction $\vec{B}$ is an antisymmetrical quantity. In the electrical force $\vec{F} = e\vec{E}$, the electrical vector $\vec{E}$ must be symmetrical too.

3. Reversibility in Physical Kinetic Laws

We shall refer now to a physical kinetic law described by the equation*:

$$\sum_{i=0}^{n} a_i(q,t)\frac{\partial^i A}{\partial t^i} + \sum_{p=0}^{k} b_p(q,t)\frac{\partial^p A}{\partial q^p} + c(q,t) = 0 . \qquad (3)$$

We consider that such an equation is general enough to cover the most cases of physical interest.

By applying the $\hat{R}$ operator, we find

$$(-1)^i \sum_{i=0}^{n} a_i(q,-t)\frac{\partial^i A^*}{\partial t^i} + \sum_{p=0}^{k} b_p(q,-t)\frac{\partial^p A^*}{\partial q^p} + c(q,-t) = 0 , \qquad (4)$$

where $A^* = \hat{R}A$.

Two situations occur:

a) $A^* = A$ i.e. A is symmetrical. In this case the equations (3) and (4) are identical in the following situations:

α) i even and a_i, b_p, c symmetrical;

β) i odd; b_p, c symmetrical and c antisymmetrical;

γ) i odd; a_i symmetrical, b_p and c antisymmetrical.

b) $A^* = -A$, i.e. A is antisymmetrical. In this case (4) leads to

$$(-1)^{i+1} \sum_{i=0}^{n} a_i(q,-t)\frac{\partial^i A}{\partial t^i} - \sum_{p=0}^{k} b_p(q,-t)\frac{\partial^p A}{\partial q^p} + c(q,-t) = 0 . \qquad (5)$$

Equations (3) and (5) are the same if:

α) i even; a_i, b_p antisymmetrical an c symmetrical;

β) i even; a_i, b_p symmetrical an c antisymmetrical;

γ) i odd; a_i, c symmetrical, b_p antisymmetrical.

If one of these situations is fulfilled, the physical evolution described by equation (3) is named reversible. In the other situations, the evolution is irreversible.

We can therefore predict the nature of a physical equation solution, without solving the equation. The existence of a reversible evolution suggests a conservation law. Each conservation law implies a reversible evolution, but the reciprocal is not generally true. When A is conserved, it means that A is

* For simplification, the p dependence of A was omitted.

symmetrical, indeed. The reversibility do not include always a conservation law, but can suggest such a conservation.

For illustration, we checked through the agency of this criterion some physical laws.

In classical mechanics, the laws are reversible when a dependence of odd powers of the velocity do not occur*. For example, the Newton, Lagrange or Hamilton equations are reversible. The wave equation in isotropic mediums is reversible, too. All these equations are of a α type.

In relativity, we shall observe that the interval between two events is a symmetrical one. Also the Lorentz transformations are invariant to the time reversal operator, so they are reversible.

In the mechanics of continuous media, the Navier - Stokes equation is irreversible, while the Lamé or Euler equations are reversible. The conservations laws are always reversible, but the laws of transport phenomena are irreversible.

In electrodynamics, some laws (for example the magnetic flux law or the Gauss law) are reversible, while orders are irreversible (like Biot - Savart or Faraday laws).

4. Reversibility in Quantum Mechanics

In quantum mechanics, the problem of time reversibility requires a separate approach since the continuous or discontinuous evolutions are involved.

The continuous evolutions are described by Schrödinger equation (in the spin - coordonates representation):

$$i\hbar\frac{\partial\psi(q,s,t)}{\partial t} = \hat{\mathsf{H}}(\hat{q},\hat{p},\hat{s})\psi(q,s,t), \qquad (6)$$

with $\hat{q},\hat{p},\hat{s}$ operators corresponding to q,p,s. We shall now define the time-reversal operator** R:

$$\hat{R} = \{q \to q' = q, p \to p' = -p, s \to s' = -s, t \to t' = -t\}.$$

In the absence of an external magnetic field, the Hamiltonian $\hat{\mathsf{H}}$ is real and symmetric:

$$\hat{R}\ \hat{\mathsf{H}}(\hat{q},\hat{p},\hat{s}) = \hat{\mathsf{H}}(\hat{q}',-\hat{p}',-\hat{s}') = \hat{\mathsf{H}}(\hat{q},\hat{p},\hat{s}).$$

The Schrödinger equation is therefore irreversible the wave function being symmetrical. It follows that:

$$P(q,p,s,t) = P(q,-p,-s,-t)$$

namely the probability to find the quantum system in the q,p,s state at t time is the same with the probability to find the system in $q,-p,-s$ state, at $-t$ time.

* Where such a dependence appears, the evolution is irreversible.

** A similar transformation occurs for $\hat{q}$, $\hat{p}$, $\hat{s}$, i.e. $\hat{q} \to \hat{q}' = \hat{q}$; $\hat{p} \to \hat{p}' = -\hat{p}$; $\hat{s} \to \hat{s}' = -\hat{s}$.

For the discontinuous evolutions (transitions) induced by simultaneous action of a small perturbation and a measuring apparatus, there exists in the first degree of nonstationary theory of perturbation:

$$\Pi(\alpha \to \beta) = \Pi(\beta \to \alpha)$$

named sometimes microversibility formula [5]. Here $\Pi(\alpha \to \beta)$ is the transition probability from state α (described by ψ_α at t_α time) to a state β (described by ψ_β at t_β time):

$$\Pi(\alpha \to \beta) = \Pi(q_\alpha, p_\alpha, t_\alpha, q_\beta, p_\beta, t_\beta).$$

As the probability transitions do not depend explicitly to *t*, it results that Π is symmetrical.

5. Statistical Irreversibility

The evolutions of real macroscopic systems are always irreversible. Generally, the statistical description of a macroscopic system can be performed in two ways: microstatistical and macrostatistical.

The microstatistical description utilizes as independent variables the q_i, p_i coordonates and moment of all particles. The macrostatistical needs a smaller number of variables (macrovariables) $\alpha_1, \alpha_2, ..., \alpha_n$ obtained by a coarse graining average in the phase space of some microscopic correspondend values [2, 5]. Since the coarse graining average do not carry of the statistical feature of the $\alpha \equiv \{\alpha_1, \alpha_2, ..., \alpha_n\}$ variables, we can introduce for α a distribution probability $P(\alpha, t)$ and also a transition probability $W(\alpha, t \mid \alpha', t')$.

For markovian systems, *P* verifies the master equation [4]:

$$\frac{\partial P(\alpha, t)}{\partial t} = \int [P(\alpha', t) w(\alpha \mid \alpha') - P(\alpha, t) w(\alpha' \mid \alpha)] \mathrm{d}\,\alpha', \qquad (7)$$

where

$$w(\alpha' \mid \alpha) = \lim_{t' \to t} \frac{W(\alpha, t \mid \alpha', t')}{t' - t}$$

is the transition probability per unit of time.

Using the Kramers - Moyal expansion it is possible to transpose the master equation into a purely differential equation (the Fokker - Planck generalized equation) [2, 5]:

$$\frac{\partial P(\alpha, t)}{\partial t} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \frac{\partial^n}{\partial \alpha^n} \left[P(\alpha, t) \left\langle (\Delta\alpha)^n \right\rangle \right] \qquad (8)$$

with

$$\left\langle (\Delta\alpha)^n \right\rangle \equiv \int (\Delta\alpha)^n w(\alpha + \Delta\alpha \mid \alpha)\, \mathrm{d}(\Delta\alpha).$$

In the second order of approximation, equation (8) leads to the main Fokker - Planck equation:

$$\frac{\partial P(\alpha,t)}{\partial t} = -\frac{\partial}{\partial \alpha}\left[P(\alpha,t)\langle \Delta\alpha \rangle\right] + \frac{1}{2}\frac{\partial^2}{\partial \alpha^2}\left[P(\alpha,t)\left\langle (\Delta\alpha)^2 \right\rangle\right]. \qquad (9)$$

We checked the behavior of these equations to the time reversal operator:

$$R = \{\alpha \to \alpha' = \alpha, t \to t' = -t\}.$$

Since $w(\alpha' \mid \alpha)$ is time independent, it follows that $\hat{R}w = w$. The probability $P(\alpha,t)$ must be also symmetrical $\left(\hat{R}P = P\right)$ because a negative probability has no sense. Therefore (8) and (9) are reversible. This result is in agreement with other authors who studied the same problem [2, 5].

Although the equation for $P(\alpha,t)$ is reversible, the evolution of the observed macrovariables $A(t)$ is irreversible in a macroscopic system. If we put

$$A(t) = \langle \alpha(t) \rangle = \int \alpha P(\alpha,t)$$

one leads to a reversible law for $A(t)$. In our opinion, this contradiction shows that the formula $A(t) = \langle \alpha(t) \rangle$ is not generally true. We can put $A(t) = \langle \alpha(t) \rangle$ only at equilibrium, in stationary case, or in a slowly relaxation processes. In the other situations the ergodicity of the systems is not fulfilled and the pass from macrostatistical variables to observed macrovariables, involves perhaps an additional time average. This question nevertheless needs a different approach.

6. Conclusions

1. A time - reversal operator $\hat{R}$ is introduced to check up the behavior of physical quantities and kinetic laws. With regard to this operator a physical quantity can be symmetrical or antisymmetrical while a kinetic law can be reversible as irreversible.

2. A reversible law suggests the existence of a conservation law.

3. Based on the proposed kinetic law (3), the reversibility of some individual physical systems is investigated.

4. In macrostatistical investigation of the macroscopic systems, a contradiction between the reversible character of the probability evolution and the irreversible character of the evolution of macrovariables occur. This suggests that, generally, the macrovariables are not purely statistical averages of some corresponding fluctuating quantities.

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