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A SOLUTION FOR GUIDANCE EQUATIONS IN THE PLANAR CASE OF HOMING MISSILES BASED ON SERIES DEVELOPMENT

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Lucrarea este o abordare originală, bazată pe dezvoltarea în serie, a problemei ecuaţiilor de dirijare a rachetei autodirijate. Scopul acestui demers este dublu. Primul este de natura tehnica, pentru a obţine o soluţie aproximativă care să poată fi utilizată pentru modelele in timp real instalate în sistemul de armament al avioanelor de luptă. Al doilea scop este ştiinţific şi anume acela de a utiliza rezultatele în alte dezvoltări teoretice legate de sinteza legilor de dirijare. Lucrarea este focalizată pe obţinerea soluţiei ecuaţiei de dirijare şi evaluarea acurateţei in funcţie de numărul de termeni ai seriei iar în final pentru a reliefa influenţa diferiţilor parametri ai problemei asupra distanţei de ratare şi a vitezei acesteia.

The paper is an original approach to the guidance equations of homing missile problem based on series development. The purpose of this approach is double. The first is a technical one; to obtain an approximate solution which can be used for real time models employed in combat system of fight aircraft. The second purpose is a scientific one to use the results in other theoretical development in guidance low synthesis. The work is focused on obtaining the equation solution and evaluating the accuracy of the solution for different numbers of terms and finally to relieve the influence of different parameters of the problem for the miss distance and its speed.

Keywords: guidance, series development, homing missile

1. Nomenclature

The main symbols utilized in this paper, according to standard [17], are the following (see also Fig. 1):

M - Missile;
T - Target;
MT - Line of sight (LOS);
$\overline{R}$ - Range;
σ_y - Absolute angle of the line-of-sight;
γ_M - Missile climb angle;

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γ_T - Target climb angle;
μ_M - Missile aspect angle;
μ_T - Target aspect angle;
$\overline{V}_M$ - Missile velocity;
u_M - Missile velocity component along the LOS;
w_M - Missile velocity component along the z_T axis;
$\overline{a}_M$ - Missile acceleration;
a_{xM} - Missile acceleration component along the LOS;
a_{zM} - Missile acceleration component along the z_T axis;
$\overline{V}_T$ - Target velocity;
u_T - Target velocity component along the LOS;
w_T - Target velocity component along the z_T axis;

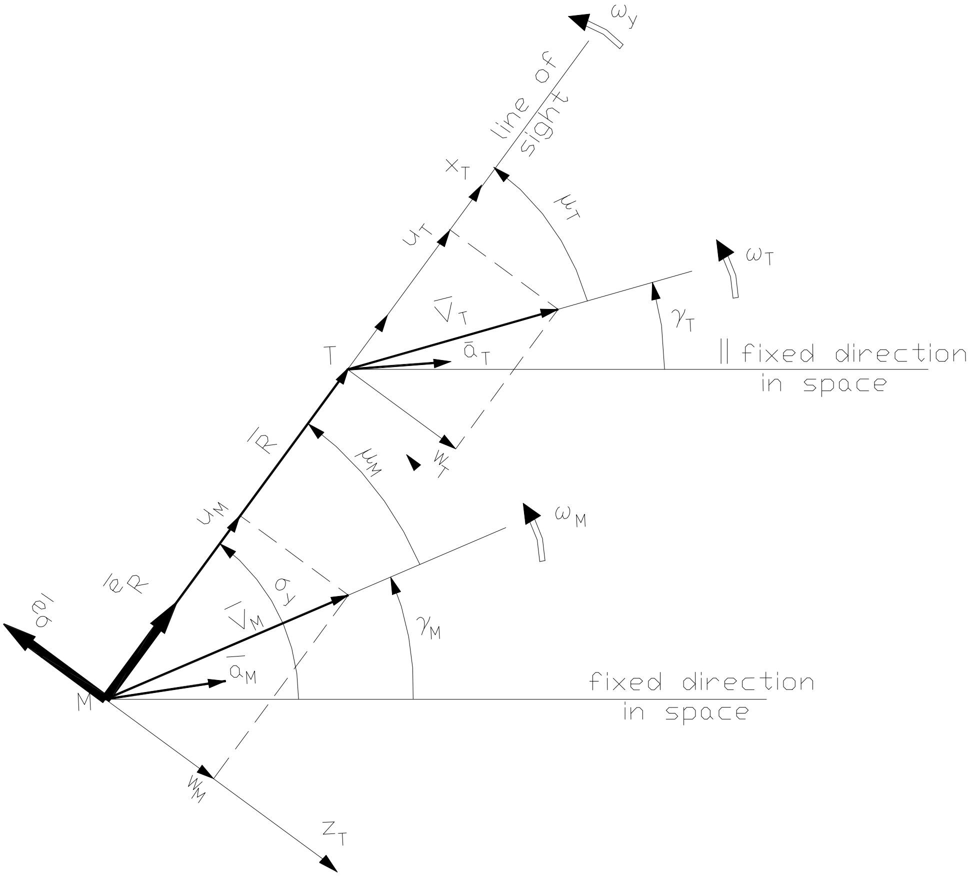

Fig. 1 Two - points guidance kinematics

$\overline{a}_T$ - Target acceleration;
a_{xT} - Target acceleration component along the LOS;
a_{zT} - Target acceleration component along the z_T axis;
ω_y - Angular velocity of LOS;
ω_M - Missile angular rate of the velocity vector;
ω_T - Target angular rate of the velocity vector;
K - Proportional navigation constant;
k - Modified navigation constant;
t_f - Time to hit the target.
t_{go} - Time to go

2. Introduction

In the dynamic flight of guided missiles domain the study of performances is particularly important because the problem has connections with the development programs for new weapon complexes and especially for missile complexes.

Generally speaking, the guided missiles systems are characterized by the technical features and tactical performances.

- The technical features have a global character because they take into consideration the complete spectrum of the missile utilization. These contain a series of specific elements such as: the type of the configuration, aerodynamic characteristics, gazodynamic characteristics, mass characteristics, flying quality parameters.

- The tactical performances particularize the technical characteristics of the missile for a specific group of targets and launch conditions and thus define how the missile is used in combat.

Following the authors in [7], tactical performances are:

- Dynamic Launch Zone (DLZ) for a given tactical situation, corresponding to a particular type of missile, integrated in a terrestrial, aerial or naval weapon complex.

- Guidance precision, hit target error and killing probability.

The problem of determining the guidance precision in general is difficult because it involves a sophisticated mathematical model, based on knowledge in the field of equations and functions with random variables. In addition, in the case of homing missile, these difficulties overlap those generated by the fact that the kinematic equations of guidance are unsteady, which requires a specific approach and an appropriate resolution in this class of problems.

In this context, we propose a method to find a solution based on the series development of the guidance differential equation in deterministic variables, for the case of two-points guidance (missile and target) in a plane. The solution ultimately allows assessment of the miss distance as an average value.

3. Problem formulation

In order to develop the guidance equations, in contrast to the three-dimensional method of constructing the kinematic equations of guidance, presented in [5], in most papers [4], [7] a planar method is used. This method is presented below.

Thus, for writing the equations for the absolute angle and the absolute angular velocity in the guidance plane[3] a polar coordinate system (R, σ_y), shown in fig. 1, is considered.

The position vector $\overline{R}$ defines the target position related to the missile and obviously coincides with the line of sight MT. The angle σ_y defines the position of the $\overline{R}$ vector (of the line of sight MT) in the guidance plane with respect to a fixed direction in space.

By absolute derivation of the unit vectors $\overline{e}_R$ and $\overline{e}_\sigma$ it can be shown that:

$$\dot{\overline{e}}_R = \omega_y \overline{e}_\sigma; \quad \dot{\overline{e}}_\sigma = -\omega_y \overline{e}_R , \tag{1}$$

with the denotation:

$$\omega_y = \dot{\sigma}_y . \tag{2}$$

Starting from the expression of the position vector:

$$\overline{R} = R\overline{e}_R , \tag{3}$$

deriving twice and taking into account relations (1) we obtain the relative velocity of *T* in respect to *M*, into an absolute referential:

$$\dot{\overline{R}} = \dot{R}\overline{e}_R + \omega_y R\overline{e}_\sigma \tag{4}$$

and yet the relative acceleration of *T* in respect to *M*:

$$\ddot{\overline{R}} = (\ddot{R} - \omega_y^2 R)\overline{e}_R + (\dot{\omega}_y R + 2\omega_y \dot{R})\overline{e}_\sigma . \tag{5}$$

We develop thereafter the left member in relations (4), (5), on the basis of motion elements of the missile and target.

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[3] Military Standard [17] define two guidance planes: the first plane $Mx_T z_T$ and the second plane $Mx_T y_T$;

The position vector derivatives can be expressed as the differences of the absolute velocities and accelerations of the missile and the target:

$$\dot{\overline{R}} = \overline{V}_T - \overline{V}_M \ , \tag{6}$$

and

$$\ddot{\overline{R}} = \overline{a}_T - \overline{a}_M \ . \tag{7}$$

Projecting (4) and (5) on the line of sight and on its normal and expressing the velocity and acceleration using the components from the coordinate system of the line of sight (fig. 1) we obtain the following scalar relations:

$$\dot{R} = u_T - u_M \, ; \tag{8}$$

$$\dot{\sigma}_y R = w_M - w_T \, , \tag{9}$$

and

$$\ddot{R} - \omega_y^2 R = a_{xT} - a_{xM} \, ; \tag{10}$$

$$\dot{\omega}_y R + 2\dot{R}\omega_y = a_{zM} - a_{zT} \, . \tag{11}$$

If we express the velocities and accelerations through the components related to velocity vectors of the missile and target the previous relations become:

$$\dot{R} = V_T \cos\mu_T - V_M \cos\mu_M \, ; \tag{12}$$

$$\dot{\sigma}_y R = V_M \sin\mu_M - V_T \sin\mu_T \, , \tag{13}$$

and

$$\ddot{R} - \omega_y^2 R = \dot{V}_T \cos\mu_T - \dot{V}_M \cos\mu_M + \omega_T V_T \sin\mu_T - \omega_M V_M \sin\mu_M \, ; \tag{14}$$

$$\dot{\omega}_y R + 2\dot{R}\omega_y = \omega_T V_T \cos\mu_T - \omega_M V_M \cos\mu_M - \dot{V}_T \sin\mu_T + \dot{V}_M \sin\mu_M \, . \tag{15}$$

The relations (12), (13), (14), (15) constitute the nonlinear form of the planar two - points guidance kinematics equations.

To obtain the linear form of the equation for the angular velocity of the line of sight in the first guidance plane, considering that the basic motion is the collision trajectory, we can start with relation (15), which, through development becomes:

$$\begin{aligned} R\Delta\dot{\omega}_y + 2\dot{R}\Delta\omega_y &= V_T \cos\mu_T \Delta\omega_T - V_M \cos\mu_M \Delta\omega_M - (\dot{V}_T \cos\mu_T + \\ &+ V_T\omega_T \sin\mu_T)\Delta\mu_T + (\dot{V}_M \cos\mu_M + V_M\omega_M \sin\mu_M)\Delta\mu_M \, . \end{aligned} \tag{16}$$

Taking into account the relations between angles (fig. 1):

$$\Delta\mu_M = \Delta\sigma_y - \Delta\gamma_M ; \quad \Delta\mu_T = \Delta\sigma_y - \Delta\gamma_T \, . \tag{17}$$

from (16) results:

$$R\Delta\dot{\omega}_y + 2\dot{R}\Delta\omega_y + (\ddot{R} - \omega_y^2 R)\Delta\sigma_y = u_T\Delta\omega_T - u_M\Delta\omega_M + a_{xT}\Delta\gamma_T - a_{xM}\Delta\gamma_M \, . \tag{18}$$

Considering a stationary base motion in which the missile and the target have an evolution with constant velocity modulus the terms $\ddot{R}-\omega_y^2 R$, the derivative of the velocity modulus $\dot{V}_T$; $\dot{V}_M$ and terms containing products of small parameters can be neglect, thus relation (18) becomes:

$$R\Delta\dot{\omega}_y + 2\dot{R}\Delta\omega_y = u_T\Delta\omega_T - u_M\Delta\omega_M \ . \tag{19}$$

If we add the linear form of the proportional navigation law and consider the first order approximation for missile dynamic:

$$\Delta\omega_M = \frac{K}{1+\tau_1 s}\Delta\omega_y , \tag{20}$$

we obtain the equation:

$$R\tau_1\Delta\ddot{\omega}_y + (3\tau_1\dot{R} + R)\Delta\dot{\omega}_y + (2\dot{R} + 2\tau_1\ddot{R} + u_M K)\Delta\omega_y = u_T(1+\tau_1 s)\Delta\omega_T \tag{21}$$

Furthermore, if we accept that the two mobile velocities are constant, it can be written:

$$R = -(t_f - t)\dot{R}; \quad \dot{R} = -u_c; \ k = \frac{Ku_M}{u_c}. \tag{22}$$

From relation (21) we obtain:

$$(t_f - t)\tau_1\Delta\ddot{\omega}_y + (t_f - t - 3\tau_1)\Delta\dot{\omega}_y + (k-2)\Delta\omega_y = \frac{u_T}{u_c}\Delta\omega_T . \tag{23}$$

Introducing a new argument:

$$\tau = 1 - \frac{t}{t_f}, \tag{24}$$

with the de notation:

$$\bar{\tau} = \frac{\tau_1}{t_f} \tag{25}$$

equation (23) becomes:

$$\tau\bar{\tau}\frac{\mathrm{d}^2\Delta\omega_y}{\mathrm{d}\tau^2} - (\tau - 3\bar{\tau})\frac{\mathrm{d}\Delta\omega_y}{\mathrm{d}\tau} + (k-2)\Delta\omega_y = \frac{u_T}{u_c}\Delta\omega_T . \tag{26}$$

If we define the miss distance:

$$\Delta h = R(t_f - t)\Delta\omega_y = u_c t_f^2\tau^2\Delta\omega_y , \tag{27}$$

we can express the angular velocity derivatives of the line of sight in terms of miss distance:

$$\Delta\omega_y = \frac{\Delta h}{u_c t_f^2\tau^2}; \frac{\mathrm{d}\Delta\omega_y}{\mathrm{d}\tau} = \frac{1}{u_c t_f^2\tau^2}\left(\frac{\mathrm{d}\Delta h}{\mathrm{d}\tau} - \frac{2\Delta h}{\tau}\right);$$

$$\frac{\mathrm{d}^2\Delta\omega_y}{\mathrm{d}\tau^2}=\frac{1}{u_c t_f^2\tau^2}\left(\frac{\mathrm{d}^2\Delta h}{\mathrm{d}\tau^2}-\frac{4}{\tau}\frac{\mathrm{d}\Delta h}{\mathrm{d}\tau}+\frac{6\Delta h}{\tau^2}\right). \tag{28}$$

Replacing (28) in relation (26) it results:

$$\tau\overline{\tau}\frac{\mathrm{d}^2\Delta h}{\mathrm{d}\tau^2}-\left(\tau+\overline{\tau}\right)\frac{\mathrm{d}\Delta h}{\mathrm{d}\tau}+k\Delta h=u_T t_f^2\tau^2\omega_T\ . \tag{29}$$

or in a compact form:

$$\overline{\tau}\tau\,\Delta h''-\left(\tau+\overline{\tau}\right)\Delta h'+k\Delta\, h=c_T\tau^2 \tag{30}$$

where:

$$c_T=u_T t_f^{\;2}\omega_T \tag{31}$$

If in this equation the response time of the system $(\tau_1=0)$ is neglected, it results:

$$\frac{\mathrm{d}\Delta h}{\mathrm{d}\tau}-\frac{k\Delta h}{\tau}=-u_T t_f^2\tau\Delta\omega_T\,, \tag{32}$$

and the solution can be written directly:

$$\Delta h=e^{\int_1^\tau\frac{k}{\tau}\mathrm{d}\tau}\left(-\Delta\omega_T u_T t_f^2\int_1^\tau\tau e^{\int_1^\tau\frac{k}{\tau}\mathrm{d}\tau}\,\mathrm{d}\tau+\Delta h_0\right). \tag{33}$$

If we take into account that: $e^{\int_1^\tau\frac{k}{\tau}\mathrm{d}\tau}=\tau^k$ and that $\Delta h_0=-\widetilde{u}_T t_f^2\Delta\omega_{y0}\cong u_T t_f\Delta\gamma_T$, the solution can be brought to the form:

$$\Delta h=u_T t_f^2\tau^2\left[\frac{\Delta\omega_T}{k-2}(1-\tau^{k-2})+\frac{\Delta\gamma_T}{t_f}\tau^{k-2}\right], \tag{34}$$

where $\Delta\gamma_T$ is due to the initial launch error.

We will continue dealing with the case where the response time of the missile is not neglected in equation of the miss distance (30) and a solution based on series development will be searched.

To this purpose, as recommended in [12], we seek a solution of the form for the equation (30):

$$\Delta h(\tau)=c_1 h_1(\tau)+c_2 h_2(\tau)+h_p \tag{35}$$

Where $c_1 h_1(\tau)+c_2 h_2(\tau)$ represents the solution of the homogeneous equation in which c_1 and c_2 are constant of integration and h_p represents the particular solution.

4. The general solution of the homogeneous equation

Next we intend to determine the two terms of the homogeneous solution separately. As recommended in [12], we will search the first term of the homogeneous solution of the form:

$$h_1(\tau) = \tau^{\delta}\sum_{i=0}^{\infty} k_i \tau^i = \sum_{i=0}^{\infty} k_i \tau^{\delta+i} \tag{36}$$

where k_i and the exponent δ are constants and will be determined below.

Deriving (36) with respect to τ we obtain:

$$h_1'(\tau) = \sum_{i=0}^{\infty}(\delta+i)k_i\,\tau^{\,\delta+i-1} \tag{37}$$

$$h_1''(\tau) = \sum_{i=0}^{\infty}(\delta+i)(\delta+i-1)k_i\tau^{\,\delta+i-2} \tag{38}$$

Substituting (36), (37) and (38) in equation (30) it results that:

$$\bar{\tau}\sum_{i=0}^{\infty}(\delta+i)(\delta+i-1)k_i\tau^{\,\delta+i-1} - (\tau+\bar{\tau})\sum_{i=0}^{\infty}(\delta+i)\,k_i\tau^{\,\delta+i-1} + k\sum_{i=0}^{\infty}k_i\tau^{\,\delta+i} = 0 \tag{39}$$

Processing this equation it follows:

$$\sum_{i=0}^{\infty}\bar{\tau}k_i(\delta+i)(\delta+i-2)\,\tau^{\delta+i-1} + \sum_{i=0}^{\infty}k_i(k-\delta-i)\,\tau^{\,\delta+i} = 0 \tag{40}$$

or

$$\bar{\tau}k_0\,\delta(\delta-2) + \sum_{i=0}^{\infty}\left[\bar{\tau}\,k_{i+1}(\delta+i+1)(\delta+i-1) + k_i(k-\delta-i)\right]\tau^{\,\delta+i} = 0 \tag{41}$$

This relation is true at any value of τ if δ is 0 or 2.

For the term h_1 of the solution we consider $\delta = 2$, and for h_2 , $\delta = 0$. For $\delta = 2$, the second term of the relation (41) becomes:

$$\bar{\tau}\,k_{i+1}(i+3)(i+1) - k_i(i+2-k) = 0 \tag{42}$$

From this we can obtain a recurrence relationship in order to generate the k_i constants:

$$k_{i+1} = \frac{(i+2-k)}{\bar{\tau}(i+1)(i+3)}k_i \tag{43}$$

If we consider the first constant $k_0 = 1$, the others can be obtained from (43):

$$k_1 = \frac{2-k}{3\bar{\tau}},\ k_2 = \frac{(2-k)(3-k)}{24\,\bar{\tau}^2},\ k_3 = \frac{(2-k)(3-k)(4-k)}{360\bar{\tau}^3},$$
$$k_4 = \frac{(2-k)(3-k)(4-k)(5-k)}{8460\bar{\tau}^4} \tag{44}$$

To summarize the results obtained so far, the development in a series of the first-term of the solution of the homogeneous equation is:

$$h_1(\tau)=\tau^2\left\{1+\frac{2-k}{3}\left(\frac{\tau}{\bar{\tau}}\right)+\frac{(2-k)(3-k)}{24}\left(\frac{\tau}{\bar{\tau}}\right)^2+\frac{(2-k)(3-k)(4-k)}{360}\left(\frac{\tau}{\bar{\tau}}\right)^3+\ldots\right.$$
$$\left.+\frac{(2-k)(3-k)(4-k)(5-k)}{8640}\left(\frac{\tau}{\bar{\tau}}\right)^4+\cdots\right\} \tag{45}$$

For the second term of the homogeneous solution, as described in [12], we will search a development of the form:

$$h_2(\tau)=h_1(\tau)\ln\tau+\sum_{i=0}^{\infty}l_i\tau^i \tag{46}$$

Similar to the first term, by derivation, we get:

$$h_2'(\tau)=h_1'\ln\tau+h_1\frac{1}{\tau}+\sum_{i=0}^{\infty}i\,l_i\,\tau^{i-1} \tag{47}$$

$$h_2''(\tau)=h_1''\ln\tau+2h_1'\frac{1}{\tau}-h_1\frac{1}{\tau^2}+\sum_{i=0}^{\infty}i\,(i-1)l_i\,\tau^{i-2} \tag{48}$$

Substituting (46), (47) and (48) in equation (30) it results that:

$$2\bar{\tau}\left(h_1'-\frac{1}{\tau}h_1\right)-h_1+\bar{\tau}\sum_{i=0}^{\infty}i\,(i-1)l_i\,\tau^{i-1}-(\bar{\tau}-\tau)\sum_{i=0}^{\infty}i\,l_i\,\tau^{i-1}+k\sum_{i=0}^{\infty}l_i\,\tau^i=0 \tag{49}$$

from this

$$2\bar{\tau}\sum_{i=0}^{\infty}(i+1)k_i\,\tau^{i+1}-\sum_{i=0}^{\infty}k_i\,\tau^{i+2}+\bar{\tau}\sum_{i=0}^{\infty}i(i-2)l_i\,\tau^{i-1}-\sum_{i=0}^{\infty}(i-k)l_i\,\tau^i=0 \tag{50}$$

or yet:

$$2\bar{\tau}\,k_0\,\tau+\sum_{i=0}^{\infty}\left[\bar{\tau}(i+2)k_{i+1}-k_i\right]\tau^{i+2}+\sum_{i=0}^{\infty}\left[\bar{\tau}(i+1)(i-1)l_{i+1}-(i-k)l_i\right]\tau^i=0 \tag{51}$$

in the end:

$$2\bar{\tau}k_0\,\tau-\bar{\tau}l_1+k\,l_0-(1-k)\,l_1\tau+$$
$$+\sum_{i=0}^{\infty}\left[\bar{\tau}(i+2)k_{i+1}-k_i+\bar{\tau}(i+3)(i+1)l_{i+3}-(i+2-k)l_{i+2}\right]\tau^{i+2}=0 \tag{52}$$

In order this relation to be true at any value of τ , the first part of the relation must satisfy:

$$2\bar{\tau}\,k_0=(1-k)l_1$$
$$k\,l_0=\bar{\tau}l_1 \tag{53}$$

If we consider $l_2=1$ we can determine from (53) the first terms of the series

$$l_1=\frac{2}{1-k}\bar{\tau}\ ;\ \ l_0=\frac{2}{k(1-k)}\bar{\tau}^2 \tag{54}$$

From the second part of the relation (52) the following recurrence relationship can be determined:

$$l_{i+3}=\frac{1}{\tau}\frac{i+2-k}{(i+3)(i+1)}l_{i+2}+\frac{1}{\tau}\frac{1}{(i+3)(i+1)}k_i-\frac{2(i+2)}{(i+3)(i+1)}k_{i+1} \tag{55}$$

From (55) we can generate the terms of the series:

$$l_3=\frac{k+1}{9}\frac{1}{\tau}\ ;\ l_4=\frac{-13k^2+45k-18}{288}\frac{1}{\tau} \tag{56}$$

To summarize the development in series of the second term of the solution of the homogeneous equation is:

$$h_2(\tau)=h_1(\tau)\ln\tau+(\overline{\tau})^2\frac{2}{k(1-k)}+\overline{\tau}\frac{2}{1-k}\tau+\tau^2+\frac{k+1}{9\overline{\tau}}\tau^3+\frac{-13k^2+41k-18}{288\overline{\tau}^2}\tau^4+\cdots \tag{57}$$

5. The particular solution

We will determine a particular solution of the non homogeneous equation of the form:

$$h_p=a_1\tau^2+a_2\tau+a_3 \tag{58}$$

Substituting (58) in equation (30) we obtain:

$$\overline{\tau}\,\tau\,h_p''-(\tau+\overline{\tau})h_p'+k\,h_p=c_T\tau^2 \tag{59}$$

where:

$$h_p'=2a_1\tau+a_2\,,\quad h_p''=2a_1 \tag{60}$$

By identification it follows:

$$a_2=0,\ a_3=0,\ a_1=\frac{c_T}{k-2} \tag{61}$$

the general solution is of the form:

$$\Delta h(\tau)=c_1\,h_1(\tau)+c_2\,h_2(\tau)+a_1\tau^2 \tag{62}$$

6. The constants of integration

We intend to determine the constants of integration c_1 and c_2. By derivation, the general solution (36), can be written:

$$\begin{aligned}\Delta h(\tau)&=c_1\,h_1(\tau)+c_2\,h_2(\tau)+a_1\tau^2\\ \Delta h'(\tau)&=c_1\,h_1'(\tau)+c_2\,h_2'(\tau)+2a_1\tau^2\end{aligned} \tag{63}$$

Assuming the values of the miss distance and its derivative at the initial moment ($\tau=1$) the equations (63) become:

$$\begin{aligned} \Delta h_0 - a_1 &= c_1 h_{10} + c_2 h_{20} \\ \Delta h_0' - 2a_1 &= c_1 h_{10}' + c_2 h_{20}' \end{aligned} \tag{64}$$

Where by $h_{10}, h_{20}, h_{10}', h_{20}'$ are denoted the values of the functions h_1 and h_2 and their derivatives at the initial moment, values that will be defined in the next sections.

Solving the system (64) with respect to the two constants it results:

$$\begin{aligned} c_1 &= \frac{h_{20}(\Delta h_0' - 2a_1) - (\Delta h_0 - a_1)h_{20}'}{h_{20}\, h_{10}' - h_{10}\, h_{20}'} \\ c_2 &= \frac{(\Delta h_0 - a_1)h_{10}' - h_{10}(\Delta h_0' - 2a_1)}{h_{20}\, h_{10}' - h_{10}\, h_{20}'} \end{aligned} \tag{65}$$

Finally the equations of the homogeneous solutions and their derivatives at the initial moment ($\tau = 1$), used in relation (65), can be defined.

$$\begin{aligned} h_{10} &= h_1(1) = \sum_{i=0}^{\infty} k_i = k_0 + k_1 + k_2 + \cdots \\ h_{10}' &= h_1'(1) = \sum_{i=0}^{\infty} k_i (i+2) = 2k_0 + 3k_1 + 4k_2 + \cdots \end{aligned} \tag{66}$$

$$\begin{aligned} h_{20} &= h_2(1) = \sum_{i=0}^{\infty} l_i = l_0 + l_1 + l_2 + l_3 + \cdots \\ h_{20}' &= h_2'(1) = h_1(1) + \sum_{i=0}^{\infty} i l_i = \sum_{i=0}^{\infty} (k_i + i l_i) = k_0 + k_1 + k_2 + \cdots + l_1 + 2l_2 + 3l_3 + 4l_4 + \cdots \end{aligned} \tag{67}$$

7. The numerical solution

Obviously, the problem may be formulated so as to allow a numerical solution. For this purpose we transform the second order differential equation in a system of first order differential equations. For this we define the variables:

$$y_1 = \Delta h \;\; ; \; y_2 = y_1' \tag{68}$$

with which the equation (30) is put in the following form:

$$\begin{cases} y_1' = y_2 \\ y_2' = \dfrac{\tau + \overline{\tau}}{\tau\overline{\tau}} y_2 - \dfrac{k}{\tau\overline{\tau}} y_1 + c_T \dfrac{\tau}{\overline{\tau}} \end{cases} \tag{69}$$

with initial conditions:

$$\begin{cases} y_{10} = \Delta h_0 \\ y_{20} = \Delta h_0' \end{cases} \tag{70}$$

8. Applications, results

In this section we will present a series of results obtained by using the previous theoretical developments, also we will compare the results achieved by using the series development method with the results obtained by numeric solving of the system (69).

A first group of results aims to reveal the number of terms required for the series development in order to obtain similar results. Thus, in figures 2 and 3 where we presented the miss distance and its derivative is observed that after about 45 terms of the resulted series, the two methods give very close values.

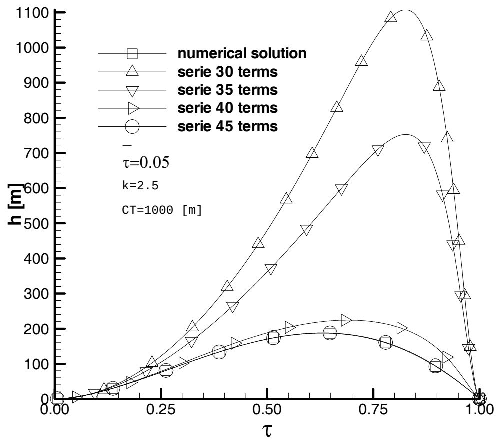

Fig. 2 The influence of the number of terms of the series on the accuracy of estimation of the miss derivative

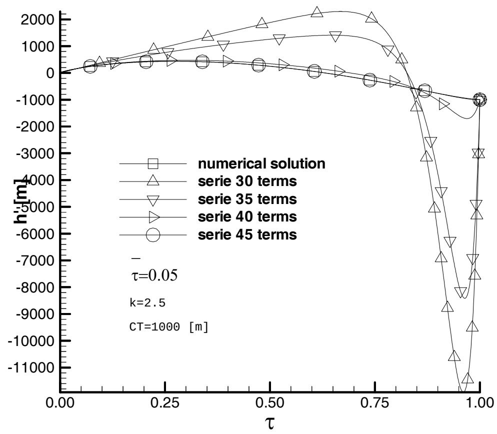

Fig. 3. The influence of the number of terms of the series on the accuracy of estimation of the miss distance speed

Next, using the series development method we will make an analysis on how the main parameters of the equation affect the obtained results. Thus, the figures 4 and 5 present the influence of the non dimensional time constant on the miss distance and its derivative. It can be observed that for the same initial conditions and the same maneuver of the target the miss distance increases as the non dimensional time constant is greater. Note that, because the non dimensional time constant defined by the relation (25) represents the ratio between the response time of the missile and the time to hit the target, the increase of the response time of the missile itself does not automatically lead to an increase of the miss distance, but only the ratio between it and the time to hit may affect the miss distance.

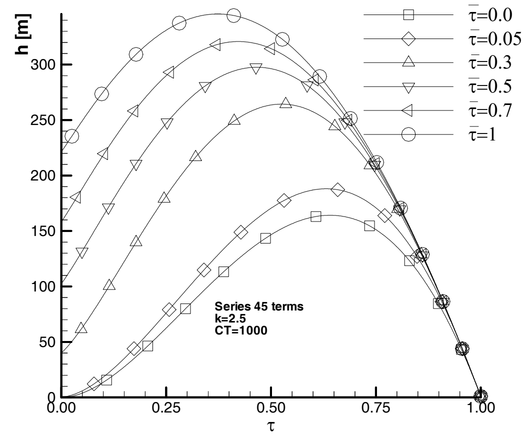

Fig. 4 The influence of the non dimensional time constant on the miss distance

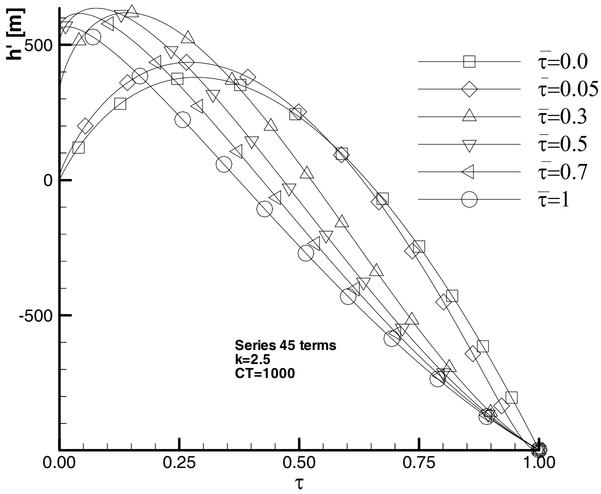

Fig. 5 The influence of the non dimensional time constant on the miss distance speed

Note – the previous results obtained in the case $\bar{\tau} = 0$ were obtained using the analytical solution (34) and the relation (32) for the derivative, the solution based on the series development is not usable for null values of the non dimensional time constant.

In the figures (6) and (7) the influence of the target maneuver on the miss distance and its speed is presented. As expected we observe that with the increase of the target maneuvers the miss distance and its speed increase. Finally for non maneuvering target and initial error zero, the miss distance remain zero any time.

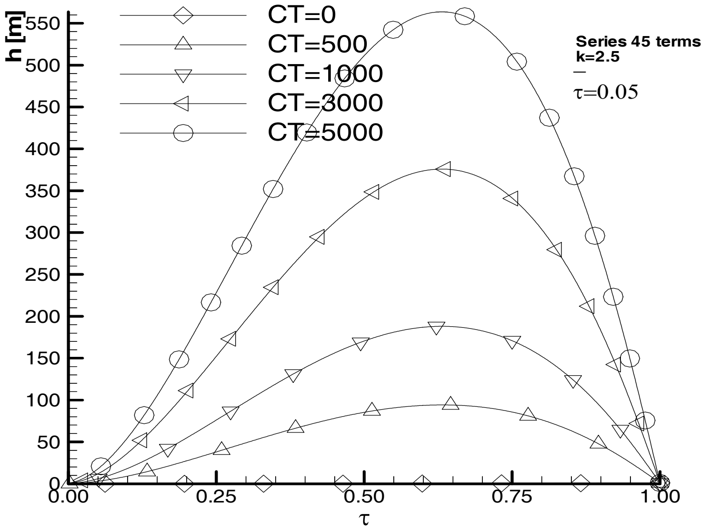

Fig. 6 The influence of the target maneuver on the miss distance

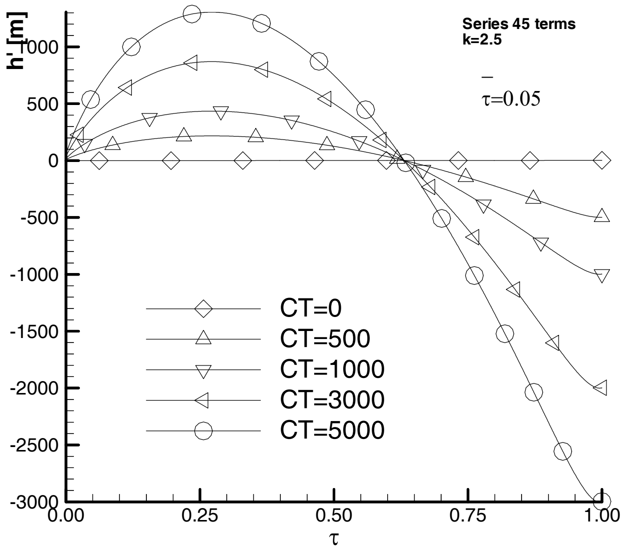

Fig. 7 The influence of the target maneuver on the miss distance speed

9. Conclusions

A first conclusion related to the obtained results shows that a reasonable number of terms of series solution equation based on the series development coincide with the numerical solution, thereby verifying each other. On the other hand, if we retain from the solution the first 4 terms pointed by relations (44), (54) and (56) we obtain an approximate solution good enough to be used in various models necessary for high-speed computing processes. For example the models used for the airplane weapons systems or missile complexes should give a real-time solution on the possibility of using the missile for the interception of a particular target, problem known as „Dynamic Launch Zone – DLZ". Furthermore, an analytical solution of the guidance problem, even with an approximate character, can be of real help in synthesis problems of the guidance system, with the various forms that this problem can take, ranging from the optimal differential games theory and continuing with the implementation of robust systems or with error detection (fault detection) in accordance to the latest developments in the field of control theory.

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