WEIBULL STATISTICS APPLICATION TO THE LOW CYCLE FATIGUE TEST OF ZIRCALOY-4

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The two parameters Weibull law for the analysis of the cycle fatigue tests has been adapted. Also it has been realized the Weibull probabilistic network. Using this method, the results of the low cycle fatigue tests on Zircaloy-4 specimens were interpreted.

Keywords: Weibull distribution, cycle fatigue, failure rate, reliability, Zircaloy-4.

1. Introduction

In the physics of materials, or generally in materials science, the fatigue terminology represents the material debilitation into continuous process produced by action of repeatedly applied loads. More precisely, if a material is subjected to cyclic loading procedure, in its structure the progressive and localized structural damage will be produced. In other words, fatigue occurs when a material is subject to repeated loading/unloading succession.

Experimental study of special alloys behavior in the low-cycle fatigue test was traditionally performed for certain amongst them, which were used in manufacturing of pressure vessels, thin-walled tubes loaded and subject to high hydrostatic pressure, that are exposed in their lifetime service to a heat source/sink which induces thermal expansion or thermal stress to the structure.

The associated characteristics of “Low Cycle Fatigue” are the moderate time event per test, samples moderate deformation and fracture dependence on time/temperature. It can also be said that, for this mechanical model, the stress level usually remains into plastic range.

From the initial utilizations offered by W. Weibull himself regarding problems and tests concerning the resistance of materials, later time behavior of electronic tubes, in the last period the Weibull statistics method has found countless applications in other fields. Amongst them we can enumerate the protection of environment against pollution, the control of products reception found in a quality assurance system, as well as chemistry and medicine applications. The advantages of this procedure reside in its relatively simple

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analytical form, in its elastic structure, offered by the existence of two or three parameters and in the ease of getting the conclusions. Differing from the classical exponential model, in this method we cannot impose anymore the hypothesis of constancy in time and the superior number of parameters raises the fidelity degree in describing complex processes.

As known from literature, the Weibull model has several forms, which are equivalent to each other [1-3]. We have chosen one of them, which is more suitable for our requirements.

We shall present, briefly enough, the Weibull model, which will be used to the results interpretation of the “Low Cycle Fatigue” test for samples of Zircaloy-4 sheets at 300 °C. In particular, we remind the reader that this material has been tested extensively by one of the authors, and the results have already been published [4, 5].

2. The Weibull statistics – normal form

In this paper, we will take a closer look at a specific distribution that is widely used in life data analysis, the well-known Weibull distribution. So called for its inventor, Waloddi Weibull, this distribution is widely used primarily in reliability engineering and in other important fields due to its polyvalent nature, multiple purpose and relative straightforwardness [2, 3].

The general expression of the frequency function of the “two parameters Weibull law”, for $x \geq 0$ is

$$f\left(\frac{x}{\theta},\beta\right)=\frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1}e^{-\left(\frac{x}{\theta}\right)^{\beta}} \tag{1}$$

or

$$f(x)=\frac{\beta}{\theta^{\beta}}x^{\beta-1}e^{-\left(\frac{1}{\theta}x\right)^{\beta}}, \tag{2}$$

where the Greek lowercase θ, respectively β are two parameters requested by definition [1, 2]. In particular, if $\beta > 0$, it is named the ***shape*** parameter and if $\theta > 0$, it is named the ***scale*** parameter.

In mathematical language, $f(x)$ is the probability density function.

According to common perception, the Weibull shape parameter is assimilated as the Weibull slope. This is based on the fact that the value of β is equal to the slope of the line in a probability plot.

The banal graphic representations - see Figures 1 and 2 - demonstrate the separate effect of the scale parameter θ (teta) or the shape parameter β (beta), on the Weibull distribution.

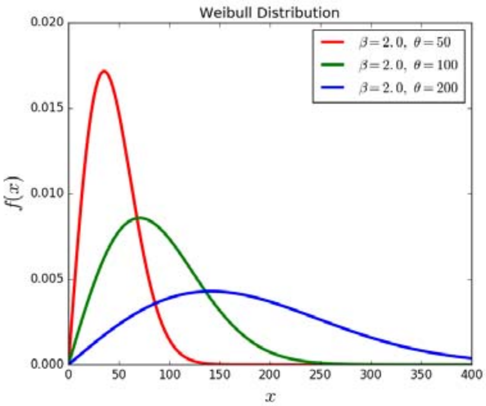

Fig. 1. Effect of the scale parameter

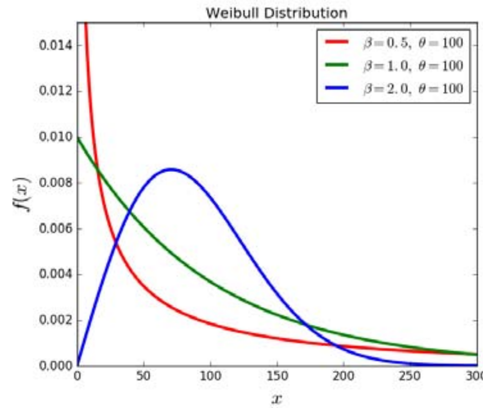

Fig. 2. Effect of the shape parameter

The distribution function for this model is

$$F_x\left(\frac{x}{\theta},\beta\right)=\int_{-\infty}^{x} f\left(\frac{y}{\theta},\beta\right)dy \tag{3}$$

or the probability function

$$F(x)=\int_{-\infty}^{x} f(y)dy\,. \tag{4}$$

A simple calculation leads to the expression

$$F_x\left(\frac{x}{\theta},\beta\right)=\begin{cases}0, x\le 0\\ 1-e^{-\left(\frac{x}{\theta}\right)^{\beta}}, x>0\end{cases} \tag{5}$$

In continuation, the form

$$R_x\left(\frac{x}{\theta},\beta\right)=1-F\left(\frac{x}{\theta},\beta\right)=e^{-\left(\frac{x}{\theta}\right)^{\beta}} \tag{6}$$

represents the probability that the event will take place in the time interval *(0,x)* or, as it is often said in the theory of reliability, the probability of functioning without losses/failure in a given time. Another denomination of the function provided above is the cumulative distribution function

$$F(x)=1-e^{-\left(\frac{x}{\theta}\right)^{\beta}}, x>0\,. \tag{7}$$

The inverse cumulative distribution function is

$$I(F(x))=\theta(-\ln(1-F(x))^{1/\beta}\,. \tag{8}$$

The principal statistical properties of the Weibull distribution such as the "mean", the "median", the "mode" and the "standard deviation" σ, are given by the next formulae:

- $\text{Mean} = \theta \cdot \Gamma(1 + \frac{1}{\beta})$
- $\text{Median} = \theta(\ln 2)^{1/\beta}$
- $\text{Mode (when } \beta > 1) = \theta(1 - \frac{1}{\beta})^{1/\beta}$
- $\sigma = \theta \cdot \sqrt{\Gamma(1 + \frac{2}{\beta}) - \left[\Gamma(1 + \frac{1}{\beta})\right]^2}$.

The gamma function, present in the above expressions, is defined as

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx \qquad (9)$$

3. The Zircaloy-4 samples rupture by “Low Cycle Fatigue” test

There are three commonly recognized forms of fatigue. They were given the names "high cycle fatigue (HCF)", "low cycle fatigue (LCF)" and "thermal mechanical fatigue (TMF)". The essential distinction between HCF and LCF is the region of the stress-strain curve where the repetitive application of load (and resultant deformation or strain) is occurring.

Finite or Infinite Fatigue Life, that is the question! In the case of HCF experiment, either infinite fatigue or finite fatigue life is possible and can be analyzed. Conversely, for LCF experiment, only finite fatigue life is possible and should be analyzed using LCF-criteria.

As terminology, we mention that the "American Society for Testing and Materials" (ASTM) defines *fatigue life*, as the number of stress cycles of a specified character that a specimen sustains before failure of a specified nature occurs. By virtue of a full understanding of the text peculiarity, in all notations, terminology and measurements, the names involved with their definitions used are in compliance with ASTM A370 – 15, surnamed “Standard Test Methods and Definitions for Mechanical Testing of Steel Products”.

Another important question relates to whether the technical option is for Stress or Strain test type, respectively.

In the HCF model, we have elastic material and a small strain increment involves a large stress increment. By comparison, in the LCF model, we have stresses close to (or at) the yield limit. The small stress increment involves large strain increment. Best “resolution” is obtained if strains are employed in the fatigue model [6, 7].

In our paper, the cyclic temperature rupture tests were conducted on a special alloy (known for its applications in nuclear power plants), named Zircaloy-4 (Zy-4) [4, 5, 8]. The specimens were subject to a constant load and different types of temperature cycles, but for this study only the fixed 300 °C value is considered. The size of the microsamples and method of heating were such that thermal stresses were considered to be negligible.

Weibull is the one who established, by interpreting the results of a cycling test, that the percentage S of a group of samples standing to N cycles, for a given loading and under established constant experimental conditions is given by the relation

$$S = e^{-\left(\frac{N}{N_0}\right)^m} . \tag{10}$$

where N_0 is a certain constant which features the endurance characteristics of the material taken into consideration. It is easier to deal with the rupture fraction F, which can be obtained from the fraction S, of the "surviving" samples, i.e.

$$F = 1 - S = 1 - e^{-\left(\frac{N}{N_0}\right)^m} , \tag{11}$$

As it can be easily noticed, the equation (11) is similar to (7), for $x > 0$, where N plays the role of the variable x.

Further on, we present the application of these results to the "Low Cycle Fatigue" test [6]. For tests made at the same load, let us consider 10 experiments, the rupture appearing after $N_1, N_2, \ldots, N_{10}$ cycles. We arrange the number of cycles in a crescent order, as shown in Table 2, in the next chapter.

In order to find the weight, we use an estimation of the distribution function, of the form:

$$\hat{F} = 100(1 - 0.5)\frac{1}{n} . \tag{12}$$

4. Weibull probabilistic network

Practically, the parameters estimation of the Weibull distribution can be made analytically or graphically via probability plotting paper, named Weibull paper. In analytical manner we obtain the same results, using either least squares algorithm (rank regression) or maximum likelihood estimation (MLE) [9, 10].

Starting from the function $F(x)$, expressed by equation (7), after applying twice the logarithm we get, step by step:

$$\ln(1 - F(x)) = \ln e^{-\left(\frac{x}{\theta}\right)^\beta} = -\left(\frac{x}{\theta}\right)^\beta \tag{13, a}$$

$$-\ln(1-F(x)) = \left(\frac{x}{\theta}\right)^{\beta}, \tag{13, b}$$

or after decimal logs in both sides

$$\lg[-\ln(1-F(x))] = \lg\left(\frac{x}{\theta}\right)^{\beta} = \beta \lg\frac{x}{\theta}. \tag{13, c}$$

Immediately, noting the left-hand side member with y, we have

$$y = \lg[-\ln(1-F(x))] = \beta \lg\frac{x}{\theta} \tag{13, d}$$

and equation (13, d) becomes

$$y = \beta(\lg x - \lg\theta). \tag{14}$$

Taking into discussion for second time the equation (13, b), we have relationships

$$[-\ln(1-F(x))]^{1/\beta} = \left(\frac{x}{\theta}\right) \tag{15, a}$$

and

$$x = \theta[-\ln(1-F(x))]^{1/\beta}, \tag{15, b}$$

or taking decimal logs on both sides

$$\lg x = \lg\left\{\theta[-\ln(1-F(x))]^{1/\beta}]\right\}. \tag{15, c}$$

At the end of calculation, noting the left-hand side member with z, we have

$$z = \lg x = \lg\theta + \frac{1}{\beta}\lg[-\ln(1-F(x))] \tag{15, d}$$

Thus $z = \lg x$ when plotted against $\lg[-\ln(1-F(x))]$ should follow a straight line pattern with intercept $a = \lg\theta$ and slope $b = 1/\beta$. Ultimately, the parameter values $\theta = 10^{a}$ and $\beta = 1/b$ are obtained.

Plotting y against $\lg x$ as is usually done in a Weibull plot, one should see the following linear relationship (14) with slope $B = \beta$ and intercept $A = -\beta\lg\theta$. Through a simple calculation, the values $\beta = B$ and $\theta = 10^{-A/B}$ are obtained.

Remarks. In many books the parameter θ is called the scale parameter or characteristic life. The latter specified appellation is motivated by the evident property $F(\theta) = 1-\exp(-\theta/\theta) = 1-\exp(-1) = 0.632$, regardless of the shape parameter β. There are different manners for estimating the parameters θ and β. One of the simplest is through the method of Weibull plotting, which used to be very popular due to its simplicity, graphical appeal, and its informal check on the Weibull model assumption. As immediately observed, according to the same reasons, we resorted here to this method.

Weibull paper scales

The Weibull p-quantile x_p is defined by the following property

$$p = F_X(x_p) = P(X \le x_p) = 1 - e^{-\left(\frac{x_p}{\theta}\right)^{\beta}}, x \in (0,1) . \quad (16)$$

In this short presentation, the results are due to the power transformation property of the Weibull distribution function.

Let be a coherent notation $X \approx F_X(\theta, \beta / x) = F(\theta, \beta)$, where X has a Weibull distribution with parameters θ and β. The mention made allows us to write that $X^{'} = X^{a} \approx F(\theta^{a}, \beta / a) = F(\theta^{'}, \beta^{'})$.

Certainly, it is a correct wording in virtue of next formal demonstrations

$$P(X^{'} \le x) = P(X^{a} \le x) = P(X \le x^{1/a}) =$$

$$1 - e^{-\left(\frac{x^{1/a}}{\theta}\right)^{\beta}} = 1 - e^{-\left(\frac{x}{\theta^{a}}\right)^{\beta/a}} = 1 - e^{-\left(\frac{x^{1/a}}{\theta^{'}}\right)^{\beta^{'}}} . \quad (17)$$

At this point we say that we can adjust parameter values, obviously. Even more, it is possible to bring the scale up or down (but mainly down), for an ideal situation of data representation, respectively into the proper range by an appropriate power transformation. After estimating $(\theta^{'}, \beta^{'})$ one can easily transform back to (θ, β) using the known value *a*, namely $\theta = (\theta^{'})^{1/a}$ and $\beta = a\beta^{'}$.

A development of the subject, for complete samples and for the Weibull special kind of censoring known as type II censoring, can be read in the study of Fritz Scholz, titled "Weibull Probability Paper" (2008), freely available on Internet.

One example to determine the adequate scale of Weibull probability paper is shown below, for a complete sample of size $n = 10$, see Table 2.

The value of the function y linearly depends on lg x, whence we obtain one of the principles on which the logarithmic paper relies: on the horizontal axis we build the logarithmic scale according to equation $S_x = k_x \lg x$, where k_x is the proportionality factor. From equation (14) we can also establish the size of the form parameter β . If we take x=1, then we get $y_1 = -\beta \lg \theta$ and $S_\theta = k_\theta \lg \theta$.

For $y = \lg[-\ln(1 - F(x))]$, corresponding to consecrate values set F_{max}=0.999, respectively F_{min}=0.001, we quickly compute y_{min} and y_{max} , extreme values of the border. Afterwards, based on the values $y_{min} = -2.99$ and $y_{max} = 0.83$, a simple calculus leads to $\Delta y = y_{max} - y_{min} = 0.83 - (-2.99) = 3.82$.

Further, by analogy, on the vertical axis we build the linear scale according to equation $S_y = k_y y$, where k_y is the proportionality factor.

We have thus established the modulus of the ordinates scale $S_y(F) = (H/\Delta y)y = (H/3.82)y$, where H is the length of the probabilistic network in millimeters. In the above indicate example $H = 100$ (see Fig. 4) and find the ordinates scale modulus $S_y(F) = (100/3.82)y = 26.17y$.

The drawing of the network was facilitated by using computational software, developed by the authors.

5. Results and discussion

In the real world of engineering, the Weibull statistics, a mathematical tool for processing experimental results [11, 12], is so powerful that the Weibull Analysis is surnamed the Life Data Analysis.

The main goals of this paper are to apply a Weibull model to the Zircaloy-4 thin-walled tubes reliability analysis, and then explore the failure rate over service time.

Even more, in our study we disseminate and comment the results of the low cycle fatigue test at 300 °C, on Zircaloy-4 microsamples. The experimental results of the fatigue fracture tests are presented in Tables 1 and 2.

The strain amplitude ε_a (or total strain) can be written as $\varepsilon_a = \varepsilon_a^{el} + \varepsilon_a^{pl}$, while the strain elastic amplitude is ε_a^{el} (or total elastic strain) and the strain plastic amplitude is ε_a^{pl} (or total plastic strain).

The fatigue life in the plastic deformation regime can be approximated by experimental classic formula

$$\varepsilon_a^{pl} = 0.5 \cdot \left(\frac{N}{D}\right)^{-0.6}. \tag{18}$$

In fact, this empirical rule is the Coffin – Manson relationship [13, 14], in the plastic behaviour part of material. In equation (18), *N* is the number of load cycles to failure and *D* is the ductility, in accordance with the theory and notations of Table 1. In context of mechanical testing, the ductility is defined as

$$D = \ln\left(\frac{A_0}{A_{fra}}\right) \approx \varepsilon_{fra} \tag{19}$$

where ε_{fra} is the fracture strain amplitude, A_0 is the value of initial transversal surface area (or cross-sectional area) and A_{fra} is the final transversal surface area, named fracture transversal surface area of microsample.

Observation. Low-cycle fatigue is usually characterized by the Coffin-Manson relation (published independently by L. F. Coffin in 1954 and S. S. Manson in 1953). Similar relationships for materials such as Zirconium and its alloys (Zircaloy-2, Zircaloy-4, used in the nuclear industry) [14] are already published. We felt the need to give a thorough explanation because we opted for material rule introduced above and its verification.

Table 1

Low cycle fatigue test on Zircaloy-4 microsamples at 300 °C

Curvature radii	Number of samples	ε_a (%)	$\varepsilon \times 10^2$ (S^{-1})	ε_a^{pl} (%)	*N* cycles	*N/D*
4	38	5.44	1.8	5.25	459	169
	42	5.41	1.8	5.22	593	218
	47	5.08	1.7	4.89	691	254
	48	5.06	1.7	4.87	577	212
	49	5.26	1.7	5.07	540	198
	67	5.62	1.9	5.43	515	189
	66	5.81	1.9	5.62	406	149
	61	5.74	1.9	5.55	700	257
	64	5.58	1.9	5.39	555	204
	59	5.71	1.9	5.52	604	222
6.5	31	3.58	1.2	3.40	742	273
	44	3.42	1.1	3.24	647	238
	46	3.40	1.1	3.22	868	319
	50	3.25	1.1	3.07	551	202
	35	3.21	1.1	3.03	748	275
	45	3.31	1.1	3.13	625	229
	56	3.69	1.2	3.51	679	250
	57	3.55	1.2	3.37	539	197
	55	3.77	1.2	3.59	405	149
	58	3.61	1.2	3.43	585	219
8	51	2.68	0.9	2.51	975	358
	41	2.62	0.9	2.45	698	257
	36	2.71	0.9	2.54	797	293
	37	2.73	0.9	2.56	1209	444
	43	2.60	0.9	2.43	739	272
	60	2.82	0.9	2.65	831	305
	63	2.82	0.9	2.65	703	258
	69	3.06	1.0	2.88	649	238
	65	3.04	1.0	2.86	371	136
	68	3.29	1.1	3.03	968	356

We used results coming up from three cycling tests for samples strained at different curvature radii, at 300 °C, namely the number of cycles at which the samples broke down.

By putting these numbers under the form of an increasing order row and taking into account the weight, we got Table 2, while Table 1 shows also other data, representing the basis which the formalism we have built relies on.

Table 2

Number of the cycles in a crescent order

F(x) (%)	N cycles	R=4	R=6.5	R=8
6.70	10	406	405	371
16.23	20	459	536	649
25.86	30	516	551	698
35.51	40	540	585	703
45.17	50	555	625	739
54.83	60	577	647	797
64.49	70	598	679	831
74.14	80	604	742	968
83.77	90	691	748	975
93.30	100	700	868	1209

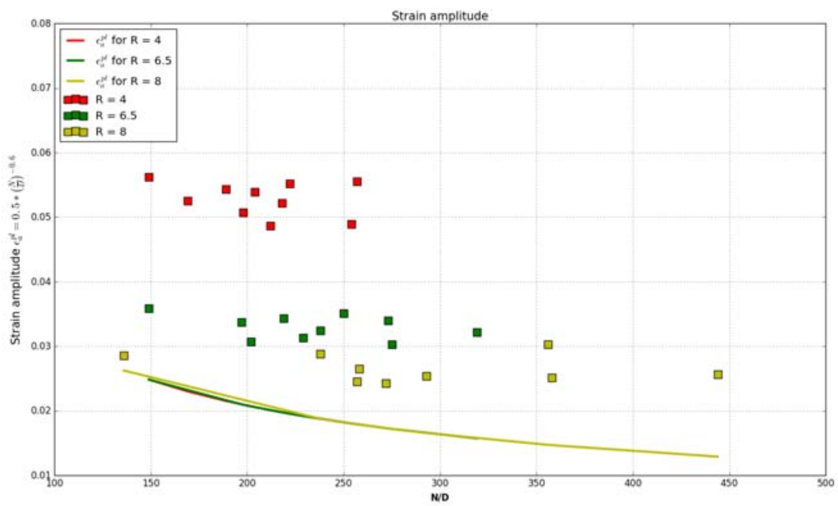

Fig. 3. The dependence of plastic strain amplitude ε_a^{pl} on the ratio (N / D)

In Fig. 3 the plastic strain amplitude ε_a^{pl} depending on the ratio (N / D) (number of load cycles to failure N and ductility D) through both experimental values (Table 1) and the classical empirical curve from equation (18), are plotted. As shown, the experimental data (colored points) is above the classical curve (continuous full curve), which makes us say that it is not verified (valid) for the material tested, respectively Zy-4.

Although it did not give the expected results, this task has been accomplished to show that the only powerful tool in processing experimental results of a low cycle fatigue test is Weibull Analysis, successfully used in our study.

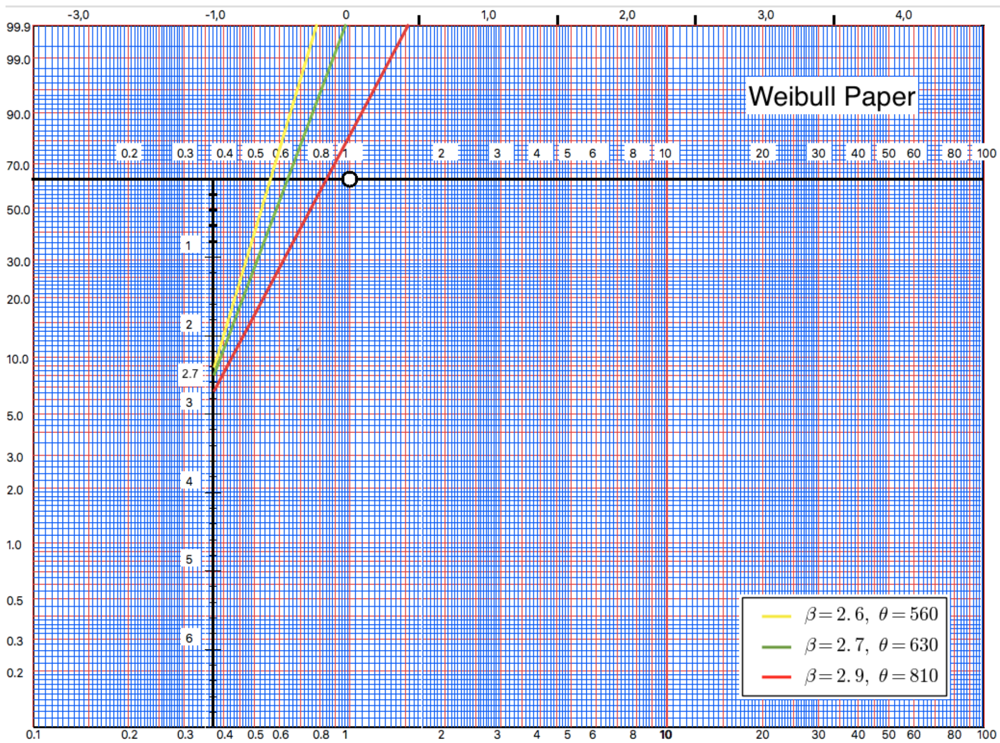

Fig. 4. Weibull probabilistic network

By means of the probabilistic network, we represented the experimental data. From the corresponding curves (in fact straight lines) we got the values of parameters, as depicted in Figure 4.

Thus, from $\beta_i \in \{2.6, 2.7, 2.9\}$ in all three cases we have the formula

$$F_i(N) = 1 - e^{-\left(\frac{N}{N_{0i}}\right)^{\beta_i}}, i = 1,2,3, \tag{20}$$

and $N_{0i} \in \{560, 630, 810\}$, or $N_{01} = 560, \; N_{02} = 630, \; N_{03} = 810$.

This way we can estimate the rupture function *F*. This function, assuming that *N* is a "continuous" variable and passing to *x*, helps us in computing the statistical properties, namely to get a better statistical interpretation of the results.

Taking into account the statements made before, the rupture function has now the expression

$$F(x) = 1 - e^{-\left(\frac{x}{\theta i}\right)^{\beta_i}}, i = 1,2,3, \tag{21}$$

where the ***scale*** parameter has the values $\theta_{01} = 560, \; \theta_{02} = 630, \; \theta_{03} = 810$ and the ***shape*** parameter successively gets the values $\beta_{01} = 2.6, \; \beta_{02} = 2.7, \; \beta_{03} = 2.9$.

The reconstruction of experimental Weibull distribution, in all three cases, for $\beta_i \in \{2.6, 2.7, 2.9\}$ and $\theta_{0i} \in \{560, 630, 810\}$, is presented in Fig. 5.

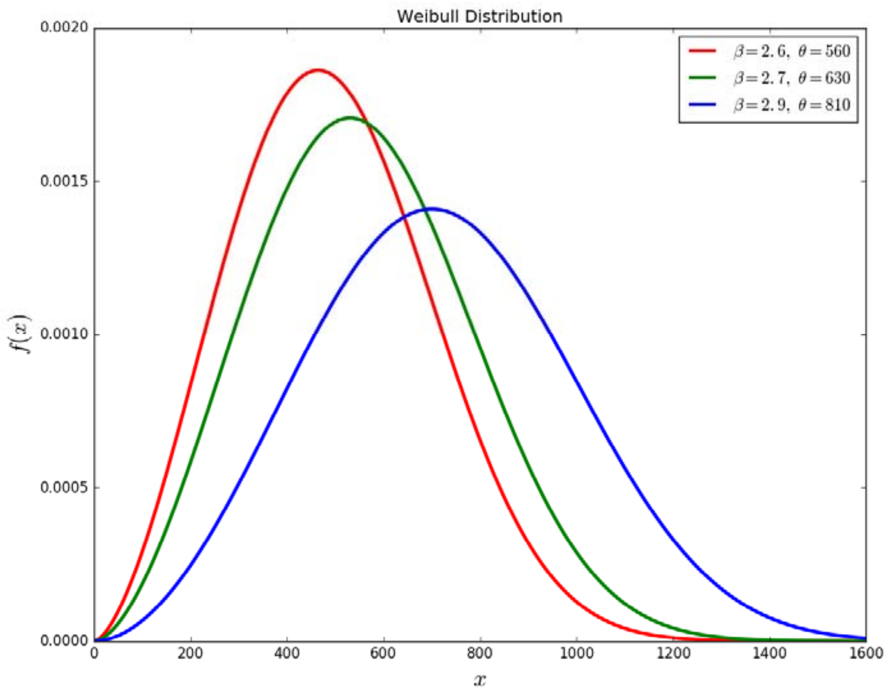

Fig. 5. The reconstruction of experimental Weibull distribution

Table 3

Weibull statistical properties

θi	β_i	Mean	Median	Mode	σ
560	2.6	497.39	486.37	464.61	6.72
630	2.7	560.24	550.03	530.79	6.81
810	2.9	722.26	713.83	700.09	7.01

The Weibull statistical properties are presented in Table 3.

In this chapter, we have explained the Weibull law plotting and its motivation, in rapport with reliability data. It also shows that the two Weibull parameter estimates are easily read from the Weibull paper, in the manner used here (see the Weibull probabilistic network - Fig. 4).

After all these experimental results have been properly processed, we may say the goals targeted *ab initio* were fully achieved. Moreover, they are in good agreement with situations reported in the scientific literature.

On the other hand, in the near future, we will propose a new perspective on the physical processes involved in the material fracture, so that we can maximize the fracture time period until tear depending on the parameters involved. The same concept has been successfully used in articles [15, 16], for seemingly distinct topics, but mathematically united under a common philosophy.

Finally, we can assume that a reader uninformed in the field, but also the specialist in training or new to this area got a better understanding of the fatigue major problems and how fatigue life is practically determined. In counterpart, for specialists (physicists, engineers, statisticians and computer scientists), with expertise of Materials Behavior or Life Prediction, it is hoped that some of this information has been helpful as well.

6. Conclusions

In this paper, the results of the low cycle fatigue tests, on the standard specimens sampled from Zircaloy-4 tubes, were interpreted.

As a first observation, we can say that the experimental data is far from the classical curve, which makes us declare that for Zircaloy-4, the empirical relation (18) is not verified.

The Weibull law with two parameters is applied to fit real reliability data in different test conditions.

Inter alia, this article explains and builds the Weibull plotting and provides its mathematical support. In addition, it shows how the two Weibull parameter estimates are easily read from the Weibull plot.

The analysis results of the low cycle fatigue test on Zircaloy-4 microsamples, are in good agreement with real test data, and provide reasonable prediction of future failure trends.

REFERENCES

[1]. *Al. Isaic-Maniu,* The Weibull Method: Applications, Romanian Academy Publishing, Bucharest, 1983.

[2]. *D. N. P. Murthy, R. Jiang,* Mixture of Weibull Distribution Parametric Characterization of Failure date Function, John Wiley and Sons Ltd., 1998.

[3]. *P. Xiuyun, Y. Zaizai,* Estimation and application for a new extended Weibull distribution, Reliability Engineering and System Safety, **vol. 121**, 2014, pp. 34–42

[4]. *V. P. Paun, C. Ciucu,* Hydrogen Concentration Effect in The Tangential Deformation at Failure of the Thin Walled Zircaloy-4 Tubes, Revista de Chimie, **vol. 54**, no. 12, 2003, pp. 957-958

[5]. *V. P. Paun,* A Critical Study of the Ring Expansion Test on Zircaloy Cladding Tubes, Materiale Plastice, **vol. 40**, no. 3, 2003, pp. 127-128

[6]. *S. Farfán, C. Rubio-González, T. Cervantes-Hernández, G. Mesmacque,* High cycle fatigue, low cycle fatigue and failure modes of a carburized steel, International Journal of Fatigue, **vol. 26**, no. 6, 2004, pp. 673–678

[7]. *Y. Zhou, B. Devarajan, K.L. Murty*, Short-term rupture studies of Zircaloy-4 and Nb-modified Zircaloy-4 tubing using closed-end internal pressurization, Nuclear Engineering and Design, **vol. 228**, no. 1–3, 2004, pp. 3–13

[8]. *V. P. Paun*, Fractal surface analysis of the Zircaloy-4 SEM micrographs by time series method, Central European Journal of Physics, **vol. 7**, no. 2, 2009, pp. 264-269

[9]. *Y. L. Lee, J. Pan, R. Hathaway, M. Barkey*, Fatigue Testing and Analysis, Elsevier Butterworth – Heinemann, Oxford, UK. 2005.

[10]. *X. Wang, J.-K. Kang, Y. Kim, P. H. Wirsching,* Low Cycle Fatigue Analysis of Marine Structures, Proceedings of OMAE 2006, 92268, Hamburg, Germany, 2006

[11]. *S.-P. Zhu, H.-Z. Huang, Y. Liu, R. Yuan, L. He,* An efficient life prediction methodology for low cycle fatigue–creep based on ductility exhaustion theory, International Journal of Damage Mechanics, **vol. 22**, no.4, 2012, pp. 556–571

[12]. *M. Hafeez, S. Kamal, M. Q. Shahbaz*, The Multivariate Order Statistics for Exponential and Weibull Distributions, Pak.j.stat.oper.res., **vol. X** , no.3, 2014, pp. 361-368

[13]. *R.I. Stephens, H.O. Fuchs*, Metal Fatigue in Engineering (Second ed.), John Wiley & Sons, 2001

[14]. *W.J. O'Donnell, B.F. Langer,* Nuclear Science and Engineering, **vol. 20**, 1964, pp. 1-12

[15]. *M. A. Paun, J. M. Sallese, M. Kayal,* Geometry Influence on the Hall Effect Devices Performance, University Politehnica of Bucharest Scientific Bulletin-Series A-Applied Mathematics and Physics, **vol. 72**, no. 4, 2010, pp. 257- 271

[16]. *M. A. Paun, J. M. Sallese, M. Kayal,* Offset and Drift Analysis of the Hall Effect Sensors. The Geometrical Parameters Influence, Digest Journal of Nanomaterials and Biostructures, **vol. 7**, no. 3, 2012, pp. 883-891