A MATHEMATICAL GENERALIZED APPROACH TO ESTIMATE SOIL MOISTURE RETENTION CHARACTERISTICS FROM TEXTURE CLASSES

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Curbele de retenție caracteristice, ale solului, sunt funcții foarte importante în procesul de modelare matematică a curgerii apei prin medii poroase nesaturate .

În această lucrare deducem o formă generalizată a caracteristicii de retenție folosind valorile conținutului volumic de umezeală calculate cu ajutorul parametrilor optimizați ai lui Mualem-Van Genuchten.

Conținutul volumic de umezeală poate fi obținut folosind o interpolare bidimensională polinomială de tipul Lagrange între curbele van Genuchten Mualem, pentru 5 clase de textură de sol şi două clase pedologice (topsoil şi subsoil)

Forma analitică, generalizată permite calcularea conținutului volumic de apă ca funcție de sucțiunea solului şi de clasa texturală a acestuia.

The soil moisture retention characteristics are very important functions in water flow through unsaturated media modeling processes.

We derived here a more general form of soil moisture retention characteristic, using volumetric moisture contents, calculated with the optimized Mualem-van Genuchten parameters. The volumetric moisture content can be obtained using a two dimensional polynomial Lagrange interpolation between the Mualem-van Genuchten curves, for five soil's texture classes and two pedological classes (topsoil and subsoil). Our general analytical form allows to compute the volumetric moisture content as a function of soil suction and of its texture class.

Keywords: Unsaturated soils, moisture content, soil database HYPRES

1. Introduction

In groundwater hydrology, unsaturated flow is important for downward vertical flow (natural and artificial recharge), upward vertical flow (evaporation and transpiration), movement of pollutants from ground surface, and flow in the capillary zone above the water table.

Soil hydraulic characteristics, like the soil water retention curve and hydraulic conductivity, are indispensable input data for water flow simulation in

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agriculture, landscape management, and water resources engineering, and to determine the spatial distribution of soil water availability. The direct measurement is troublesome, time-consuming and expensive. That is the reason for the lack of easily accessible and representative soil hydraulic properties, in different countries of the world.

HYPRES (Hydraulic Properties of European Soils) database [1] established by the collaboration of 12 European countries, provides general data and information concerning 5521 soil horizons, located in the West and Central Europe. Using the available measured hydraulic properties held by different institutions in Europe, standardization of particle-size classes and parameterization of hydraulic data have been achieved by fitting the Mualem-van Genuchten model parameters to the individual soil volumetric water content θ and hydraulic conductivities, K, like functions of soil water pressure head h. One of the goals of HYPRES database is to determine the spatial distribution of soil water availability using the Soil Map of Europe and the pedotransfer class functions.

Each soil horizon was allocated to one of two pedological classes (topsoil and subsoil) and to one of the six FAO texture classes: five mineral (coarse, medium, medium-fine, fine, very fine) and organic. For each of 11 possible soil textural/pedological classes, continuous pedotransfer functions for the prediction of hydraulic properties have been developed [2].

The goal of our work is, taking into account the results obtained for moisture contents and conductivities at 14 pressure heads, using the optimized Mualem-van Genuchten parameters [2], to determine a class of polynomial two-dimensional functions which describe the volumetric soil water content θ like a function of texture/pedological class and of soil suction pF ($pF = \lg|h|$, with the pressure head h in centimeters).

That result can be a more general approach to determine the pedotransfer functions $\theta(h)$, $K(h)$, and a useful tool in modeling the water transfer through unsaturated soils.

2. Soil water retention and hydraulic conductivity relationship

Unsaturated flow in the zone of aeration can be analyzed by Darcy's law, considering unsaturated hydraulic conductivity $K(\theta)$ like a function of the volumetric water content, θ, and of the negative pressure head (tension), h, in the soil's pores [3].

Hydraulic conductivity, K, is a measure of the ability of soils to transmit water and depends upon the soil properties (total porosity, pore-size distribution, and pore continuity) and fluid properties (viscosity and density).

Mathematical relationship for soil water retention and hydraulic conductivity proposed by van Genuchten [4] describes the total soil water-retention curve (even for bubbling pressure ay which air will enter the soil). The soil water retention curve is given by:

$$\theta(h) = \theta_r + (\theta_s - \theta_r)\left[\frac{1}{1 + (\alpha|h|)^n}\right]^m , \quad (1)$$

where: θ (cm^3cm^{-3}) is the soil volumetric water content, θ_r (cm^3cm^{-3}) – the residual soil water content θ_s (cm^3cm^{-3}) – saturated soil water content, α, n, m are parameters defining the moisture retention characteristic's (MRC) shape, and h (cm)is the pressure head in the soil's pores. Mualem proposed a relation between m and n ($m = 1 - \frac{1}{n}$).

Unsaturated soil hydraulic conductivity, K, can be expressed as a function of soil volumetric water content:

$$K(\theta) = K_S\left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)^{\frac{1}{2}}\left[1 - \left(1 - \left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)^{\frac{1}{m}}\right)^m\right]^2 . \quad (2)$$

θ_r, θ_s, , α, n, m are parameters which have to be estimated from observed soil water retention data.

Sampling the soil profiles MRC can be estimated at reasonable level of accuracy from simple soil properties as particle – size distribution (percentage of clay, silt, send), dry bulk density, percentage organic matter, using Mualem-van Genuchten parameters fitting on the geometric mean curves and continuous pedotransfer functions for the prediction of hydraulic properties [2].

3. A mathematical generalized approach to estimate soil moisture retention characteristics from texture classes

The texture of a soil is defined by the relative proportions of sand, silt, and clay present in the particle-size analysis.

To achieve compatibility within HYPRES and with other European soil databases, it was decided to standardize the particle-size data to three size limits. Clay is defined as the particle-size fraction < 2 µm, silt as the fraction between 2 and 50 µm, and sand as the fraction between 50 and 2000 µm (FAO, 1990; USDA, 1951) [5].

According to their particle-size composition 11 textural classes have been standardized: topsoil with coarse, medium, medium-fine, fine, very fine, subsoil with coarse, medium, medium-fine, fine, very fine, and organic class.

Table 1

Moisture contents at different pressure head using the optimized Mualem-van Genuchten parameters, for TOPSOIL's texture classes

h(cm)	0	-10	-20	-50	-100	-500	-1000	-5000	-16000
pF	0	1	1.3	1.7	2	2.7	3	3.7	4.2
θ^*_{Coarse}	0.403	0.379	0.352	0.294	0.243	0.148	0.120	0.077	0.059
θ^*_{Medium}	0.439	0.425	0.410	0.379	0.347	0.270	0.240	0.182	0.150
$\theta^*_{\text{MediumFine}}$	0.430	0.426	0.421	0.406	0.383	0.293	0.252	0.173	0.132
θ^*_{Fine}	0.520	0.507	0.495	0.472	0.448	0.388	0.364	0.311	0.278
$\theta^*_{\text{VeryFine}}$	0.614	0.602	0.592	0.567	0.541	0.470	0.439	0.374	0.334

Table 2

Moisture contents at different pressure head using the optimized Mualem-van Genuchten parameters, for SUBSOIL's texture classes

h(cm)	0	-10	-20	-50	-100	-500	-1000	-5000	-16000
pF	0	1	1.3	1.7	2	2.7	3	3.7	4.2
θ^*_{Coarse}	0.366	0.338	0.304	0.233	0.179	0.094	0.073	0.046	0.036
θ^*_{Medium}	0.392	0.382	0.372	0.349	0.324	0.258	0.231	0.179	0.149
$\theta^*_{\text{MediumFine}}$	0.412	0.409	0.405	0.392	0.373	0.297	0.261	0.189	0.149
θ^*_{Fine}	0.481	0.475	0.470	0.456	0.441	0.394	0.373	0.327	0.297
$\theta^*_{\text{VeryFine}}$	0.538	0.533	0.529	0.517	0.503	0.459	0.438	0.392	0.361

The volumetric soil water content θ, and hydraulic conductivity, K, as functions of pressure head, h, were parameterized with the equation derived by van Genuchten [2]. Using continuous pedotransfer functions for the prediction of hydraulic properties, water content and hydraulic conductivity values have been calculated, and stored in HYPRES. The tables 1 and 2 could characterizes the hydraulic properties of a soil horizon with a known texture class if the pressure head, h, is measured: θ(texture class, pF) with $pF = \lg|h|$.

For our modeling purposes [3,6] we need analytical functions to describe the water content and the hydraulic conductivity, for different soil textural classes.

We have applied a two dimensional Lagrange polynomial interpolation method for discrete input data parameters pF and TS (TS =Topsoil/Subsoil having the meaning: Topsoil or Subsoil) to obtain the given values (Table 1 or Table 2) for $\theta(pF, TS)$.

If Topsoil or Subsoil is one of the soil pedological classes, and $pF = \lg|h|$., we derived a continuous analytical function $\theta(pF, TS)$ with TS=1 for coarse soil, *TS*=2 for medium, *TS*=3 for medium-fin, *TS*=4 for fine, and *TS*=5 for very fine soil.

Between two classes of texture we can define subclasses of texture considering the percentage of the particle-size composition. For example between coarse and medium we can calculate $\theta_{coarse\text{-}medium}(pF, TS \in (1;2))$, with *TS* values between 1 and 2. If the soil sample has 50% coarse and 50% medium, *TS*=1.5.

If $\Phi\left(pF_i, TS_j\right), i = \overline{0:m}, j = \overline{0:n}$ is the input data matrix, we compute the Lagrange interpolation polynomial function:

$$\theta_{mn}(pF, TS) = \sum_{i=0}^{m}\sum_{j=0}^{n}\left[\Phi\left(pF_i, TS_j\right)\alpha_{m,i}(pF)\,\beta_{n,j}(TS)\right] \tag{3}$$

where:

$$\alpha_{m,i}(pF) = \prod_{\substack{k=0\\k\neq i}}^{m}\frac{(pF - pF_k)}{(pF_i - pF_k)} \quad \text{for} \quad i = \overline{0:\mathrm{m}} \tag{4}$$

$$\beta_{n,j}(TS) = \prod_{\substack{k=0\\k\neq j}}^{n}\frac{(TS - TS_k)}{(TS_j - TS_k)} \quad \text{for} \quad j = \overline{0:\mathrm{n}} \tag{5}$$

Due to the great number of θ (*i*, *j*) function's values, the polynomial form is very complicated. We split the data matrix (5, 14) in three sets of data, three matrix (5, 5), (5, 5), and (5, 4). For each matrix we have obtained a polynomial function.

The two dimensional Lagrange interpolation polynomial form $\theta(pF, TS)$ is different for Topsoil and Subsoil.

With the polynomial function (6) we can compute the volumetric water content for all textural classes corresponding to pedological class "Topsoil", for suction values between *pF* =0 and *pF*=2. Similar function we have obtained for the other two regions (*pF* =2 and *pF*=3), (*pF* =3 and *pF*=4.2).

Continuous pedotransfer functions $\theta(pF, TS)$ for Topsoils are:

$$\begin{aligned}\theta_G(pF, TS) = {} & (0.0030 * pF - 0.0022 * pF^4 + 0.0074 * pF^3 - 0.0059 * pF^2 - \\ & 0.0099) * TS^4 + (-0.0343 * pF + 0.0249 * pF^4 - 0.0828 * pF^3 + 0.0661 * pF^2 \\ & + 0.1235) * TS^3 + (0.1246 * pF - 0.0914 * pF^4 + + 0.2965 * pF^3 - 0.2283 * pF^2 \\ & 0.5151) * TS^2 + (-0.1448 * pF + 0.1121 * pF^4 - 0.3337 * pF^3 + 0.2203 * pF^2 \\ & + 0.8654) * TS - 0.0071 * pF - 0.0135 * pF^4 - 0.0221 * pF^3 + 0.0876 * pF^2 - 0.061\end{aligned} \tag{6}$$

with *TS* between 1 and 2.

Continuous pedotransfer functions $\theta(pF, TS)$ for Subsoils are:

$$\theta_G(pF,TS) = (-1.7956/10^4 * pF^4 + 1.8449/10^4 * pF^3 - 3.3824\text{e}-004 * pF^2$$
$$+ 8.7497\text{e}-004 * pF - 0.0048) * TS^4 + (-0.0024 * pF^4 + 0.0165 * pF^3$$
$$- 0.0176 * pF^2 - 0.0017 * pF + 0.0575) * TS^3 + 0.0396 * pF^4 - 0.1908 * pF^3 \quad (7)$$
$$+ 0.2122 * pF^2 - 0.0486 * pF - 0.2271) * TS^2 + (-0.1476 * pF^4 + 0.6517 * pF^3$$
$$- 0.7236 * pF^2 + 0.2288 * pF + 0.3775) * TS + 0.1622 * pF^4 - 0.6996 * pF^3$$
$$+ 0.7683 * pF^2 - 0.2758 * pF + 0.1630$$

with *TS* between 1 and 2.

3. Results and Conclusions

Table 3 and Table 4 contain the computed values obtained with the analytical functions (6) and (7). These polynomial forms can be used to calculate intermediary values for known mixtures between different texture classes.

Table 3

Moisture contents at different pressure head using polynomial form (6), for TOPSOIL's texture classes

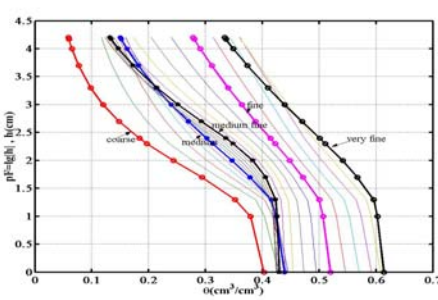
	Topsoils	***h*=0**	***h*=-10**	***h*=-20**	***h*=-50**	***h*=-100**
TS		***pF*=0**	***pF*=1**	***pF*= 1.3**	***pF*= 1.7**	***pF*=2**
1	θ_{Coarse}	0.403	0.3793	0.3522	0.2943	0.2438
1.2	$\theta_{Coarse-Medium}$	0.4287	0.4038	0.3880	0.3236	0.2776
1.4		0.4420	0.4180	0.4035	0.3450	0.3032
1.6		0.4461	0.4246	0.4118	0.3603	0.3223
1.8		0.4442	0.4260	0.4153	0.3710	0.3364
2	θ_{Medium}	0.439	0.4244	0.4101	0.3785	0.3469
2.2	$\theta_{Medium-MediumFine}$	0.4328	0.4217	0.4150	0.3841	0.3550
2.4		0.4274	0.4194	0.4143	0.3889	0.3619
2.6		0.4244	0.4188	0.4150	0.3937	0.3684
2.8		0.4250	0.4209	0.4179	0.3993	0.3753
3	$\theta_{\text{MediumFine}}$	0.430	0.4263	0.4216	0.4063	0.3832
3.2	$\theta_{MediumFine-Fine}$	0.4397	0.4355	0.4326	0.4151	0.3926
3.4		0.4541	0.4485	0.4450	0.4261	0.4037
3.6		0.4728	0.4651	0.4606	0.4392	0.4168
3.8		0.4952	0.4848	0.4791	0.4545	0.4317
4	θ_{Fine}	0.520	0.5068	0.4954	0.4717	0.4484
4.2	$\theta_{Fine-VeryFine}$	0.5457	0.5300	0.5218	0.4904	0.4665
4.4		0.5705	0.5530	0.5440	0.5101	0.4856
4.6		0.5919	0.5741	0.5649	0.5301	0.5050
4.8		0.6075	0.5913	0.5828	0.5495	0.5239
5	θ_{VeryFine}	0.614	0.6024	0.5916	0.5673	0.5414

Table 4

Moisture contents at different pressure head using polynomial form (7), forSUBSOIL's texture classes

	Subsoils	***h*=0**	***h*=-10**	***h*=-20**	***h*=-50**	***h*=-100**
TS		***pF*=0**	***pF*=1**	***pF*= 1.3**	***pF*= 1.7**	***pF*=2**
1	θ_{Coarse}	0.3660	0.3380	0.3041	0.2330	0.1790
1.2 **1.4** **1.6** **1.8**	$\theta_{Coarse-Medium}$	0.3782 0.3855 0.3893 0.3911	0.3543 0.3655 0.3730 0.3782	0.3380 0.3526 0.3629 0.3703	0.2705 0.2991 0.3206 0.3367	0.2259 0.2617 0.2887 0.3088
2	θ_{Medium}	0.392	0.382	0.376	0.349	0.324
2.2 **2.4** **2.6** **2.8**	$\theta_{Medium-MediumFine}$	0.3930 0.3950 0.3986 0.4042	0.3855 0.3894 0.3943 0.4008	0.3808 0.3858 0.3914 0.3983	0.3587 0.3671 0.3749 0.3830	0.3358 0.3455 0.3544 0.3633
3	$\theta_{\text{MediumFine}}$	0.4120	0.4090	0.4067	0.3920	0.3730
3.2 **3.4** **3.6** **3.8**	$\theta_{MediumFine-Fine}$	0.4222 0.4345 0.4488 0.4645	0.4191 0.4310 0.4446 0.4595	0.4169 0.4288 0.4422 0.4568	0.4022 0.4138 0.4268 0.4410	0.3839 0.3962 0.4100 0.4251
4	θ_{Fine}	0.4810	0.4750	0.4722	0.4560	0.4410
4.2 **4.4** **4.6** **4.8**	$\theta_{Fine-VeryFine}$	0.4974 0.5126 0.5254 0.5344	0.4906 0.5053 0.5181 0.5278	0.4876 0.5022 0.5151 0.5251	0.4713 0.4861 0.4994 0.5102	0.4571 0.4726 0.4863 0.4970
5	θ_{VeryFine}	0.538	0.533	0.5308	0.517	0.503

The figs. 1 and 2 contain the two dimensional polynomial form of volumetric moisture content $\theta(pF, TS)$ for unsaturated soil as a function of the soil texture class TS, and the suction values (pF between 0 and 4.2.).



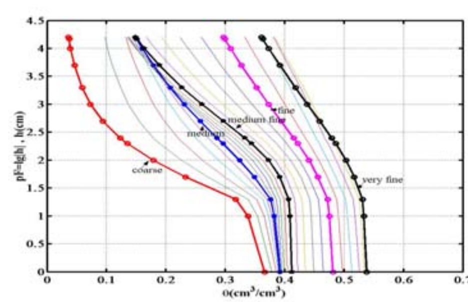


Fig. 1. Correlations between pF and θ, for Topsoils. Fig. 2. Correlations between pF and θ, for Subsoils.

The two dimensional Lagrange polynomial interpolation have been done with standardized HYPRES values of moisture content at 14 pressure head using the optimized Mualem-van Genuchten parameters.

We derived polynomial functions for hydraulic conductivities, too.

Such kind of functions can be successfully used (despite there long polynomial form) because the moisture content and hydraulic conductivity can be compute using only the soil textural class and the pressure head values.

4. Acknowledgements

This research was developed in the frame of a Romanian National Research Program IDEI, grant 189/2007, sponsored by CNCSIS, Romania.

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