MINIMIZATION OF HIGH-FREQUENCY OSCILLATIONS OF TROLLEY MOVEMENT MECHANISM DURING STEADY TOWER CRANE SLEWING

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This article describes the optimizing of the trolley movement mechanism during steady tower crane slewing. It provides minimization of high-frequency oscillations of the links of the mechanism. The optimization was carried out on the basis of a dynamic model of the trolley movement mechanism, which is represented by a system of three second-order differential equations. This system is reduced to one differential equation of the sixth order, which describes the change of the driving torque as an expression of load position and its higher time derivatives.

The variational problem of optimal control of the trolley movement mechanism, where the root mean square value of the rate of the driving torque change (the optimization criterion) was stated and solved. In the optimization, high-frequency oscillations of the elements of the mechanism were eliminated during the start-up process. Low-frequency oscillations that are caused by the oscillation of the load on the flexible suspension, were eliminated at the beginning of the steady motion. It was achieved due to the selection of the proper boundary conditions.

The condition of the minimum of the integral functional, which is represented by a linear differential equation of the fourteenth order, was solved by the analytical method in the process of the variational problem solving.

Keywords: tower crane, trolley movement mechanism, rate of driving torque change, variational problem, optimization criterion.

1. Introduction

In order to increase the capacity of the tower cranes, their operators often combine several operations (slewing and hoisting, slewing and the trolley movement, etc.). However, this increases the dynamic loads of oscillatory nature

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in the elements of the crane mechanisms. For instance, low-frequency oscillations that are caused by deviations from the vertical of the flexible suspension with a load and high-frequency oscillations that depend on the nature of the change in the driving torque of the drive mechanisms are observed.

The low- and high-frequency oscillations of the elements and the metal structure of the crane lead to increasing the loads and, as a result, may cause damages. In addition, the non-optimal manner of the mechanisms movement control increases the energy losses.

Therefore, the development of optimal control (the driving torque law) of the trolley movement mechanism, which leads to the reduction of high-frequency oscillations, is an important goal to achieve.

2. Analysis of recent studies

A significant number of scientific works have been dedicated to the study of dynamic loads during the operation of the cranes' mechanisms. Among them, we can outline the following ones [1-14].

General issues of hoisting machines dynamics are described in the following fundamental studies [1-3]. Dynamic processes during the operation of the mechanisms of the trolley movement and hoisting considering different types of cranes were investigated and the factors, which influence the load oscillations, were established in studies [4-7]. Investigations [8, 9] include the research of joint movement of mechanisms of the trolley movement and slewing of the crane. Here control of the trolley movement mechanisms was found, it provides the reduction of the pendulum load oscillations.

In the study [10] the dynamic processes of the joint motion of the mechanisms of the trolley movement and tower crane slewing were investigated. The driving torques were changed in both of the mechanisms. The work revealed the tendencies in kinematic, dynamic, and power characteristics of the mechanisms. The loads that act on the elements of these mechanisms were established as well as pendulum oscillations of the load.

Several optimization problems were solved [11-14] in order to reduce the oscillations of the load on the flexible suspension during the operation of crane mechanisms. The problem of reduction of the pendulum load oscillations during the slewing of the tower crane was solved in [11]. Here, a complex dynamic criterion was used in the calculations. The acceleration of the crane movement mechanism was optimized in the study [12]. It was grounded on the controlled drive. The obtained in the work result provides the minimum duration of the mechanism acceleration with the elimination of the load oscillations.

The solution of the time-optimal problem for the dynamical system „crane-load" has the form of the „on-off" function [13, 14]. This leads to the appearance of high-frequency oscillations and, as a consequence, additional loads on the elements of the drive mechanisms and the metal structure of the crane.

Therefore, the problem of eliminating high-frequency oscillations in the elements of crane mechanisms during their joint operation is relevant and requires further study.

The purpose of the current study is to reduce the high-frequency component of the oscillations of the trolley movement mechanism during the steady slewing of the tower crane.

2. Statement of the optimal control problem

A dynamic model of the trolley movement mechanism during tower crane steady slewing (fig. 1) was developed in order to conduct this research. In this model, the crane boom system is presented as a holonomic mechanical system. It consists of absolutely rigid bodies, except for the traction rope, which has stiffness properties with a stiffness coefficient C or C' depending on the direction of the trolley movement as well as a flexible suspension of the load, which oscillates in the plane of the trolley movement (along the boom).

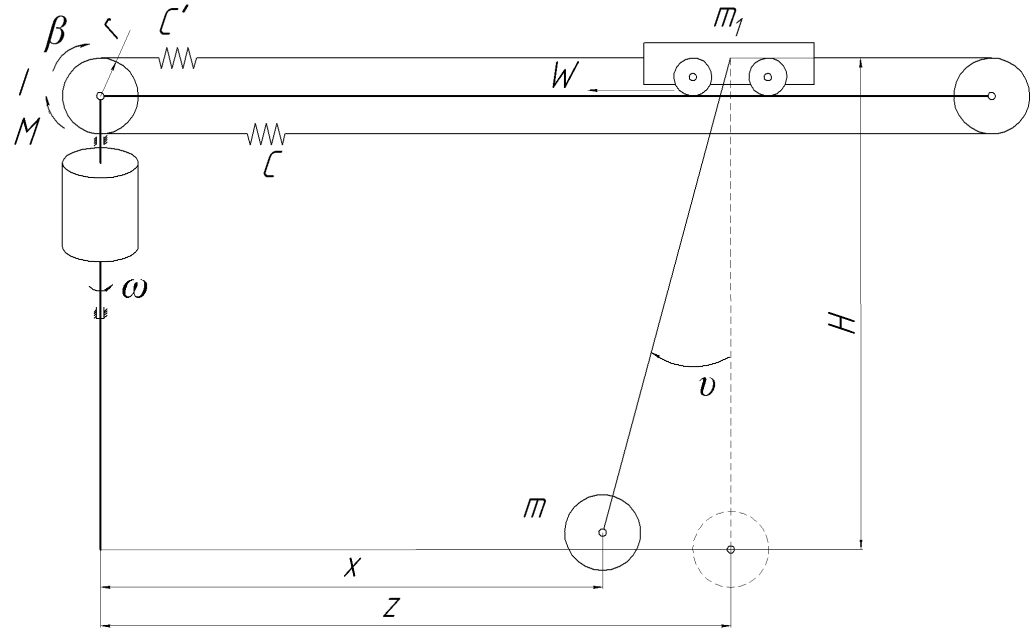

Fig. 1. The dynamic model of the trolley movement mechanism in case of the steady slewing of the crane

In the accepted dynamic model, the linear coordinates of the centers of mass of the trolley z and the load x, as well as the angular coordinate of the rotation of the drive pulley β are used as the generalized coordinates. In addition, the crane boom system slews around the crane axis of rotation with a constant angular velocity ω. The length of the suspension of the load H in the studied case is a constant, i.e. $H=const$.

The following differential equations describe the motion of the dynamical system:

$$\begin{cases} I\ddot{\beta} = M - cr(\beta r - z); \\ m_1\ddot{z} - m_1\omega^2 z = c(\beta r - z) - \frac{mg}{H}(z - x) - W; \\ m\ddot{x} - m\omega^2 x = \frac{mg}{H}(z - x), \end{cases} \tag{1}$$

where m_1 and m are the reduced masses of the trolley and the load, respectively;
I and M – the moment of inertia and the driving torque of the trolley movement mechanism, respectively (they are reduced to the axis of the pulley rotation);
r – pulley radius;
g – acceleration of gravity;
W – force of resistance to the trolley movement;
c – coefficient of stiffness (its numerical value depends of the direction of the trolley movement: $c=C$ – the case when the trolley moves towards the tower, $c=C'$ – the case when the trolley moves away the tower).

From the last equation of system (1) the coordinate of the center of mass of the trolley and its time derivatives may be calculated:

$$z = \left(1 - \frac{H}{g}\omega^2\right)x + \frac{H}{g}\ddot{x}; \tag{2}$$

$$\dot{z} = \left(1 - \frac{H}{g}\omega^2\right)\dot{x} + \frac{H}{g}\dddot{x}; \tag{3}$$

$$\ddot{z} = \left(1 - \frac{H}{g}\omega^2\right)\ddot{x} + \frac{H}{g}\overset{IV}{x}. \tag{4}$$

From the second equation of the system (1) taking into account expressions (2)-(4) we might express the angular coordinate of the pulley:

$$\beta = \frac{1}{cr}\left\{\left[\left(c - m_1\omega^2\right)\left(1 - \frac{H}{g}\omega^2\right) - m\omega^2\right]x + \left[\left(c - 2m_1\omega^2\right)\frac{H}{g} + m_1 + m\right]\ddot{x} + m_1\frac{H}{g}\overset{IV}{x} + W\right\}. \tag{5}$$

Then the angular velocity and acceleration of the pulley may be obtained:

$$\dot{\beta} = \frac{1}{cr}\left\{\left[\left(c - m_1\omega^2\right)\left(1 - \frac{H}{g}\omega^2\right) - m\omega^2\right]\dot{x} + \left[\left(c - 2m_1\omega^2\right)\frac{H}{g} + m_1 + m\right]\dddot{x} + m_1\frac{H}{g}\overset{V}{x}\right\}. \tag{6}$$

$$\ddot{\beta} = \frac{1}{cr}\left\{\left[\left(c - m_1\omega^2\right)\left(1 - \frac{H}{g}\omega^2\right) - m\omega^2\right]\ddot{x} + \left[\left(c - 2m_1\omega^2\right)\frac{H}{g} + m_1 + m\right]\overset{IV}{x} + m_1\frac{H}{g}\overset{VI}{x}\right\}. \quad (7)$$

Substitution the expressions (2)-(7) in the first equation of the system (1) brings the formula for driving torque calculation:

$$M = a_0 + a_1 x + a_2\ddot{x} + a_3\overset{IV}{x} + a_4\overset{VI}{x}, \quad (8)$$

where

$$\begin{cases} a_0 = Wr; \\ a_1 = -\left[m - m_1\left(1 - \frac{H}{g}\omega^2\right)\right]\omega^2 r; \\ a_2 = \frac{I}{cr}\left[\left(c - m_1\omega^2\right)\left(1 - \frac{H}{g}\omega^2\right) - m\omega^2\right] + \left[m + m_1\left(1 - 2\frac{H}{g}\omega^2\right)\right]r; \\ a_3 = \frac{I}{cr}\left[\left(c - 2m_1\omega^2\right)\frac{H}{g} + m + m_1\right] + m_1\frac{H}{g}r; \\ a_4 = \frac{I \cdot m_1}{cr}\frac{H}{g}. \end{cases} \quad (9)$$

Taking into account the expression (8) we may write down the time derivative of the driving torque:

$$\dot{M} = \frac{dM}{dt} = a_1\dot{x} + a_2\dddot{x} + a_3\overset{V}{x} + a_4\overset{VII}{x} = \left[a_1\frac{d}{dt} + a_2\frac{d^3}{dt^3} + a_3\frac{d^5}{dt^5} + a_4\frac{d^7}{dt^7}\right]x. \quad (10)$$

Since the high-frequency oscillations of the links of the trolley movement mechanism depend on the rate of the driving torque change, we set the root mean square value of the $\dot{M}$ as a criterion to minimize:

$$\dot{M}_{ск} = \left[\frac{1}{t_1}\int_0^{t_1}\dot{M}^2 dt\right]^{1/2} \rightarrow \min, \quad (11)$$

where t – time;
t_1 – duration of the start (acceleration) of the mechanism.

Criterion (11) should be minimized. Indeed, it reflects the undesirable property of the trolley movement mechanism. Therefore, we consider a variational problem in which it is necessary to minimize the functional (11), where the desired solution of the problem $x=x(t)$, $0 \le t \le t_1$, must satisfy the boundary conditions:

$$t=0: x=x_0;\ \dot{x}=0,\ \ddot{x}=x_0\omega^2,\ \dddot{x}=0,\ \overset{IV}{x}=x_0\omega^4,\ \overset{V}{x}=0,\ \overset{VI}{x}=x_0\omega^6;$$
$$t=t_1: x=x_1=x_0+\frac{vt_1}{2},\ \dot{x}=v,\ \ddot{x}=x_1\omega^2,\ \dddot{x}=v\omega^2,\ \overset{IV}{x}=x_1\omega^4,\ \overset{V}{x}=v\omega^4,\ \overset{VI}{x}=x_1\omega^6, \tag{12}$$

where x_0 – the initial position of the trolley and load;
v – the steady velocity of the trolley and the load.

3. Solving of the optimal control problem

Variation problem (11) can be rewritten in equivalent form as follows:

$$\int_0^{t_1}\dot{M}^2 dt \to \min. \tag{13}$$

The Euler-Poisson equation is the condition of the minimum of the functional (13), which for the case (13) may be presented is as follows:

$$\frac{d}{dt}\frac{\partial\dot{M}^2}{\partial\dot{x}}+\frac{d^3}{dt^3}\frac{\partial\dot{M}^2}{\partial\dddot{x}}+\frac{d^5}{dt^5}\frac{\partial\dot{M}^2}{\partial\overset{V}{x}}+\frac{d^7}{dt^7}\frac{\partial\dot{M}^2}{\partial\overset{VII}{x}}=0.$$

The use the rule of a complex function differentiation and substitution an explicit expression for $\dot{M}$ gives the following:

$$\frac{d}{dt}\left(2\dot{M}a_1\right)+\frac{d^3}{dt^3}\left(2\dot{M}a_2\right)+\frac{d^5}{dt^5}\left(2\dot{M}a_3\right)+\frac{d^7}{dt^7}\left(2\dot{M}a_4\right)=0 \Leftrightarrow$$
$$\Leftrightarrow a_1\frac{d\dot{M}}{dt}+a_2\frac{d^3\dot{M}}{dt^3}+a_3\frac{d^5\dot{M}}{dt^5}+a_4\frac{d^7\dot{M}}{dt^7}=0 \Leftrightarrow \tag{14}$$
$$\left[a_1\frac{d}{dt}+a_2\frac{d^3}{dt^3}+a_3\frac{d^5}{dt^5}+a_4\frac{d^7}{dt^7}\right]^2 x=0.$$

The obtained equation (14) is, in fact, a linear, homogeneous differential equation of the 14-th order with respect to the unknown function x. In order to solve it, we have to calculate the roots of a characteristic polynomial, which may be expressed as follows:

$$Q(\lambda)=\left[a_1\lambda+a_2\lambda^3+a_3\lambda^5+a_4\lambda^7\right]^2=\lambda^2\left[a_1+a_2\lambda^2+a_3\lambda^4+a_4\lambda^6\right]^2$$

The polynomial $Q(\lambda)$ is a square of a polynomial of the 7-th order. That's why it has 7 roots of the 2-nd order. One of them, obviously, is a zero $\lambda_0=0$. The following equation has to be solved in order to calculate other roots:

$$a_1 + a_2\lambda^2 + a_3\lambda^4 + a_4\lambda^6 = 0.$$

In order to reduce the degree of the previous equation, we use the designation $\lambda^2=\mu$. As a result, we obtain an algebraic equation of the 3-rd order

$$a_1 + a_2\mu + a_3\mu^2 + a_4\mu^3 = 0, \tag{15}$$

the roots of which can be calculated analytically by the Cardano method or in an approximately manner with one of the numerical methods.

In the frame of the current research the following numerical values were used: m=5000 kg, I=30 kgm^2, H=10 m, ω=0.075 rad/s, r=0.15 m, c=1.65·10^5 N/m, V=0.85 m/s, x_0=7 m, t_1=5 s, W=5500 N. Then the approximate solutions of equation (15) are as follows: $\mu_1\approx$-687.17, $\mu_2\approx$-3.9041, $\mu_3\approx$-0.0044954.

Using the equality $\lambda^2=\mu$, we may find the roots of the characteristic polynomial $Q(\lambda)$:

$$\lambda_{1,2} = \pm\sqrt{\mu_1} \approx \pm i\cdot 26.214 = \pm i\cdot\alpha_1,\ \lambda_{3,4} = \pm\sqrt{\mu_2} \approx \pm i\cdot 1.91759 = \pm i\cdot\alpha_2,$$

$$\lambda_{5,6} = \pm\sqrt{\mu_3} \approx \pm 0.067048 = \pm\alpha_3,$$

where i is the imaginary unit.

Let's remind that all the roots $\lambda_{1,2,3,4,5,6}$ of characteristic polynomial $Q(\lambda)$ are the roots of the second order. Then the general solution of the linear and homogeneous differential equation (14) can be written as follows:

$$\begin{aligned} x(t) &= (C_1 + C_2 t)\cos(\alpha_1 t) + (C_3 + C_4 t)\sin(\alpha_1 t) + (C_5 + C_6 t)\cos(\alpha_2 t) + \\ &+ (C_7 + C_8 t)\sin(\alpha_2 t) + (C_9 + C_{10} t)e^{\alpha_3 t} + (C_{11} + C_{12} t)e^{-\alpha_3 t} + C_{13} + C_{14} t,\ 0 \le t \le t_1, \end{aligned} \tag{16}$$

where $C_{1,\ldots,14}$=*const*.

In order to calculate the coefficients $C_{1,\ldots,14}$ we shall substitute the image (16) in the boundary conditions (12) of the original problem. As a result, we will receive a system of linear algebraic equations of the 14-th order with respect to $C_{1,\ldots,14}$. The approximate solution of this will look as follows: C_1=0, C_2=0, C_3=0, C_4=0, $C_5\approx$0.0026020, $C_6\approx$-0.0031223, $C_7\approx$0.027422, $C_8\approx$-0.0014111, $C_9\approx$-2279.6, $C_{10}\approx$34.549, $C_{11}\approx$6406.3, $C_{12}\approx$172.48, $C_{13}\approx$-4119.7, $C_{14}\approx$375.28.

Substitution of the values $C_{1,\ldots,14}$ in (16) brings the final solution of the variational problem (13), which, in fact, coincides with the original problem (11).

4. Brief results analysis

As a result of solving the variational problem the plots of kinematic (fig. 2-4), power (fig. 5, fig. 6), and energy (fig. 7) characteristics were built (in fig. 2-4. broken curves refer to the trolley, the continuous curves – to the load).

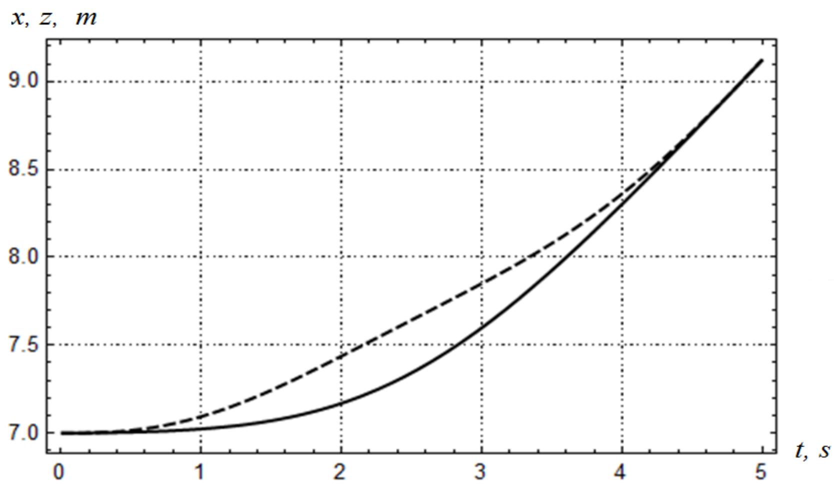

Fig. 2. Plots of the trolley and the load positions

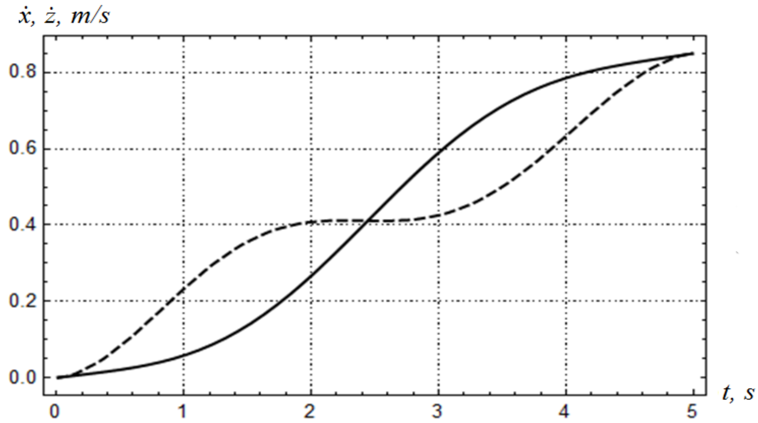

Fig. 3. Plots of the trolley and the load velocities

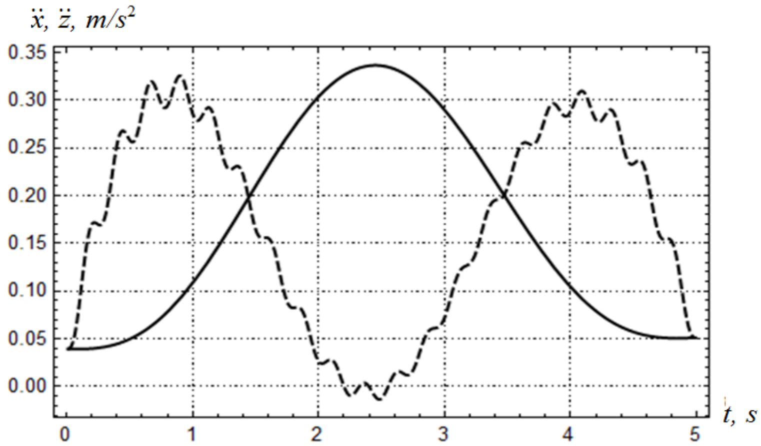

Fig. 4. Plots of the trolley and the load acceleration

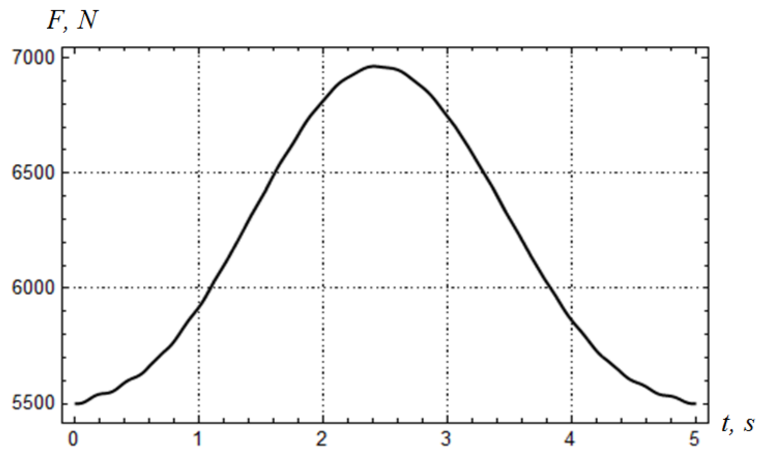

Fig. 5. Plot of the trolley driving force

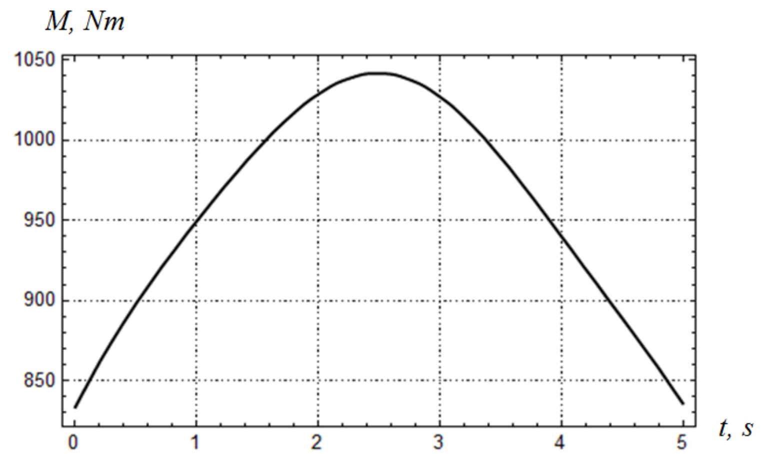

Fig. 6. Plot of the driving torque of the trolley movement mechanism

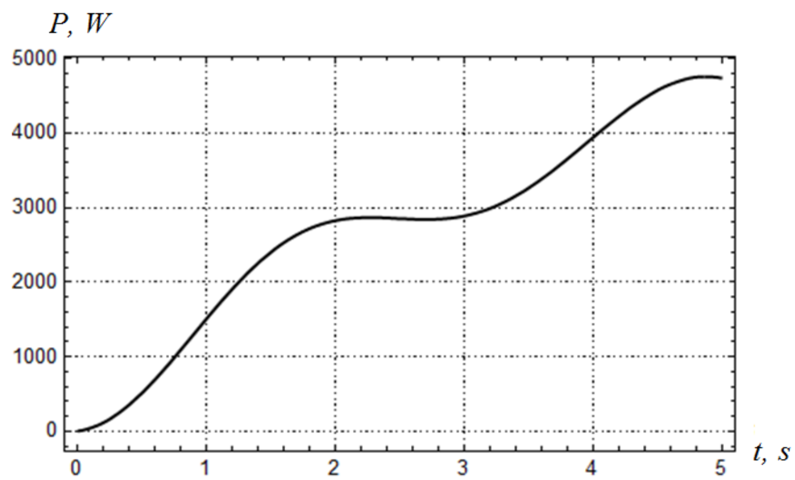

Fig. 7. Plot of the power of the trolley movement mechanism

These plots indicate that low-frequency oscillations of the trolley and the load. They are eliminated at the beginning of the steady mechanism movement. Indeed, at this moment the coordinates and velocities of the trolley and the load coincide.

There are slight high-frequency oscillations of the acceleration of the trolley, which attenuate during the steady movement. As an example, we show low- and high-frequency oscillations of the trolley and the load accelerations during the optimal mode of movement (fig. 8). In the shown case the criterion is a root mean square value of the driving torque. It might be obtained by substituting in expression (16) following coefficients $C_{1,\ldots,14}$: $C_1\approx-1.354\cdot10^{-6}$, $C_2\approx1.0536\cdot10^{-7}$, $C_3\approx2.0921\cdot10^{-8}$, $C_4\approx1.8714\cdot10^{-7}$, $C_5\approx3.6064\cdot10^{-2}$, $C_6\approx1.4894\cdot10^{-2}$, $C_7\approx2.0835\cdot10^{-2}$, $C_8\approx-3.1770\cdot10^{-3}$, $C_9\approx-7.8196\cdot10^{1}$, $C_{10}\approx2.9682\cdot10^{0}$, $C_{11}\approx-9.9386\cdot10^{1}$, $C_{12}\approx-4.4152\cdot10^{0}$, $C_{13}=0$, $C_{14}=0$. They correspond to the same boundary conditions (12).

From the given plots it is possible to see that high-frequency oscillations of accelerations of the load are absent, and the trolley has a considerable amplitude 0.60 m/s^2. In the case of optimization by criterion (13) the similar value is 0.33 m/s^2. Optimization by the criterion of the root mean square value of the driving torque leads to the following results: the maximum value of the load acceleration is 0.25 m/s^2, and the trolley acceleration is 1.3 m/s^2.

Plots of the force in the traction body (rope) of the trolley and the driving torque of the trolley movement mechanism are shown in fig. 5 and fig. 6. These plots indicate that there are virtually no low- or high-frequency oscillations. The maximum value of force is 6900 N, the maximum value of the torque is 1040 Nm. Optimization by the criterion of the root mean square of the driving torque value, both low- and high-frequency oscillations of traction force (fig. 9) are observed. Their maximum value is 6650 N. Curve of driving torque includes an insignificant high-frequency component (fig. 10) at a maximum value 1027 Nm.

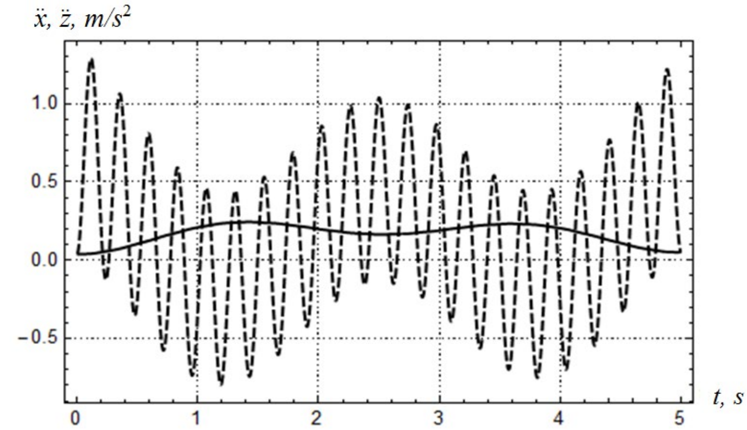

Fig. 8. Plots of the trolley and the load acceleration (case of optimization by the root mean square criterion of the driving torque)

The power of the drive mechanism (fig. 7) includes an insignificant low-frequency component. The high-frequency oscillations are absent. The maximum value of the power is 4800 W. Optimization by the criterion of the root mean square of the driving torque value leads to the power with increased low-frequency component of oscillations and a rather small high-frequency component at a maximum value of 5800 W the (fig. 11).

The analysis of the obtained results indicates that in case of optimization of the trolley movement mechanism by the criterion of root mean square value of the driving torque the maximum values of traction force and the driving torque are bigger than similar values corresponding to the criterion (13). In addition, during this mode of movement significant low- and high-frequency oscillations of links of the mechanism may be observed.

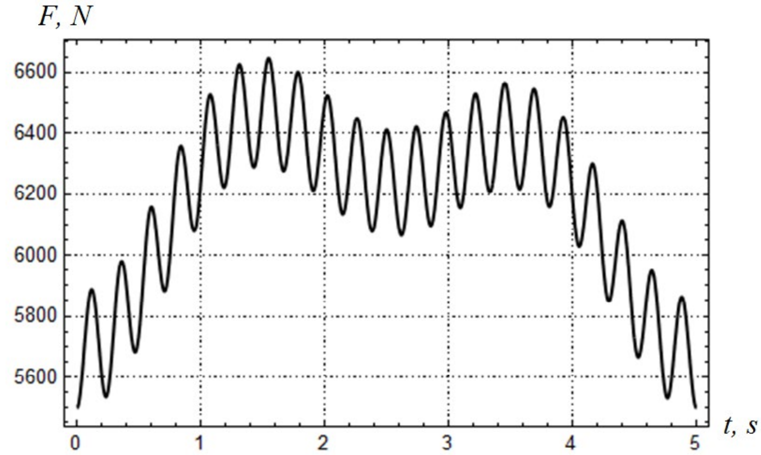

Fig. 9. Plot of the trolley traction force (case of optimization by the root mean square criterion of the driving torque)

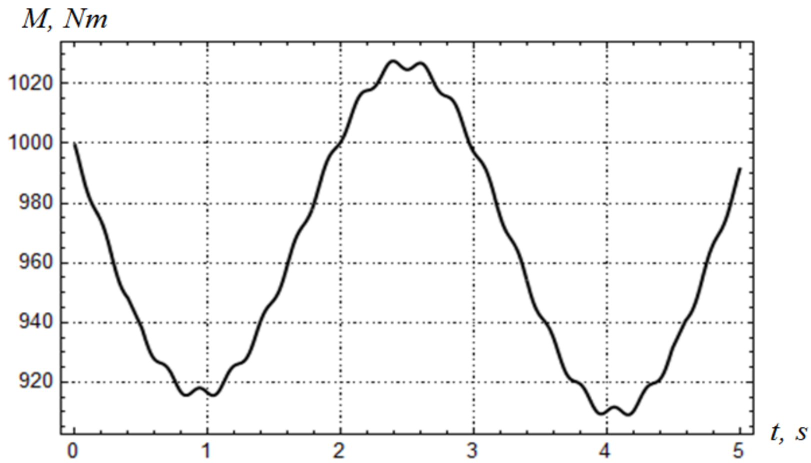

Fig. 10. Plot of the driving torque of the trolley movement mechanism (case of optimization by the root mean square criterion of the driving torque)

Optimization by the criterion (13) brings a slightly increase in the maximum values of traction force (by 3.7%), driving torque (by 1.3%), and load acceleration (by 32%). However, it significantly reduces the maximum values of the drive power (by 21%), the acceleration of the trolley (by 394%), and, most importantly, virtually eliminates low- and high-frequency oscillations of the trolley movement mechanism elements.

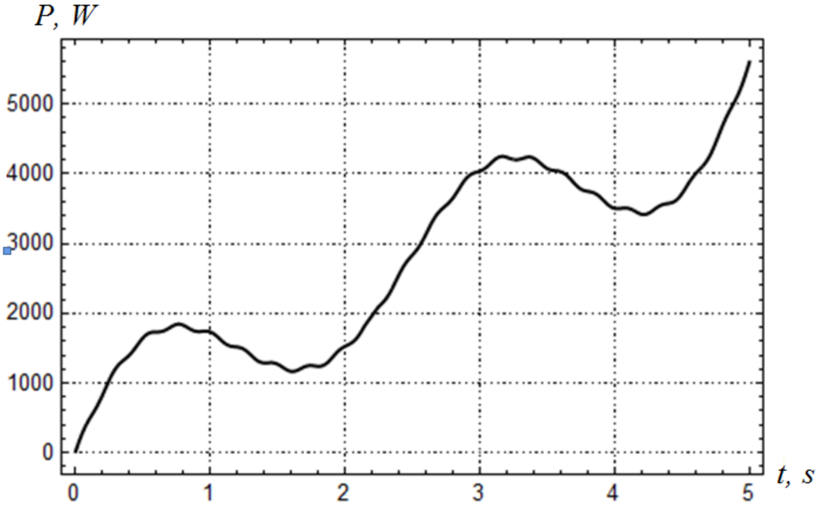

Fig. 11. Plot of the power of the trolley movement mechanism (case of optimization by the root mean square criterion of the driving torque)

In order to support the statement about high-frequency oscillations of drive torque minimization we have carried out calculations of numerical values of criterion (11) for both cases: considered in the article and the referred (case of optimization by the root mean square criterion of the driving torque). For the first case $\dot{M}_{ск}$=70.27 Nm/s, for the second one $\dot{M}_{ск}$=127.87 Nm/s. Such deviation might be explained by the exploitation of criterion (11). Also we have shown additional indicator $\ddot{x}_{ск}=\left[\frac{1}{t_1}\int_0^{t_1}\ddot{x}^2dt\right]^{1/2}$. For the first case $\ddot{x}_{ск}$=0.20 m/s^2, for the second one $\ddot{x}_{ск}$=0.28 m/s^2. These values support our previous conclusion about minimization of high-frequency oscillations.

5. Conclusions

In the article, the variational problem was stated and solved. It involves the criterion (the root mean square value of the rate of change of the driving torque of the trolley movement mechanism drive), the developed mathematical model of dynamics of the mechanism during steady tower crane slewing.

The initial variational problem was reduced to a homogeneous linear differential equation of the 14-th order with constant coefficients relative to the coordinate of the load. It was solved with the analytical method under given boundary conditions of the load movement.

The obtained result gives the practical effect: it eliminates low-frequency (pendulum) oscillations of the load and high-frequency oscillations of the mechanism links during controlled movement. It, in turn, improves the crane capacity, durability and increases the energy consumption of the trolley movement mechanism.

Perspectives of further investigation in this direction are connected with involving other (complex ones) criteria to minimize, constraints, improving mathematical models of the tower crane mechanisms, etc.

R E F E R E N C E S

[1]. *S.A. Kazak* Динамика мостовых кранов (Dynamics of Overhead Cranes). – Moscow: Mechanical Engineering, 1968. – p. 332.

[2]. *M.S. Komarov* Динамика грузоподъемных машин (Dynamics of Hoisting Machines). Moscow: Mechanical Engineering. – 1969. – p. 206.

[3]. *N. A. Lobov* Динамика грузоподъемных кранов (Dynamics of Hoisting Cranes). Moscow: Mechanical Engineering. – 1987. – p. 160.

[4]. *H.P. Lee* „Dynamic Responses of a Beam with a Moving Mass" in Journal of Sound and Vibration, **vol**. 191 (2), 1996, pp. 289–294.

[5]. *D.C.D. Oguamanam, J.S. Hansen, G.R. Heppler* „Dynamics of a Three-Dimensional Overhead Crane System" in Journal of Sound and Vibration, **vol**. 242 (3), 2001, pp. 411–426.

[6]. *G. Sun, M. Kleeberger* „Dynamic Responses of Hydraulic Mobile Crane with Consideration of the Drive System" in Mechanism and Machine Theory, **vol**. 38 (12), 2003, pp. 1489-1508.

[7]. *A.N. Orlov* „General Dynamic Model of Load-Lifting Cranes (Общая динамическая модель грузоподъемных кранов)" in Proceedings „Optimization of Parameters of Construction and Road Machines". – Yaroslavl: Publishing House of Yaroslavl Political Institute, 2012. pp. 13–20.

[8]. *R.P. Gerasimyak, V. A. Leshchev* Анализ и синтез крановых электромеханических систем (Analysis and Synthesis of Crane Electromechanical Systems). – Odessa: SMIL, 2008. – p. 192.

[9]. *R.P. Gerasymyak, O.V. Naidenko* „Features of Electric Drive Control of the Crane Boom Outreach Mechanism During Rotation of the Crane with Suspended Load" in Electrical Engineering and Electrical Equipment, Issue 68, 2007, pp. 11–15.

[10]. *V.S. Loveykin, Yu.O.Romasevych, A.V. Loveykin, D.I. Mushtyn* „Dynamic Analysis of the Movement of the Mechanisms of the Trolley Movement and Slewing of Tower Crane with a Boom (Динамічний аналіз руху механізмів зміни вильоту та повороту баштового крана з балочною стрілою)" in Machinery & Energetics. Journal of Rural Production Research, **vol**. 11, No 1, 2020, pp. 5-11.

[11]. *V. Loveikin, Y. Romasevych, I. Kadykalo, A. Liashko* „Optimization of the Swinging Mode of the Boom Crane upon a Complex Integral Criterion" in Journal of Theoretical and Applied Mechanics (Bulgaria), 2019, **vol**. 49(3), pp. 285–296.

[12]. *E.V. Naidenko* „Control of the Electric Drive of Mechanisms of Horizontal Movement with the Suspended Load (Управление электроприводом механизмов горизонтального перемещения с подвешенным грузом)” in Electrical Engineering and Electrical Equipment, Issue 69, 2007, pp. 17–22.
[13]. *V.S. Loveikin, Y.A. Romasevich, A.S. Khoroshun, A.G. Shevchuk* „Time-Optimal Control of a Simple Pendulum with a Movable Pivot. Part 1” in International Applied Mechanics, **vol. 54**, Issue 3, 2018, pp. 358-365.
[14]. *Y. Romacevych, V. Loveikin, O. Stekhno* „Closed-loop optimal control of a system Trolley - Payload” in UPB Scientific Bulletin, Series D: Mechanical Engineering, **vol. 81**, Issue 2, 2019, pp. 3-12.