NUMERICAL MODELLING OF THE COUPLING ELECTROMAGNETIC- HYDRODYNAMIC EQUATIONS OF AN ANNULAR INDUCTION MHD PUMP

Nassima BERGOUG[1], Fatima Zohra KADID[2], Rachid ABDESSEMED[3]

Abstract- The magnetohydrodynamic (MHD) is the study of the interaction of electrically conducting fluids and electromagnetic fields. This paper presents the numerical modelling of the coupling electromagnetic- hydrodynamic phenomena using the Finite volume Method (FVM) in cylindrical coordinate and the stream vorticity formulation ξ, ψ in two-dimensional in an annular induction MHD pump. The magnetic induction and the electromagnetic force in MHD pump are presented. The variation of the velocity in the channel of the MHD pump in harmonic mode is also done.

Keywords: Annular channel, induction pump, magneto hydrodynamic (MHD), Maxwell equations, Navier- Stokes equations, finite volume method

Nomenclature

The symbols used are defined as

$\vec{A}$: Magnetic vector potential [Tm];
$\vec{B}$: Magnetic induction [T];
$\vec{J}$: Electric conduction current density [A/m^2];
($\vec{J} = \overrightarrow{J_{ex}} + \overrightarrow{J_{in}}$)
$\overrightarrow{J_{in}}$: Induced current density [A/m^2];
$\overrightarrow{J_{ex}}$: Current density [A/m^2];
$\vec{V}$: Flow velocity [m/s];
σ : Electric conductivity [(Ωm)$^{-1}$];
μ : Permeability [H/m];

[1] L.E.B. Research Laboratory, Electrical engineering department; University of Batna, Algeria, e-mail: nassima1_elt@yahoo.fr
[2] L.E.B. Research Laboratory, Electrical engineering department; University of Batna, Algeria
[3] L.E.B. Research Laboratory, Electrical engineering department; University of Batna, Algeria

$\vec{\xi}$ (r, z): Vorticity vector;
$\vec{\psi}$ (r, z): stream function;
V_r, V_z: Components of the velocity;
P: pressure of the fluid (Pa);
υ : Kinematic viscosity of the fluid (m^2 / s);
$\vec{F}$: Electromagnetic force (N / m^3);
ρ : Fluid density (kg/m^3).

1. Introduction

Annular linear induction pumps (ALIP) are used for the propulsion of the fluids. The pumping action is obtained by Lorentz force produced by the interaction of induced current in the liquid metal and their associated magnetic field. Even though the efficiency of the ALIP is very low compared to conventional mechanical pumps, it is very useful due to the absence of moving parts, low noise and vibration level, simplicity of flow rate regulation and maintenance, and high temperature operation capability [1].

Linear induction machines use a traveling magnetic field wave created by polyphase currents. The most well-known device working on this principle is the Linear Induction Motor (LIM) such as developed for ground transportation. There is also another type of linear inductor whose application belongs to the field of magnetohydrodynamics (MHD) [2] and [3].

The magnetohydrodynamic (MHD) is the theory of the interaction of electrically conducting fluids and electromagnetic fields. Application arises in astronomy and geophysics as well as in connection with numerous engineering problems, such as liquid metal cooling of nuclear reactors, electromagnetic casting of metals, MHD power generation and propulsion [4], [5] and [6].

In the previous work [7] we studied the 2D electromagnetic phenomena in a MHD pump by the finite volume method in harmonic mode. Different characteristics of the MHD pump (vector potential, magnetic induction, currents density and the electromagnetic force) were obtained.

In this work we study the coupling electromagnetic - hydrodynamic equations of an annular induction MHD pump using the finite volume method and the stream vorticity formulation $\boldsymbol{\xi}$, $\boldsymbol{\psi}$. The proposed pump produces an axial flow. As a consequence, electromagnetic force can be obtained before calculating the velocity profiles in the channel.

2. Mathematical models and equations

An electromagnetic pump for a liquid metal is considered. A schematic view of the pump is shown in Fig.1. The liquid metal flows along a channel with a cylindrical geometry of annular cross section. A ferromagnetic core is placed on the inner and the outer side of the channel, [2].

The principle of the MHD pump (Fig.1) is similar to that of the asynchronous motor; the supply of the inductor creates a magnetic field ***B*** sliding with the velocity of synchronism, where electric currents are induced in the liquid metal by means of a magnetic field, producing an electromagnetic force **F** with the instantaneous field ensuring the flows of the fluid, [8].

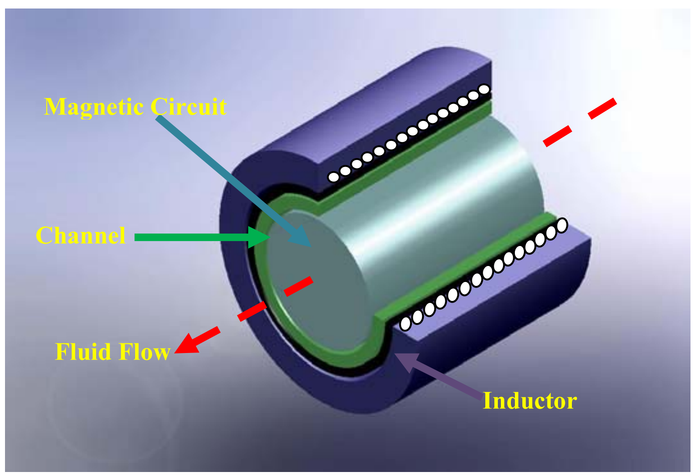

Fig.1. Schematic view of the MHD pump

The MHD pump dimensions and the properties of the mercury are given respectively in tables 1 and 2.

Table 1

Dimensions of the MHD pump

Parameters	Values
Channel's length	40 cm
Channel's width	2 cm
Inductor width	20 cm
Inductor length	40 cm
Air-gap width	4 mm

Table 2

The properties of the mercury

Parameters	Values
Density ρ	$13.6*10^3$ (kg/m^3)
Electric conductivity σ	$1.06*10^6$ (Ω. m)$^{-1}$
Viscosity μ	$0.11*10^{-6}$ (m^2/s)

The Maxwell's equations used; where to magnetic vector potential ***A*** has only one component are characterized by:

$$\overrightarrow{rot}\left(\frac{1}{\mu}\overrightarrow{rot}\,\vec{A}\right) = \vec{J}_{ex} - \sigma\left(\frac{\partial \vec{A}}{\partial t} - \vec{V}\,\Lambda\,\overrightarrow{rot}\,\vec{A}\right) \tag{1}$$

The magnetic induction and the electromagnetic force are given by:

$$\vec{B} = \overrightarrow{rot}\vec{A} \tag{2}$$

$$\vec{F} = \vec{J}\,\Lambda\,\vec{B} \tag{3}$$

After developments of the equation (1) in cylindrical coordinates we obtain:

$$\frac{\partial}{\partial z}\left(\frac{1}{r\mu}\frac{\partial A}{\partial z}\right)+\frac{\partial}{\partial r}\left(\frac{1}{r\mu}\frac{\partial A}{\partial r}\right)=-J_{ex}+\frac{\sigma}{r}\left(\frac{\partial A}{\partial t}+V\frac{\partial A}{\partial z}\right) \tag{4}$$

The equations describing the pumping process in the channel are presented as:

$$\frac{\partial \vec{V}}{\partial t}+\left(\vec{V}.\vec{\nabla}\right)\vec{V} = -\frac{1}{\rho}\overrightarrow{grad}P + \upsilon\Delta\vec{V} + \frac{\vec{F}}{\rho} \tag{5}$$

$$div\vec{V} = 0 \tag{6}$$

where V is the velocity, p the pressure, ρ the density of the liquid, υ the kinematic viscosity and $\vec{F}$ the Lorentz force which is given by (3).

The difficulty is that in the previous equations there are two unknown: the pressure and the velocity. The elimination of pressure from the equations leads to a vorticity-stream function which is one of the most popular methods for solving the 2-D incompressible Navier-Stokes equations [12]:

$$\vec{V} = \overrightarrow{rot}\vec{\psi}\ ,\ \vec{\xi} = \overrightarrow{rot}\vec{V} \tag{7}$$

$$V_r = -\frac{1}{r}\frac{\partial \psi}{\partial z}\ ,\ V_z = \frac{1}{r}\frac{\partial \psi}{\partial r} \tag{8}$$

$$\xi = \frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r} \tag{9}$$

Using these new dependent variables, the two momentum equations can be combined to give:

$$\upsilon\left[\frac{\partial^2 \xi}{\partial r^2} + \frac{\partial^2 \xi}{\partial z^2} + \frac{1}{r}\frac{\partial \xi}{\partial r} - \frac{\xi}{r^2}\right] = \frac{\partial \xi}{\partial t} + \frac{\partial \xi}{\partial r}V_r + \frac{\partial \xi}{\partial z}V_z + \frac{V_r}{r}\xi + \frac{1}{\rho}\left(\frac{\partial F_z}{\partial r}\right) \tag{10}$$

An additional equation involving the new dependent variables ξ and ψ can be obtained by substituting (10) into (7) which give:

$$\frac{1}{r}\left[\frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial r^2}\right] = -\xi \tag{11}$$

3. Numerical method and results

The method consists of discretising differential equations by integration on finite volumes surrounding the nodes of the grid.

In this method, each principal node P is surrounded by four nodes N, S, E and W located respectively at North, South, East and West (Fig.2) [11].

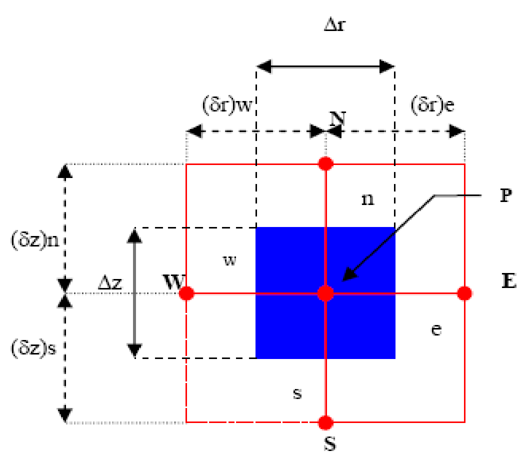

Fig.2. Discretization in finite volume method

We integrate the electromagnetic and Navier Stokes equation using the finite volume method on the domain delimited by the surfaces E, W, N and S. Finally we obtain the algebraic equation which is written as:

$$a_P\xi_P = a_E\xi_E + a_w\xi_W + a_N\xi_N + a_S\xi_S - (a_n{}'\xi_N - a_s{}'\xi_s)V_z + d_P \tag{11}$$

After the solving of the equation (11) we obtain the following system of equations:

$$[M]\,[\xi] = [D] \tag{12}$$

where:

$[M]$: Coefficients matrix,

$[\xi]$: Vorticity vector matrix,

$[D]$: Vector source matrix (dp).

The same method is used for the electromagnetic equation (4). After integration using the finite volume method we obtain the following system:

$$[N+j\,L][A] = [J_{ex}] \tag{13}$$

where:

$[N + jL]$: Coefficients matrix,

$[A]$: Vector Potential matrix,

$[J_{ex}]$: Vector source matrix.

The solution is ensured by applying boundary conditions, on the domain frontiers, as shown in Fig. 4. The Flowchart of the computation algorithm is presented in Fig. 3.

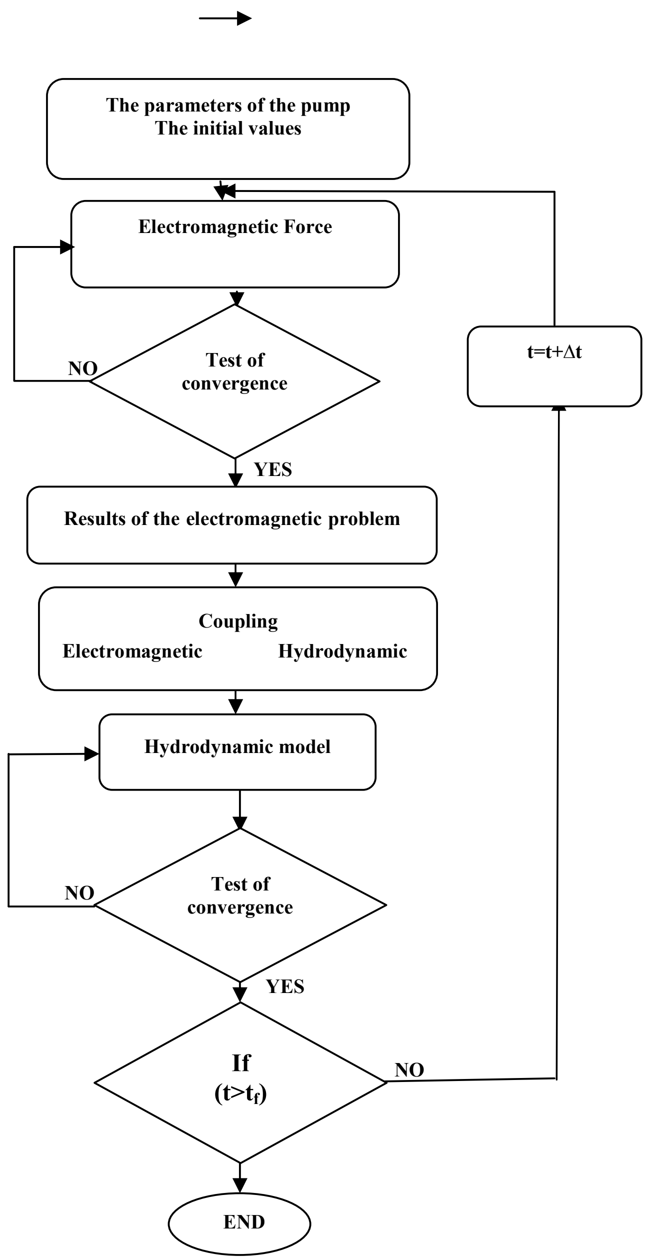

Fig.3- Computation algorithm Flowchart

Figs 4 and 5 represent respectively the annular MHD pump in the plane (r, z) with the boundary conditions and the equipotential lines of the MHD pump.

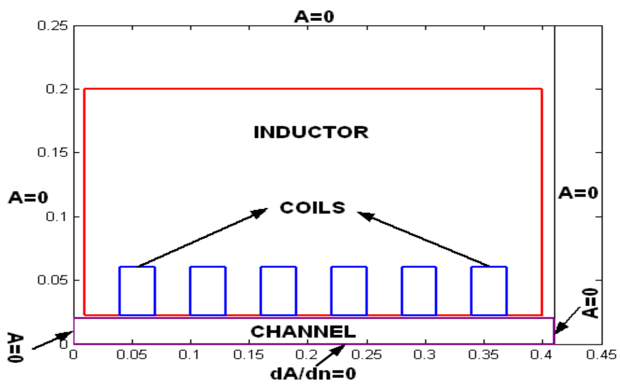

Fig.4. Geometry of the annular MHD pump

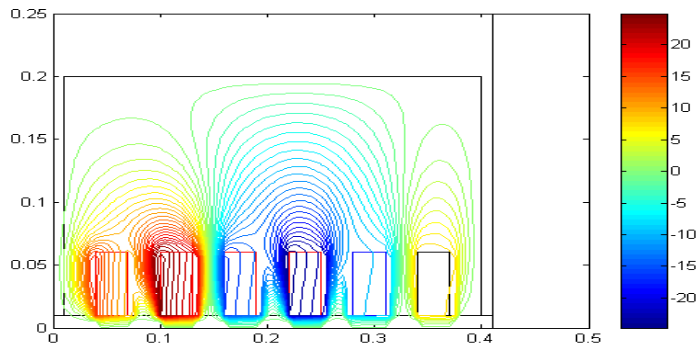
Fig.5. Equipotential lines in the MHD pump

The distribution of the vector potential **A** and the magnetic induction **B** of the MHD pump are shown in fig 6 and 7.

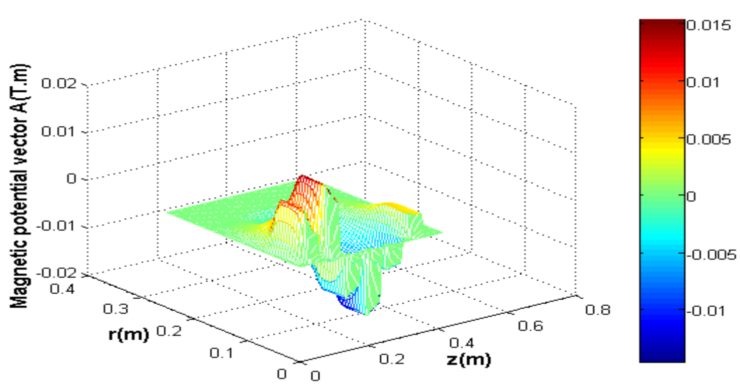

Fig.6. Distribution of the magnetic vector potential in the MHD pump

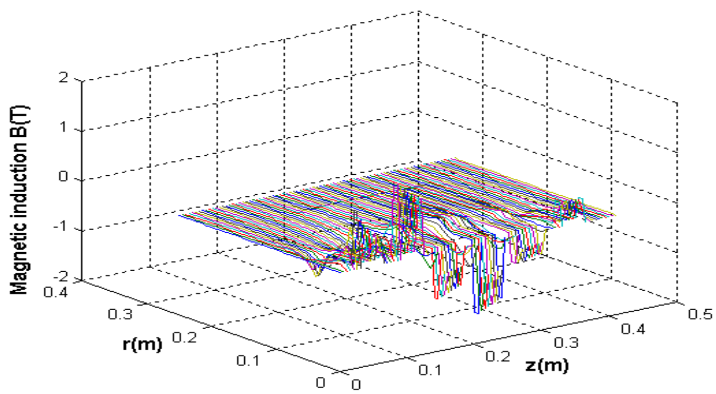

Fig.7. the magnetic induction in the MHD pump

Fig 8 is representing the variation of the velocity in the channel of the MHD pump. We note that the velocity of the fluid passes through a transitional period and then stabilizes as all the electrical machines. The velocity increases as we advance in the channel.

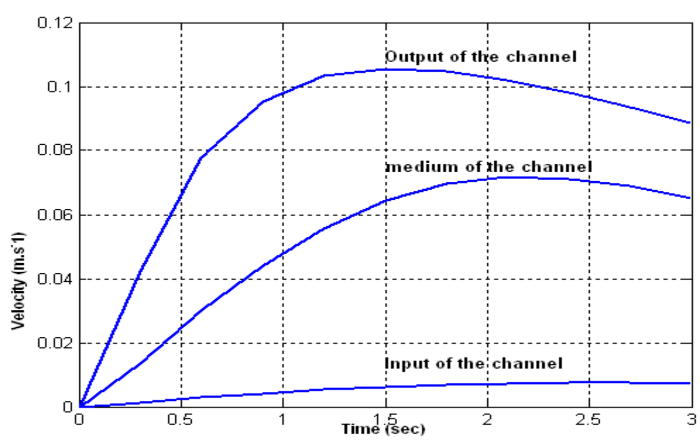

Fig.8. Variation of the velocity at the entrance, medium point and output of the channel

Fig 9 is representing the variation of the velocity in channel of the MHD pump for different frequencies values. We note that when the frequency increases, the velocity increases.

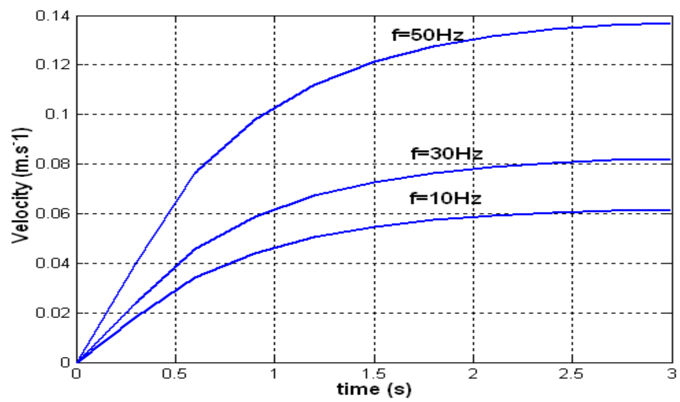

Fig.9. Velocity for different frequencies in the channel of the MHD pump

The variation of the velocity in the channel of the MHD pump channel is shown in Fig 10.

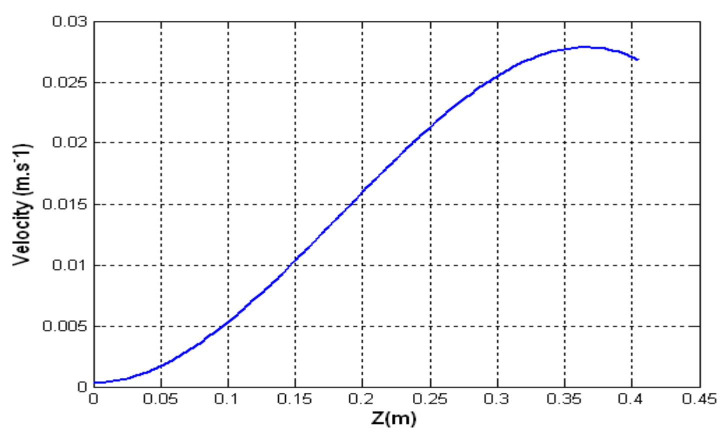

Fig.10. Velocity in the MHD pump

4. Conclusion

The flow characteristics of the liquid metal are obtained by coupling the electromagnetic and the hydrodynamic equations. In this paper, the flow characteristics of the liquid metal are analyzed under various conditions and the simulation results are investigated.

The variations of the velocity in the channel of the MHD pump for different positions and for different frequencies are obtained.

There is a need for an electromagnetic pump that works without any moving parts and produces large thrust.

The obtained results of the velocity are in perfect agreement with those presented in [5] and [8].

REFERENCES

[1] *P. L. Leboucher*, "An inverse method in electromagnetism applied to the optimization of inductors", IEEE Transactions on Magnetics, Vol. 28. No. 5, pp.3330-3332, September 1992.

[2] *R. Berton*, Magnétohydrodynamique, Editions Masson, 1991.

[3] *C. A. Borghi*, A. Cristofolini and M. Fabbri, "Study of the Design Model of a liquid Metal Induction Pump", IEEE Transactions on Magnetics, Vol. 34. No. 5, pp.2956-2959, September 1998.

[4] *N. Takorabet*, "Computation of Force Density inside the channel of an Electromagnetic Pump by Hermite Projection", IEEE Transactions on Magnetics, Vol.42, No.3, pp.430-433, March 2006.

[5] *M. Ghassemi, H. Rezaeinezhad and A. Shahidian* "Analytical Analysis of Flow in a Magnetohydrodynamic Pump (MHD)", IEEE Conference, Speedam 2008- ISCHIA (ITALY).

[6] *Antonio Affanni, Giovanni Chiorboli* "Numerical Modelling and Experimental Study of an AC Magnetohydrodynamic (MHD) Micropump", IMTC-Instrumentation and Measurement Technology, Conference Sorrento, Italy, IEEE, pp.2249-2253, 24-27, April 2006.

[7] *N. Bergoug, F.Z. Kadid, and R. Abdessemed* "The Influence of the Ferromagnetic Materials on the Performances of an Annular Induction Magnetohydrodynamic (MHD) Pump" International Conference on Electrical Engineering Design and Technologies ICEEDT'07, November 4-6, 2007, Hammamet, Tunisia.

[8] *F. Z. Kadid, R. Abdessemed, S. Drid*, "Study Of The Fluid Flow In A MHD Pump By Coupling Finite Element- Finite Volume Computation", Journal of electrical engineering, Vol. 55, No.11-12, pp.301-305, 2004.

[9] *G. S. Park and K.Seo*, "A Study on the Pumping Force of the Magnetic Fluid Linear Pump", IEEE Transactions On Magnetics, Vol. 39, No.3, pp. 1468-1471, May 2003.

[10] *Shamsuddeen N, Ankit Ashok, Rajendrakumar J, Krishna V*, "Design and Analysis of Annular Linear Induction Pump (ALIP)", IEEE conference, February 5, 2010.

[11] *S.V.Patankar*, Numerical Heat Transfer Fluid Flow, Hemisphere Publishing, 1980.
[12] *Krzeminski, S. K, A. Cala and M. Smialek* “Numerical Simulation Of 2D MHD flows ψ-ξ-A method”, IEEE Transactions On Magnetics, Vol.32, 1996.