A TECHNIQUE FOR BI-DIMENSIONAL CONTOUR CONSTRUCTION

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În lucrare este prezentată o tehnică relativ nouă de construire a unor contururi bidemensionale cu posibilă aplicație la obținerea de profile aerodinamice. Proiectantul poate obține geometrii ce urmează a fi validate în medii Computational Fluid Dynamics. Tehnica furnizează o relație biunivocă între geometria obținută şi un set de şase parametri completat de căteva constrângeri. Acest aspect puternic susține faptul că modul prezentat de a construi contururi bidemensionale poate fi utilizat în algoritmi de optimizare pentru profile aerodinamice şi în special algoritmi evolutivi.

In the paper is presented a relatively new technique for bi-dimesnional contours construction which may be applied to obtain aerodynamic profiles. The designer is able to obtain geometries to be validated into Computational Fluid Dynamics environments. The technique provides a biunivoque relation between the obtained geometry and a set of six parameters completed with some constraints. This strong point states that the presented way of constructing bi-dimensional contours may be used into optimization algorithms for airfoils and especially evolutionary algorithms.

Keywords: bi-dimensional, airfoil, parameterization, parabola, c-spline

1. Introduction

Construction of bi-dimensional contours is a mandatory step when designing geometries to be verified from different technical points of view such as aerodynamic performances. When speaking about aerodynamic shapes, there are many ways of defining bi-dimensional contours named airfoils, and those definitions are used for having the correct language while speaking about the proposed geometry [1]. In order to allow the repeatability of the proposed geometry over a large number of studies tmany ways to obtain the contours have been proposed, the most popular being the ones proposed by the National Advisory Committee for Aeronautics (NACA) from United States of America. Besides NACA, who performed extensive tests since 1935 [2], there are many others like Office National d'Études et Recherche Aérospatiales (ONERA) from

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France, Royal Aircraft Factory (RAF) from United Kingdom, Central Aerohydrodynamic Institute (TsAGI) from Russia etc. which proposed their own contours to be used as airfoils and the designer may consider one to have advantages or disadvantages. The technical development lead to changes in the design technique for these contours spotted from the last decade of the past century by many authors and even multidisciplinary design optimization [3].

In this context of constant development of design methods for airfoils based on parametric geometries, some using Computer Aided Design approaches [4,5], this paper presents a relatively new of them technique of constructing bi-dimensional contours to be verified from the aerodynamic point of view. It is believed that this technique can be used in optimization algorithms for airfoil design.

2. Geometry definition

We define the geometry of an aerodynamic profile or airfoil using the notation from Figure 1. This definition help in showing the general shape of bi-dimensional contours verified and confirmed as airfoils.

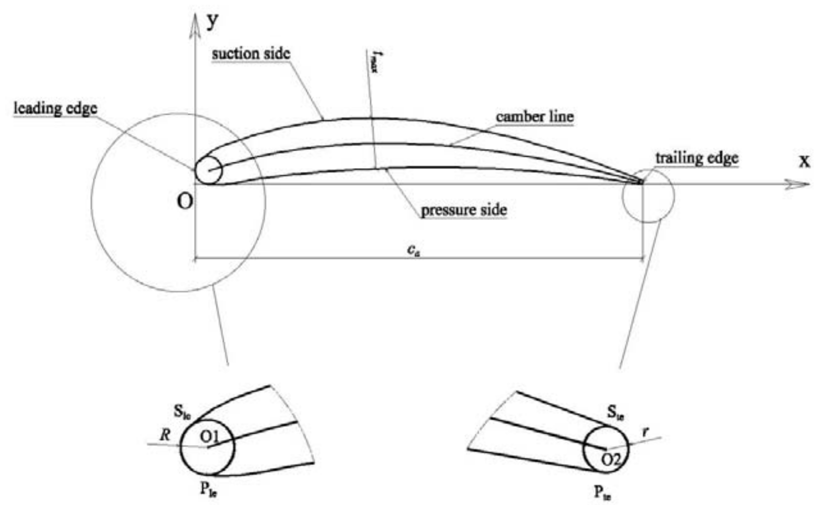

Fig. 1. Aerodynamic profile geometry definition

The geometry is constructed into the xOy Cartesian system and it is placed tangent to the two axes, Ox and Oy, by the two circles at the leading and at the trailing edge, with their respective radii, *R* and *r*, and centers, O1 and O2.

The "suction side" is the curve from S_{le} to S_{te} and it is tangent to the two circles mentioned above into these two points.

The "pressure side" is the curve from P_{le} to P_{te} and it is tangent to the two circles into these two points.

c_a is the chord of the profile having the magnitude of the segment obtained by projecting the entire profile on the Ox axis.

The “camber line” is the curve considered to be equally spaced from the suction and pressure sides. Some may call this curve “mean camber line”.

t_{max} is the maximum thickness of the profile and it is defined as the length of the biggest segment which can be placed perpendicular to the camber line from pressure side to the suction side.

3. Contour simplification and parameterization

This section is dedicated to the proposal of some simplification and parameterization to be used when proposing bi-dimensional contours to be verified from the aerodynamic point of view and confirmed as airfoils.

For the leading edge, we consider that S_{le} is placed into the second quadrant of the respective leading edge circle. Thus, the first derivate of the suction side in S_{le} is strictly positive. We consider that P_{le} is placed into the fourth quadrant of the same circle resulting. Thus, a strictly positive first derivate of the pressure side into P_{le}, Fig. 2 a).

For the trailing edge, we consider that S_{te} is placed into the first quadrant of the respective trailing edge circle. Thus, the first derivate of the suction side in S_{te} is strictly negative. We consider that P_{te} is placed into the third quadrant of the same circle resulting. Thus, a strictly negative first derivate of the pressure side into $P_{te,}$ Fig. 2 b).

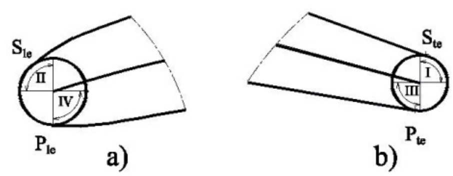

Fig. 2. Placement of the points into the quadrants of the two circles

For the proposed way of constructing bi-dimensional contours we consider a parabolic distribution for the thickness of the profile and a parabolic shape for the camber line. For both parabolic issues we use the parabola portions as displayed in Fig. 3a) and b).

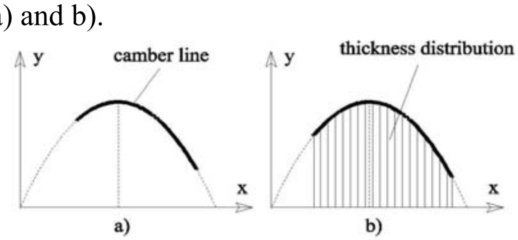

Fig. 3. Parabola portions for: a) camber line, b) thickness distribution

The camber line is computed from the respective parabola portion and the thickness distribution is computed as the absolute distance from the *y* coordinate of the current point on the parabola portion and the x axis.

We may consider that each parabola is defined by the equation:

$$y = -4A(x^2 - x)\ ,\ A \in (0;1]. \tag{1}$$

The interval for *A* is considered to limit the maximum possible curvature of the parabola.

Now, in order to obtain only a portion of the parabola, we define the starting point:

$$\begin{cases} x = B \\ y = -4A\left(B^2 - B\right), \end{cases} \tag{2}$$

with $B \in [0;1)$ and the ending point:

$$\begin{cases} x = B + C(1 - B) \\ y = -4A\left\{\left[B + C(1 - B)\right]^2 - \left[B + C(1 - B)\right]\right\}, \end{cases} \tag{3}$$

with $C \in (0;1]$.

We can see that for the three parameters *A*, *B* and *C* each parabola portion is fully defined, Figure 4.

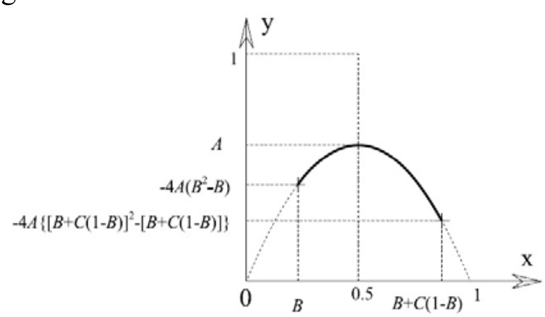

Fig 4. Fully defined parabola portion with the help of *A*, *B* and *C*

So, in order to obtain two parabola portions, we will have three parameters for the camber line $A_1 \in (0;1]$, $B_1 \in [0;1)$, $C_1 \in (0;1]$, and three parameters for the thickness distribution: $A_2 \in (0;1]$, $B_2 \in [0;1)$, $C_2 \in (0;1]$.

The parabola portion for the camber line must be first scaled and then rotated to fit between O_1 and O_2 points. The scale sc_1 can be obtained as the ratio between two Euclidian norms:

$$sc_1 = \frac{\sqrt{\left(x_{O1} - x_{O2}\right)^2 + \left(y_{O1} - y_{O2}\right)^2}}{\sqrt{\left(x_1 - x_2\right)^2 + \left(y_1 - y_2\right)^2}}, \tag{4}$$

where the coordinates x_1, x_2, y_1, y_2 can be calculated with the three parameters of the first parabola portion and equations (2) and (3), and the rotating angle α is the angle between the line passing through the start and end point of the corresponding parabola portion and the line passing through O_1 and O_2. This angle is given by:

$$\alpha = \operatorname{atan}\left(\frac{y_1 - y_2}{x_2 - x_1}\right) - \operatorname{atan}\left(\frac{y_{O1} - y_{O2}}{x_{O2} - x_{O1}}\right). \tag{5}$$

In equations (4) and (5) the coordinates of O1 and O2 are defined by:

$$x_{O1} = R;\; y_{O1} = R;\; x_{O2} = c_a - r;\; y_{O2} = r. \tag{6}$$

The signs of the angles coming out of the arctangent evaluation are easy to impose such as to make the proper difference when calculating the exact value of the angle α.

The thickness distribution must be scaled with respect to the maximum thickness t_{max}. This scale, sc_2, is computed as the ratio between t_{max} and the maximum thickness computed for the second parabola portion. The computed maximum thickness can be either the y coordinate of the peak of the respective parabola, A_2, or one of the y coordinate of the end points of the second parabola portion.

4. Contour construction

Now, we shall start the construction of the contour, with pressure and suction sides, using the values obtained before. We will consider for this the portion of the camber line placed outside the two circles from the leading and the trailing edge.

We divide this portion of the camber line into n curves of equal length obtaining n+1 points, C_1, C_2, …, C_n, C_{n+1}, on which we put the corresponding thickness obtained from the second parabola portion. C_1 and C_{n+1} are easy to be obtained knowing the two radii R and r.

The equal lengths are computed as integrals on the unrotated parabola portion which helps in finding the exact locations of C_2, …, C_n:

$$l = \frac{L}{n} = \frac{\int_{C_1}^{C_{n+1}} \left[1 + (y_1')^2\right] dx}{n} = \int_{C_i}^{C_{i+1}} \left[1 + (y_1')^2\right] dx,\;\; y_1 = -4A_1(x^2 - x),\;\; i = \overline{1,n}. \tag{7}$$

Now we only need to calculate the tangential direction onto the camber line in each of the C_1, C_2, …, C_n, C_{n+1} points and to find points on the pressure side, P_1, P_2, …, P_n, P_{n+1} and on the suction side, S_1, S_2, …, S_n, S_{n+1}.

The points on the pressure and suction sides are obtained by putting segments on the calculated direction, segments equal to half of the corresponding thickness, Figure 5.

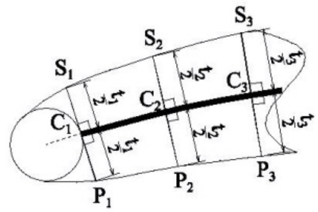

Fig. 5. Placement of points onto the pressure and suction side

The angle of the tangential direction for each S_iP_i into C_i is computed as the complementary angle β_i given by the first derivative of the unrotated and unscaled camber line:

$$\beta_i = arctg\left(\frac{\pi}{2} - y_1'(x)\Big|_{C_i}\right),\ \ y_1 = -4A_1\left(x^2 - x\right),\ \ i = \overline{1,n}. \tag{8}$$

Since the angle calculated from equation (8) can be either positive or negative, we keep in mind that S_i is above C_i which is above P_i, meaning: $y_{S_i} > y_{C_i} > y_{P_i},\ \forall i = \overline{1,n+1}.$

The thicknesses t_i are computed from the corresponding parabola portion:

$$t_i = y_2(x_i),\ \ x_i = B_2 + (i-1)\frac{C_2(1-B_2)}{n},\ \ y_2 = -4A_2\left(x^2 - x\right),\ \ i = \overline{1,n+1}. \tag{9}$$

Since problems may occur due to scaling and rotating issues we will show later in the paper an example of how all data is used into Computer Aided Design (CAD) software. Also, problems can occur for the first and the n+1 thickness so we may not consider them when drawing the geometry.

We now have n+1 points on the pressure side and n+1 points on the suction side but there are still missing some curves to completely define the bi-dimensional contour: (P_{le};P1), (P_{n+1};P_{te}), (S_{le};S1), (S_{n+1}; S_{te}). Since we have to build the entire contour anyway, we will use cubic splines curves for creating the pressure and suction sides. The cubic spline, or short c-spline, for the pressure side will contain all the following points: P_{le}, P_1, P_2, …, P_n, P_{n+1}, P_{te} and it will be tangent to the leading edge circle into P_{le} and tangent to the trailing edge circle into P_{te}. Similarly, the c-spline for the suction side will contain S_{le}, S_1, S_2, …, S_n, S_{n+1}, S_{te} and will be tangent to the leading edge circle into S_{le} and tangent to the traoling edge circle into S_{te}.

The term spline comes from the analogy to a draughtsman's approach to pass a thin metal or wooden strip trough a given set of constrained points and was first studied from an energetic point of view considering the materials properties for bending to touch the given points [6]. Since this approach is rather difficult and has limited application some other ways to construct splines were found namely here polynomials splines. C-splines are curves based on third degree polynomial interpolation between the constrained points [7].

We present the approach for the suction side, the one for pressure side being very similar. For our problem we may consider the following sets of functions which define the c-spline curve of the suction side:

$$f_i(x) = a_{0_i} + a_{1_i}x + a_{2_i}x^2 + a_{3_i}x^3,\ i = \overline{0, n+1}. \quad (10)$$

The function f_i define, the curve over the portion between S_i and S_{i+1} curves on the suction side. For i=0 we consider the curve from $S_0=S_{le}$ to S_1, and for $i=n+1$ we consider the curve from S_{n+1} to $S_{n+2}=S_{te}$.

We impose the condition that the c-spline contains all the points previously defined:

$$f_i(x_{S_i}) = y_{S_i},\ f_i(x_{S_{i+1}}) = y_{S_{i+1}}, \overline{i = 0, n+1}. \quad (11)$$

Two other conditions refer to the “smoothness” of the c-spline, namely two adjacent third degree polynomial functions have the first and second derivatives equals in the control points (tangent and curvature):

$$f_i'(x_{S_{i+1}}) = f_{i+1}'(x_{S_{i+1}}),\ f_i''(x_{S_{i+1}}) = f_{i+1}''(x_{S_{i+1}}),\ i = \overline{0, n}. \quad (12)$$

We must pay attention to the fact that the exact coordinates for the lateral points S_{le} and S_{te} are unknown but we know that they are positioned on the two leading and trailing edge circles, meaning:

$$R^2 = (R - x_{S_{le}})^2 + (y_{S_{le}} - R)^2,\ r^2 = \left[r - (c_a - x_{S_{te}})\right]^2 + (y_{S_{te}} - r)^2. \quad (13)$$

Also, the c-spline of the suction side contains these two points:

$$f_0'(x_{S_{le}}) = \frac{R - x_{S_{le}}}{y_{S_{le}} - R},\ f_{n+1}'(x_{S_{te}}) = \frac{r - (c_a - x_{S_{te}})}{y_{S_{te}} - r}. \quad (14)$$

If we calculate the number of unknowns we will count 4(n+2), the a coefficients of the polynomial functions from equation (10) and 4 coordinates (2 for S_{le} and 2 for S_{te}), equals (4n+12) unknowns. The count for the equations is now 2(n+2) from equation (11), 2(n+1) from equation (12), 2 from equation (13) and 2 from equation (14), equals to (4n+10). We see that two equations are still missing. These equations come from the conditions for S_{le} and S_{te} to be in the second and first quadrants of their respective circles, meaning:

$$\begin{aligned} &(0 < x_{S_{le}} < R)\ \&\ (y_{S_{le}} > R),\ \left[(c_a - r) < x_{S_{te}} < c_a\right]\ \&\ (y_{S_{te}} > r) \\ &or \\ &\left[f_0'(S_{le}) > 0\right]\ \&\ \left[f_{n+1}'(S_{te}) < 0\right] \end{aligned} \quad (15)$$

The conditions from equation (15) make the difference between curves 1, the correct one, and 2, the wrong one, from Figure 6.

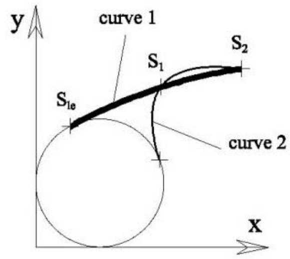

Fig. 6. Example of the condition from equation (15) for the leading edge (curve 1 is the correct c-spline for the suction side)

Having the complete system with (4*n*+12) unknowns and equations one can solve it using a predefined system of equations solver from a developer resource [8] and therefore determine the entire suction side and respectively the pressure side. It is clear that the obtained solution is unique this giving an important attribute to the technique: it creates a biunivoque relation between the six parameters defining the parabola and the resulting geometry if some additional constraints are imposed: the chord, two radii, and the maximum thickness.

5. Application

This section illustrates a short example of constructing a bi-dimensional contour to be verified and confirmed as airfoils starting from the six parameters described above and the constraints on its general dimensions. The constraints are related to technological issues when speaking about the trailing edge radius r, stress evaluation when speaking about the maximum thickness t_{max}, general dimensions of the desired geometry when speaking about the chord c_a, aerodynamic efficiency for off-design functional points when speaking about the leading edge radius R etc.

Let's consider the following values for the six parameters defining the two parabola portions:

$$A_1 = 0.3;\ B_1 = 0.6;\ C_1 = 0.9;\ A_2 = 0.8;\ B_2 = 0.3;\ C_2 = 0.9. \qquad (16)$$

The two parabola portions will have the exact shape displayed in Figure 7, with defined starting and ending points as previously shown.

We impose for the possible aerodynamic profile the chord c_a, radius of the leading edge R and of the trailing edge r and the maximum thickness t_{max}:

$$c_a = 1;\ R = 0.03;\ r = 0.005;\ t_{max} = 0.09. \qquad (17)$$

From the data obtained so far we can calculate the scales sc_1 and sc_2 and the angle of rotation α obtaining the following values:

$$sc_1 = 2.225612; \; sc_2 = 0.1125; \; \alpha = 32.417^0. \quad (18)$$

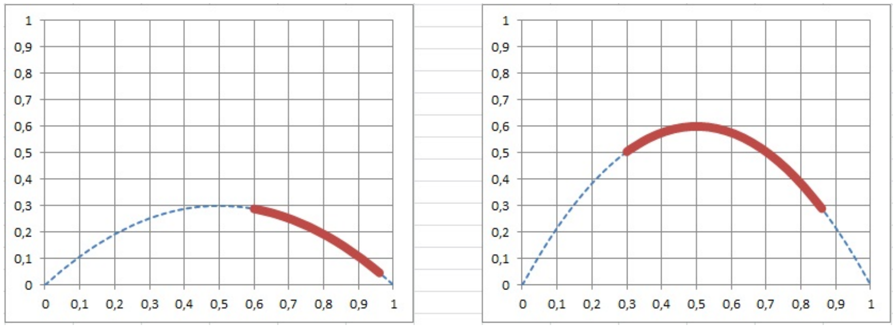

Fig. 7. Two parabola portions obtained using the parameters from the example

Now, we present the construction of the bi-dimensional contour into a CAD software using its features to draw the necessary geometry and n=12 the number of equal curves on the camber line, Figure 8.

Step 1- camber line drawing, scaling by the calculated scale sc_1 and drawing of the two circles from the leading and trailing edge;

Step 2- scaling of the thicknesses and placement of the respective segments perpendicular in the calculated points to the scaled camber line;

Step 3- drawing of the c-splines for the suction and pressure sides, rotation and translation of the geometry near the origin.

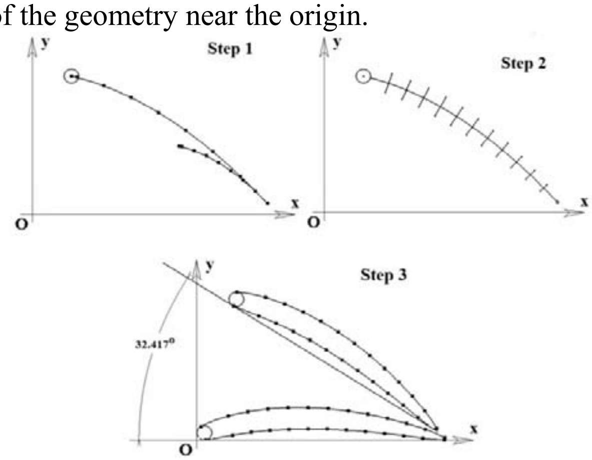

Fig. 8. Steps for creating the airfoil geometry using a CAD software

6. Conclusions

The technique presented in the paper is dedicated to the creation of bi-dimensional contours shaping possible aerodynamic profiles starting from six parameters.

An obtained contour can be evaluated from aerodynamic point of view using classical analytical methods, own algorithms based on numerical methods for fluid dynamics [9] or using Computational Fluid Dynamics tools.

Moreover, the proposed technique can be easily used for design optimization with evolutionary algorithms [10] since it provides a clear and biunivoque relation between the obtained geometry and the set of six parameters along with the chord, the leading and trailing edge radii and the maximum thickness of the possible aerodynamic profile.

In practice, before proceeding to the optimization algorithm, the design space offered by the proposed technique must be carefully examined. Some constraints onto the six parameters will result or, before calling the evaluation tool for aerodynamics, the obtained geometry must be somehow checked. These steps are necessary in order to avoid unnecessary evaluations for obvious wrong geometries.

Also, its application with the help of CAD software makes it easy to use with Computational Fluid Dynamics environments where the geometry needs to be inserted into the solver as it was created by the designer.

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