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CHAOS AND HYPERCHAOS IN A SYMMETRICAL DISCHARGE PLASMA: EXPERIMENT AND MODELLING

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În această lucrare prezentăm rezultate experimentale şi modelări computaţionale ale tranziţiei haos-hiperhaos în plasma unui sistem format din două descărcări electrice care se produc în acelaşi tub de sticlă şi au geometrii şi parametri identici. Caracteristicile plasmei din spaţiul inter-anodic sunt controlate de o tensiune de polarizare aplicată pe cei doi anozi. Dinamicile plasmei sunt investigate prin intermediul fluctuaţiilor de curent. Pentru un anumit domeniu al polarizării inter-anodice continue, se observă o tranziţie haos-hiperhaos. Prezentăm un model computaţional constând din trei oscilatori neliniari cuplaţi care conduce la rezultate în bună concordanţă cu cele experimentale.

In this work we present experimental observations and computational modelling of chaos to hyperchaos transition in a plasma system consisting of two identical discharges in a symmetrical set up. The two discharges have identical geometries and are running in the same glass tube under identical conditions. The plasma generated in the inter-anode space is controlled by a relative biasing of the two anodes. We study the changes in the dynamics of this plasma as reflected in the current fluctuations. For a particular range of the d.c. inter-anode biasing we observe a transition from chaos to hyperchaos. We propose a computational model consisting of three coupled nonlinear oscillators. The agreement between the experiment and the results of the model is a clear evidence for the correctness of our model.

Keywords: chaos, hyperchaos, plasma, nonlinear oscillators

1. Introduction

Recently, an increasing interest is shown for high-dimensional nonlinear systems. It is well known that, to generate hyperchaos from autonomous systems, the state equation must satisfy two basic conditions: the dimension of the state equation should be at least 4 and the system should have at least two positive Lyapunov exponents satisfying that the sum of all Lyapunov exponents is negative [1,2].

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The interest for hyperchaotic dynamics is justified by the rapid development of new techniques in communications as well as in complex systems studies [3,4]. Experimentally, hyperchaotic behaviors have been identified in many systems such as: an electronic circuit [5], an NMR laser [6], a semi-conductor system [7] and a chemical reaction system [8-9].

In this work, experimental observations and computational modelling of chaos to hyperchaos transition in a plasma system are presented. The system consists of two discharges, with identical geometries, running in the same glass tube under identical conditions [10]. The plasma generated in the inter-anode space is controlled by a relative biasing of the two anodes. The biasing is the superposition of a d.c. voltage and a sinusoidal one, in series. We monitor the changes in the dynamics of this plasma via the current fluctuations measured as voltage variation across a load resistor [11]. For a particular range of the d.c. inter-anode biasing we observe a transition from chaos to hyperchaos that appears as a second Lyapunov exponent increases across the zero line.

We propose a computational model consisting of three coupled nonlinear oscillators. Two of the oscillators are standing for the plasmas of the two discharges while the third one models the inter-anode plasma double layer. The coupling between these oscillators is determined by the d.c. inter-anode biasing. By changing the coupling parameter, we observe a transition from chaos to hyperchaos as a second Lyapunov exponent becomes positive.

2. Experimental set-up and results

We use the experimental set-up presented in [10]. As the d.c biasing is increased in the presence of a low amplitude sinusoidal perturbation, the time-series of the fluctuations on the load resistor show dramatic changes. In certain intervals the behavior is periodical and for high values of the inter-anode d.c. biasing the transition from chaos to hyperchaos is observed. Figure 1 shows a phase space attractor for two values of the d.c. biasing, corresponding to chaotic behaviour (U=64V) and to hyperchaotic dynamics (U=72V). The transition between the two types of dynamics is clearly observed by computation of the Lyapunov exponents spectrum. In the case of hyperchaos, there are two positive Lyapunov exponents ($\lambda_1 = 0.620$; $\lambda_2 = 0.101$; $\lambda_3 = -0.806$; $\lambda_4 = -1.137$; $\lambda_5 = -2.108$; $\lambda_6 = -3.448$; $\lambda_7 = -6.520$) while in the case of chaos, only one positive Lyapunov exponent exists ($\lambda_1 = 0.185$; $\lambda_2 = -0.318$; $\lambda_3 = -1.011$; $\lambda_4 = -1.420$; $\lambda_5 = -2.134$; $\lambda_6 = -3.340$; $\lambda_7 = -6.810$). We used the Tisean Package [12] for the computation of the Lyapunov exponents spectrum. The number of exponents was chosen in agreement with the dimension of the proposed computational model.

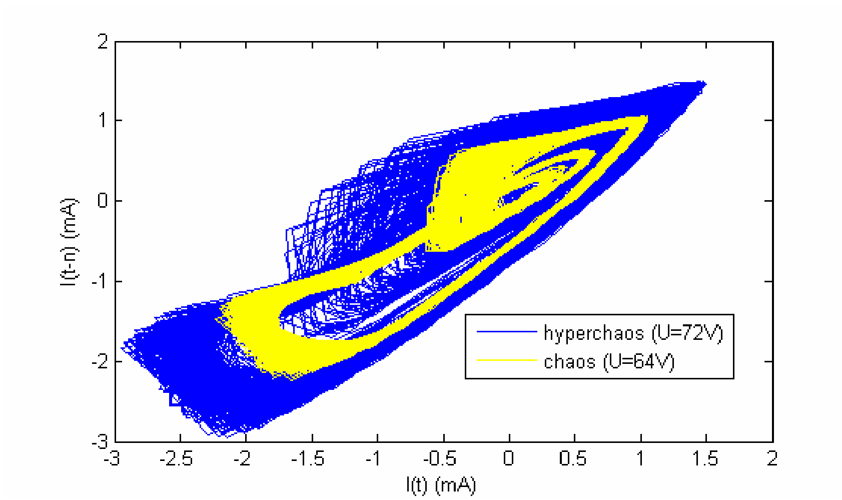

Fig. 1. Experimental phase space attractor showing chaotic and hyperchaotic dynamics.

3. Computational Model

The most popular model for the study of chaos-hyperchaos transition is a system of two or more coupled nonlinear oscillators [13]. As computational model we consider a system of three coupled nonlinear oscillators of the generalized van der Pol type [14,15]. Two of the oscillators represent the plasmas of the two discharges while the third one models the inter-anode plasma. The coupling parameter *m* (considered as control parameter) is in correspondence with the d.c. inter-anode biasing. The following system of equations, results:

$$\dot{x}_1 = x_2 \tag{1}$$

$$\dot{x}_2 = -c({x_1}^2 - 1)x_2 - {x_1}^3 + e\cos x_7 + mx_3 \tag{2}$$

$$\dot{x}_3 = x_4 \tag{3}$$

$$\dot{x}_4 = -f({x_3}^2 - 1)x_4 - {x_3}^3 - m(x_1 + x_5) \tag{4}$$

$$\dot{x}_5 = x_6 \tag{5}$$

$$\dot{x}_6 = -c({x_5}^2 - 1)x_6 - {x_5}^3 + e\cos x_7 - mx_3 \tag{6}$$

$$\dot{x}_7 = \omega \tag{7}$$

From a bifurcation diagram a transition from chaos to hyperchaos can be sometimes associated with a sudden expansion in the range of the coordinate values. This can be observed from the bifurcation diagram in Fig. 2.

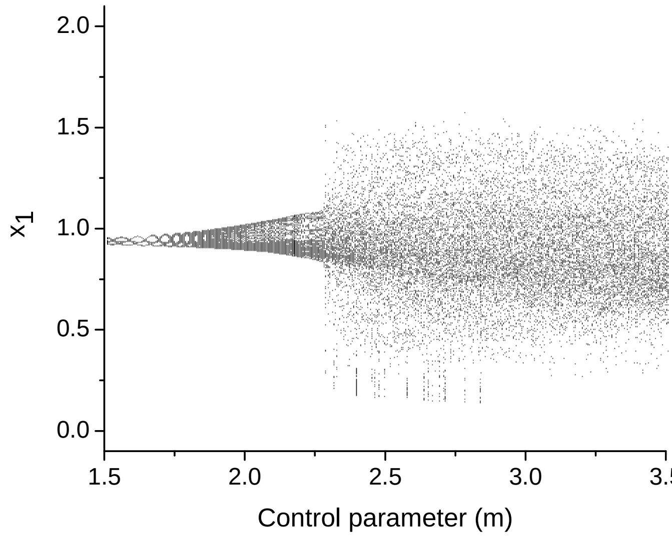

Fig. 2. Bifurcation diagram showing a possible transition from chaos to hyperchaos.

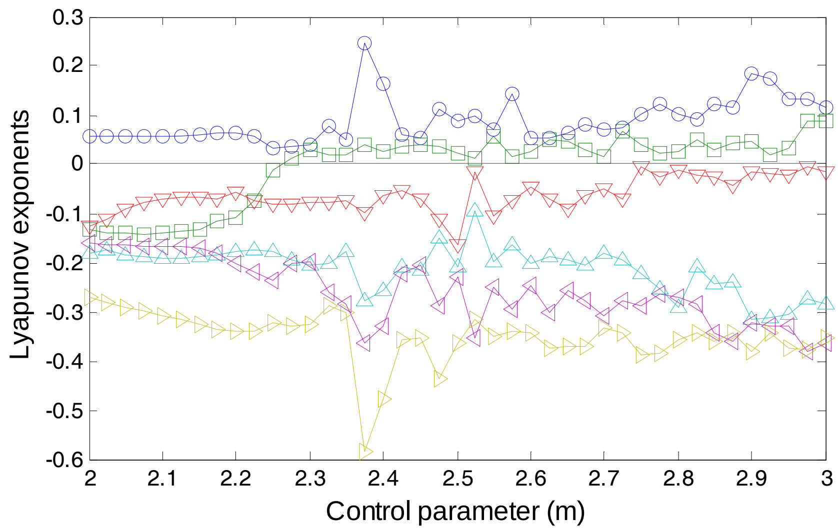

Fig. 3. Lyapunov exponents spectrum as function of the control parameter (*m*).

From both Fig.3 and 4 we observe that in the neighborhood of m=2.25 a transition from chaos to hyperchaos is observed as m is increased.

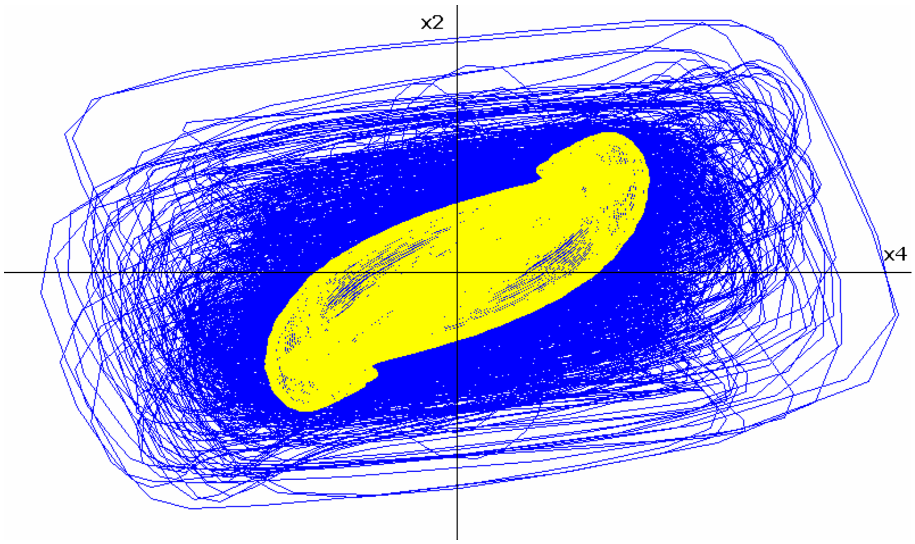

Fig. 5. Phase portrait for the values m=2.2 (yellow) showing chaos and m=2.77 (blue) showing hyperchaos in the (x_2, x_4) projection

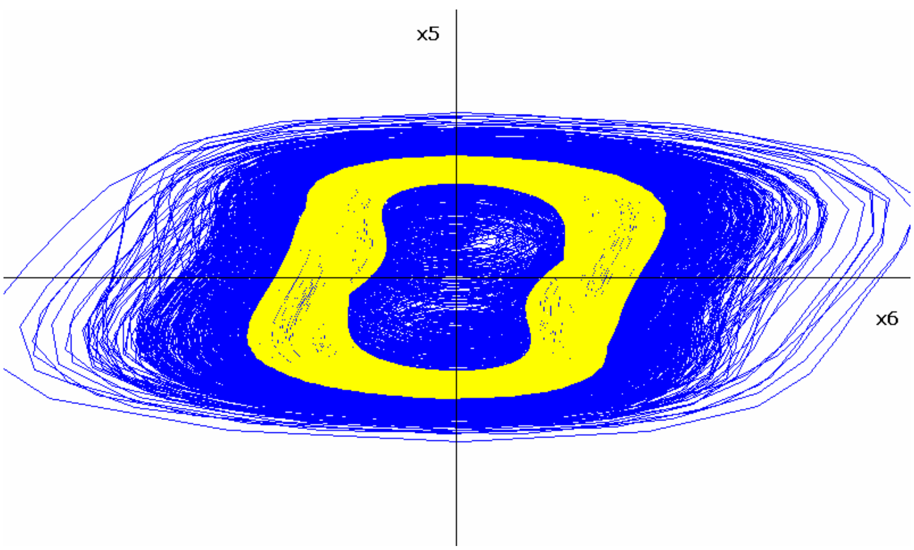

Fig. 6. Phase portrait for the values m=2.2 (yellow) showing chaos and m=2.77 (blue) showing hyperchaos in the (x_5, x_6) projection

6. Conclusions

We present experimental observations and computational modelling of a transition from chaos to hyperchaos in a plasma generated in the inter-anode space of two identical discharges. The plasma dynamics is controlled by a relative biasing of the two anodes. To our knowledge this is the first time a chaos to hyperchaos transition in an electric discharge is reported.

We propose a computational model consisting of three coupled nonlinear oscillators. Good agreement between the experiment and the results of the model is obtained.

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