THE WEAR OF GEAR TEETH WITH FUNCTIONALITY IN ABRASIVE RANDOMLY PARTICLES

Monica VLASE[1], Andrei TUDOR[2]

Se arată că particulele abrazive ce pătrund în contactul convergent dintre doi dinţi ai unui angrenaj ce formează cupla de frecare, pot deteriora suprafaţa roţilor dinţate. Uzura datorată particulelor abrazive depinde de modul în care are loc interacţiunea acestora atunci când geometria particulelor este aleatoare. Se evaluează numărul de particule din interstiţiul dintre doi dinţi ai angrenajului.Se determină uzura ca parametru determinist aleator folosind modelele de deformare corespunzătoare. Se consideră că particulele abrazive sunt ovoide cu lungimea razelor „R" şi " r" variabile aleatoare. Distribuţia variabilă aleatoare este normală şi exponenţială.

It is shown that the abrasive particles, which penetrate in the convergent joint of gear's teeth friction pairs, can damage the superficial layer of surfaces. The wear caused by individual particle depends on having both a satisfactory understanding of individual interactions and a suitable procedure for combining these under circumstances when particle geometry has randomly aspects. The particle number in the gap of gear teeth is evaluated. Wear determinist and random parameter predicted from the corresponding particle deformation models are shown.
It is considered that the abrasive particles are ovoid with random variable radii R and r. The random variable distributions are considered to be exponential and normal.

Keywords: abrasive wear, gear teeth, random geometry, durability

1. Introduction

Friction couples operating in infested environments with abrasive particles are one of durability restriction case. For the convergent-divergent gaps (interstices), with relative surfaces motion, the particles penetration and their driving into the gap, depend on the gap's geometry, the particle geometry and friction coefficient. The harder particles can appear from the exterior couple environment (dust, sand) or even from the wear particle detachment in the adhesion or fatigue process [1,2,3,4].

Abrasion can occur due to: **micro-cutting** by the hard particle sharp edges or their roughness; by **fracturing** due to the crack convergence; by **fatigue** due to repeated plastic streins; by hard granule, **plucking** out of the material [3,4].

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The way in which the abrasive particles pass on the worn surface shows two types of wear: - two elements wear; - three elements wear. Two elements abrasion supposes that particles are fixed in one of the couple elements. This can be considered similar to the abrasion of one surface by the roughness of the conjugated surface [1,2,3].

Gear mechanisms can work in abrasive environments as open gears or due to the sealing failure. The goal of this paper is to analyse the abrasive wear produced by ovoid particles with aleatory variable geometry, into a gear teeth gap.

2. Contact model between an ovoid and a cylinder

Abrasive particles attach to the gear teeth flanks, as shown in Fig.1.

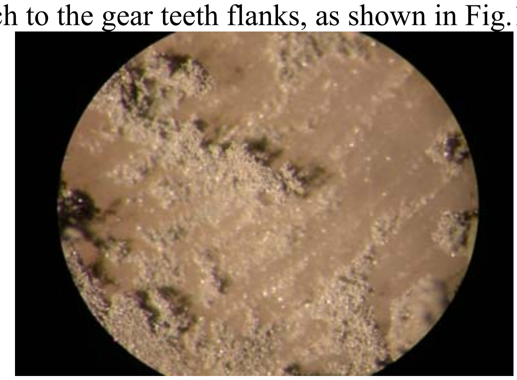

Fig.1. Abrasive particles attached to the gear tooth flank.

In order to model the wear phenomenon between the abrasive particles and gear tooth flank, the abrasive particle geometry is idealised as an ovoid, characterised by two spherical segments of radii R and r and the distance a_p between the spheres centre. This ovoid is showen in Fig.2.

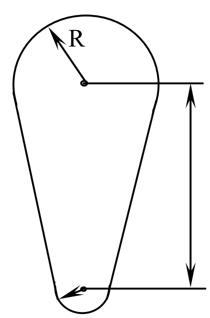

Fig.2. Geometry of the ovoid abrasive particle.

The following dimensionalless values are defined:

length coefficient $\xi = a_p / R$

rounding coefficient $\chi = r / R$

particle relative radius $R_{ap} = R / R_c$

The gap (interstice) between the gear teeth is variable, depending on the contact point on the gearing line, as showen in Fig.3.

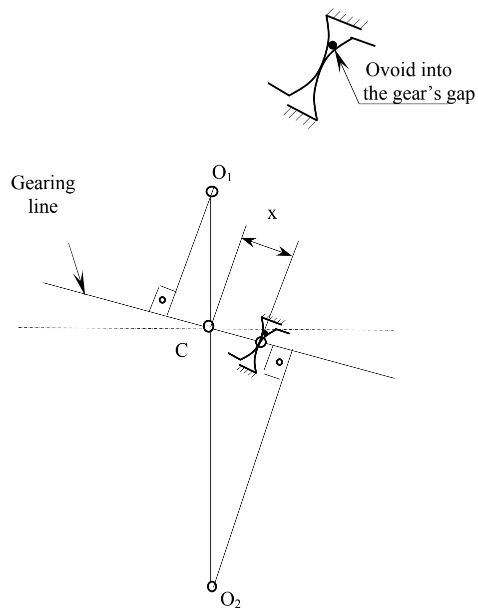

Fig.3. Mobil contact point on gearing line.

The initial hypothesis is that the gear teeth deformations are elastic and the ovoid particles are perfectly rigid.

The contact of teethes will be considered as a contact between an equivalent cylinder with a rigid plane.The equivalent cylinder is defined similar to Hertz solution of the contact pressure. In this case, the contact between ovoid and teeth will be considered a contact between the elastic or plastic cylinder and the rigid ovoid.

We consider that the ovoid particle is in contact with one tooth through the greater sphere and in contact with the next tooth through the smaller sphere of this ovoid and the direction of the ovoid is radial.

To generalize the results, the geometric parameters of the gear (rolling radius, base circle radius, gearing line length) are made dimensionless by the gear modulus.

It is considered that the teeth number of the drive gear, (z_1), and the transmision ratio, (i_t), are known. In this paper, we analyse only the case of spur gear teeth. The corelation between the gear geometry and ovoid geometry is based on the ovoid greather sphere radius, (R), gear modulus, (m), and dimensionless radius of ovoid ($R_{am}=R/m$).

To define the contact between gear tooth and the ovoid particle it is necessary to determine the number of the abrasive particles which exists in the gap. Using the geometric parameters of the teeth and the ovoid, we can calculate this number per unit teeth length (z_p):

$$z_p = \frac{\beta_{max} - \beta_{min}}{\beta_c} + 1 \quad (1)$$

where: β_{max} is the maximum contact angle between the tooth and the greater sphere of the ovoid (wrapping angle of equivalent cylinder);

β_{min} is the minimum contact angle between the tooth and the small sphere of the ovoid;

β_c is the angle between two consecutive ovoids in contact.

Figs. 4 and 5 present the abrasive particle number variation, (z_p), versus point position on the line of action, (x-dimensionless lenght of contact point relative to the pitch point), for different values of the length coefficient, (ξ), and the rounding coefficient, (χ) of abrasive particle. The pitch point C (figure 3) is considered as an origin for the contact point between teeth (figures 4 and 5).

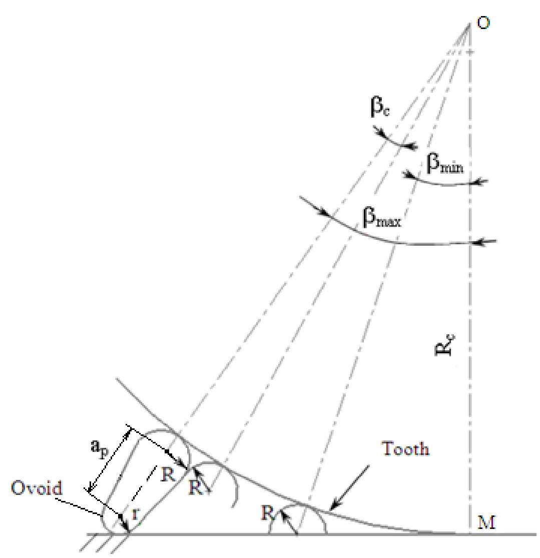

Fig. 4. The contact angle between tooth (equivalent cylinder) and ovoid

The ovoid positions into the gap between the gear teeth can be random, depending on the geometry of abrasive particle and equivalent cilinder.

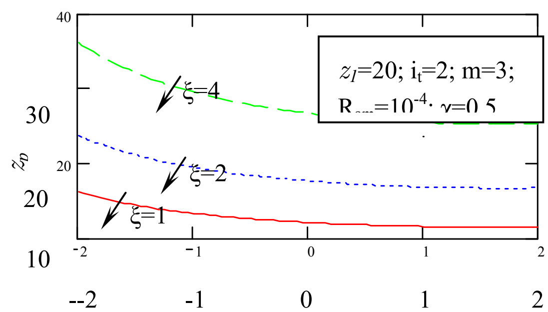

Fig.5. The variation of the abrasive particle number, (z_p), versus contact point on the line of action (x), for different values of the length coefficient, (ξ).

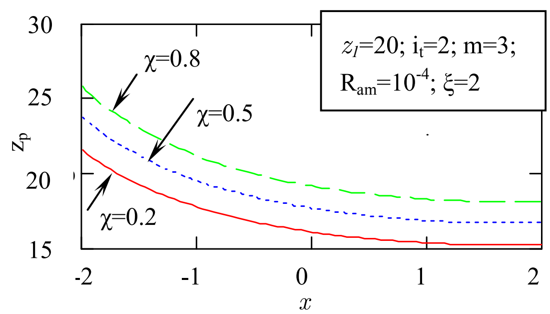

Fig.6. The variation of the abrasive particle number, (z_p), versus point position on the gearing line , (x), for different values of the rounding coefficient, (χ). z_1

Figs. 7 and 8 present the variation of the abrasive particle number, (z_p) versus abrasive particle relative dimension, (R_{am}), for different values of the length coefficient, (ξ), and the rounding coefficient, (χ).

The external F force distribution upon the abrasive particles z_p is determined using the following simplifying hypothesis:

- the cylinder strains are elastic;
- the abrasive particles are rigid, thus, the distribution circle of the ovoid big spheres centers is concentric with the defined circle of the cylinder specific to the friction couple;
- the abrasive particles number on the contact unit length of the couple is elevated (10^4 order of magnitude) and the wrapping angle is small (2°), thus, it is accepted that the abrasive particles number, (z_p), loaded by the external force (F) is a (β) contact angle continuous function.

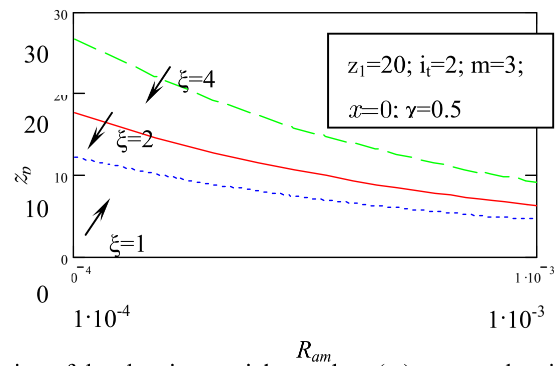

Fig.7. The variation of the abrasive particle number, (z_p), versus abrasive particle relative dimension, (R_{am}), for different values of the length coefficient, (ξ).

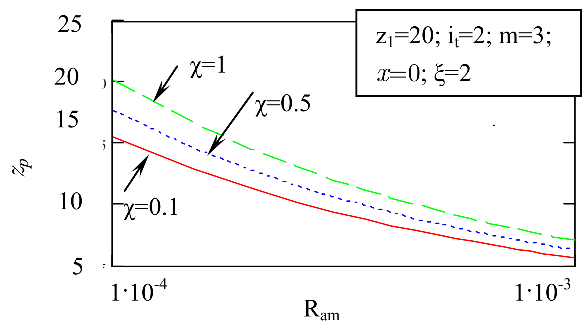

Fig.8. The variation of the abrasive particle number, (z_p), versus abrasive particle relative dimension, (R_{am}), for different values of the rounding coefficient, (χ).

It is known that between the force in a certain point, situated at the angular distance β and the elastic strain, there is a hertzian relation as:

$$F_\beta = c\,\delta^{3/2} \tag{2}$$

where: c is the hertzian rigidity in the specific point, being accepted as constant all along the circular contact length of the cylinder with the hard particles;
δ is the strain law.

The specific force can be deduced if the strain law, (δ), is known.

For the accepted case, with the greather spheres centers of the ovoid situated on a circle, the following expression is proposed:

$$\delta = A\tan(\beta + \gamma) \tag{3}$$

where the constants A and γ can be determined from the limit conditions:

$$\delta = \delta_1 \quad for \quad \beta = \beta_{\min}$$

$$\delta = 0 \quad for \quad \beta = \beta_{\max}$$

Thus,

$$A = \frac{\delta_1}{\tan(\beta_{\min} - \beta_{\max})}$$

and

$$\phi = -\beta_{\max}$$

From the condition of mechanical equilibrium,

$$F_{a1} = \frac{1}{\sum_{i=1}^{z_p} \left[\frac{\tan\left[\beta_{\max} - \beta_{\min} - (i-1)\beta_c\right]^{\frac{3}{2}} \cdot \cos\left[\beta_{\min} + (i-1)\beta_c\right]}{\tan(\beta_{\max} - \beta_{\min})^{\frac{3}{2}}} \right]} \quad (4)$$

The maximum non-dimensional elastic force ($F_{a1} = F_1/F$) supported by the cylinder is determined.

Thus, Figs. 9 and 10 present the variation of the maximum dimensionless force versus relative dimension of the abrasive particle for different values of the length coefficient, (ξ), and rounding coefficient, (χ).

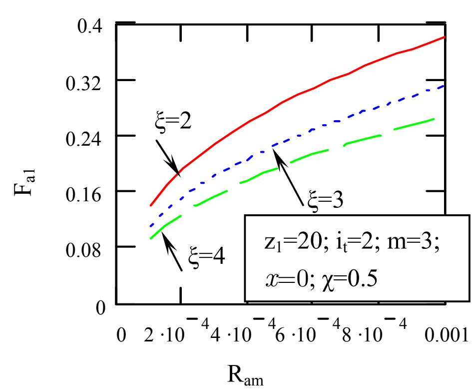

Fig.9. The variation of the maximum dimensionless force, (F_{a1}), versus relative dimension of the abrasive particle, (R_{am}), for different values of the length coefficient, (ξ).

Fig. 9 shows that the maximum dimensionless force, (F_{a1}), from the abrasive particle-equivalent tooth contact decreases with the increase of the abrasive particle length coefficient (ξ).

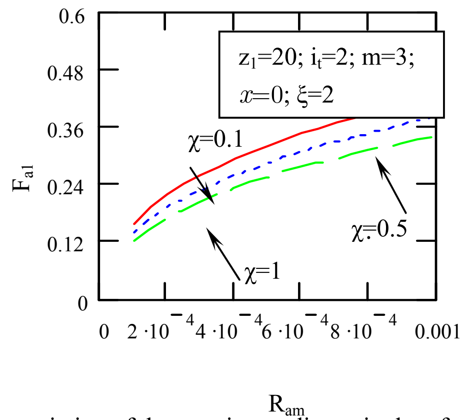

Fig.10. The variation of the maximum dimensionless force, (F_{a1}), versus relative dimension of the abrasive particle, (R_{am}), for different values of the rounding coefficient, (χ).

Fig. 10 shows that the rounding coefficient, (χ), has an important effect on the force distribution in the abrasive particle-equivalent tooth contact area.

3. The elastic loading capacity of the cylinder

In order to evaluate the elastic loading capacity between the rigid, ovoid particle and the elastic cylinder contact one shall use the Tresca plasticity criteria [2,3].

$$I_T = \max\left(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\right) \quad (5)$$

where: σ_1, σ_2 , σ_3 are the principal dimensionless stresses.

The condition for flow avoidance, evaluated with axial flow strength σ_c, is:

$$\frac{p_o}{\sigma_c} \le \frac{1}{I_T}$$

where p_o is the pressure in the center of hertzian contact.

The elastic loading capacity of the sphere-rigid ovoid contact can be defined as ratio $\frac{p_o}{\sigma_c}$. For the function characteristics (friction coefficient *f*, contact angle *β*) situated on the $1/I_M$ curve, the elastic loading capacity $\frac{p_o}{\sigma_c}$ can be evaluated. For the function characteristics situated above the $1/I_T$ curve, the deformation states is plastic and the stresses and deformation state can be determined with the plasticity theory. For the function characteristics situated beneath the $1/I_T$ curve, the wear particle appearance is defined by the elastic fatigue after a specific cycle number (Wöhler curve type).

Regarding the critical deformations of the transition from the elastic state to the plastic one, one considers the penetration (the interference) of the p_o pressure, for which the Tresca invariant reaches the limit value. Thus, using the Hertz equation system one can obtain:

$$h_{acr} = \left(\frac{h}{R}\right)_{cr} = (hk) = \left(\frac{\pi}{12}\frac{p_o}{E_o}\right)^2 = \left(\frac{\pi}{12}\frac{\sigma_c / E_o}{I_T}\right) \qquad (6)$$

where h is maximum interference (penetration) of ovoid to the elastic cylinder, R-radius of equivalent cylinder, E_o – elasticity modulus of cylinder.

For higher interference between abrasive particle and gear's teeth the deformations are plastic and they are determined with the Hencky slip-lines.

When operating, due to working conditions, abrasive particles (rigid ovoids) occur, then these particles can create plastic deformations on the **counter piece** (cylinder or sphere). These plastic deformations occur when the equivalent stress evaluated by the von Mises or Tresca parameter equalizes or becomes grater than the counter piece yield limit.

The elastic loading capacity is defined that the maximum contact pressure (p_{max}) for which the elastic limit of the material occurs. We consider that dimensionless parameter (p_{omax}) as a ratio of the maximum contact pressure and the yield limit of material.

The durability of friction pair with elastic strain can be evaluated using the Wohler fatigue curve.

The rigid abrasive particle has a plastic has a plastic penetration into gear teeth for the first contact and until the area contact becomes large, the deformation are elastic [4, 5].

4. Wear model for plastica contact

To evaluate the contact surface of a rigid particle with a plastically plane deformed, the Hencky slip-line method is used.

The plastic strain state is considered. The ovoid particles from the gap determine different contact angles with the cylinder. The particles effects upon the strain state are determined by the adhesion angle of abrasive particle to the tooth flank, defined by friction angle (f) and attack angle of the abrasive particle (α) [6,7,8].

The following angles are defined, which are specific to Hencky strain lines:

$$\varepsilon = 0.5\,\mathrm{acos}(\mathrm{f}); \qquad \phi = \alpha - \varepsilon; \qquad \eta = a\sin\left[\frac{\sin(\alpha)}{\sqrt{1-f}}\right]$$

For the specific conditions of the contact between a small sphere (micrometric radius) and a cylinder (millimeter radius), the wear process is determined by the Archard coefficient.

Depending on the cutting (α) and adhesion (ε) angles, two cases can be analyzed. When the cutting angle α < ε, the Archard wear parameter has the expression [5].

$$k_{A1} = \frac{\sqrt{3}}{f_2}\left(\frac{\gamma}{\gamma_r}\right)^2\left[\sin(\varepsilon) - \sin(\alpha) + 0.5\frac{p_{an}}{f_2}\left(\sin(\alpha)\cos(\alpha) + \sin(\alpha)^2\tan\left(\frac{\pi}{2} - \eta\right)\right)\right] \quad (7)$$

where: $f_1 = A_n \sin(\alpha) + \cos(2\varepsilon - \alpha)$ $\qquad f_2 = A_n \cos(\alpha) + \sin(2\varepsilon - \alpha)$

$$A_n = 1 + \frac{\pi}{2} + 2\varepsilon - 2\alpha - 2\eta$$

The shearing deformation

$$\gamma = \frac{f_1}{\sin(\varepsilon)} - f\frac{\sin(\phi)}{\sin(\varepsilon) - \sin(\alpha)}$$

and yield strain by shear stress

$\gamma_c = \dfrac{E_m}{\tau_c}$ with the melting energy per unite of volume (E_m), and yield limit (τ_c)

The Archard wear parameter, for the cutting angle α > ε is

$$k_{A2} = \frac{\sqrt{3}\,0.5}{f_2^{\,2}}\left(\frac{\gamma}{\gamma_c}\right)^2\left(0.5\sin(2\alpha) + \sin(\alpha)^2\right) \quad (8)$$

Fig. 11 shows the Archard wear parameter as a function of the cutting angle of abrasive particle for the yield strain corresponding to three materials.

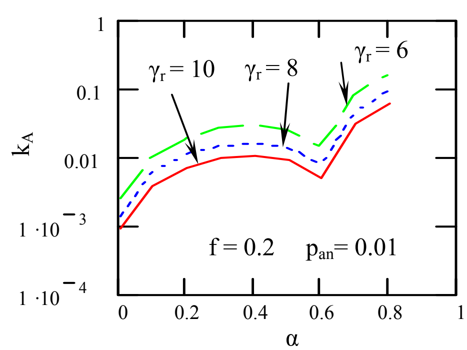

Fig. 11. Archard wear parameter vs. cutting angle of the abrasive particle.

When the cutting angle of the abrasive particle or other parameter is a random variable, it is necessary to analise the effect of probabilistic parameter on the wear parameter or friction coefficient [2].

We define the wear random parameter (I_w),

$$I_w = \int_{x_{\min}}^{x_{\max}} k_A(x_1, x_2, ..., x_n) f_p(x) x_m \, dx \tag{9}$$

where: $k_A(x_1,x_2,...,x_n)$ is the Archard deterministic wear parameter for the x_1, x_2,..., x_n variable;

$f_p(x)$ – frequency function of stochastic x variable;

x_m – statistic media of x variable;

x_{min}, x_{max} – minimum and maximum of stochastic x variable.

In this paper, we apply this concept to the cutting angle, that a stochastic variable. The exponential and normal probabilistic laws appear in some tribological phenomena [2,7,8].

a) The exponential law

The frequency function of cutting angle (α) is

$$f_e(\alpha) = \exp(-\alpha B) \tag{10}$$

where the constant B will be evaluted by the integrated law of frequency function (the sume of all probabilistic events is 100%).

Fig. 12 shows the effect of friction coefficient on the wear random parameter, for some dimensionless nominal contact pressure, when the yield strain is γ_c=10.

Fig. 13 shows that the contact pressure increases proportional to the random wear parameter.

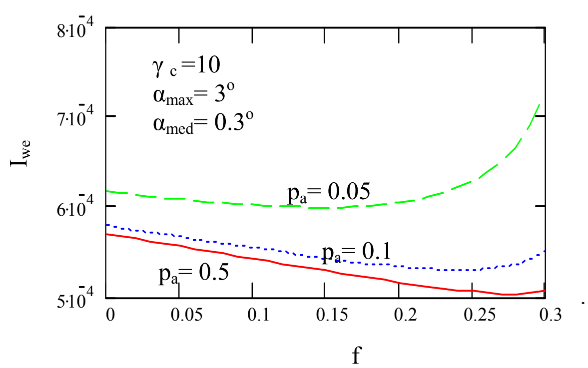

Fig.12. Wear exponential random parameter vs. adhesion parameter.

The wear parameter decreas or increas, when the statistic parameters of cutting angle are variable. For example, Fig. 13 shows the effect of variation of the maximum cutting angle, when the minimum angle is constant.

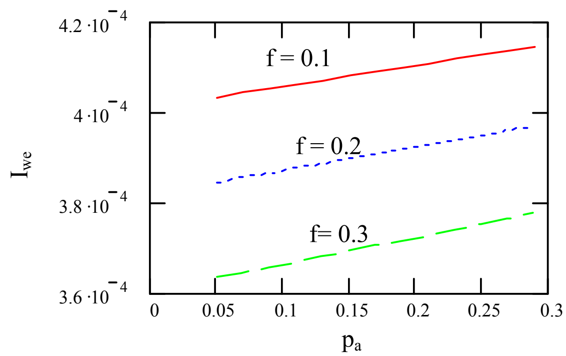

Fig.13. Wear exponential random parameter vs. dimensionless contact pressure.

The wear parameter decreas or increas, when the statistic parameters of cutting angle are variable. For example, Fig. 13 shows the effect of variation of the maximum cutting angle, when the minimum angle is constant.

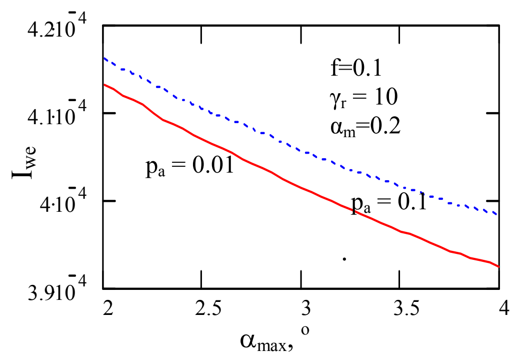

Fig.14. Effect of maximum statistic angle about wear random parameter.

b) The Gauss normal law

The frequency function of cutting angle (α) is

$$f_{pN}(\alpha) = \frac{1}{\sqrt{2\pi}} \exp\left[-\left(\frac{\alpha - \alpha_m}{\sqrt{2}\,\sigma_\alpha}\right)^2\right] \qquad (11)$$

when σ_α is standard deviation of cutting angle.

We accept the rule of the „ 6^{th} σ_α" for the maximum and minimum statistic cutting angle, when the media and standard deviation are known

$$\alpha_{min} = \alpha_m - 3\sigma_\alpha \qquad \alpha_{max} = \alpha_m + 3\sigma_\alpha$$

Thus, for example, the wear random parameter as a function to the adhesion parameter and the contact pressure are shown in Figure 15 and Figure 16.

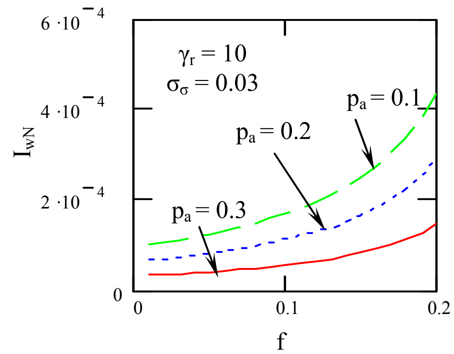

Fig.15. Wear normal Gauss random parameter vs. adhesion parameter.

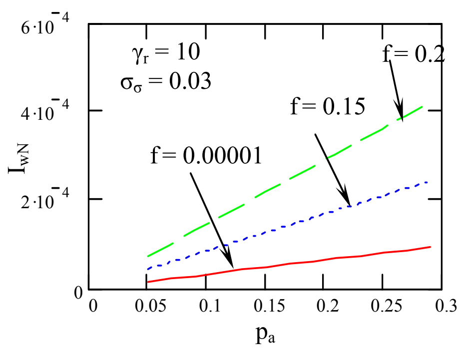

Fig.16. Wear normal Gauss random parameter vs. contact pressure.

The effect of standard angle deviation (σ_α) about the Archard wear parameter is shown in Figure 17.

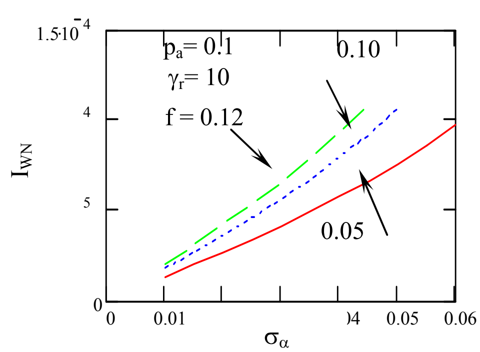

Fig. 17. Wear normal Gauss random parameter vs. standard angle deviation.

It can be seen that the Arhard wear parameter increases verry rapidly with the standard deviation.
If the wear random parameter is known, it is possible to estimate the durability of the friction pairs, function to geometric aspects.

5. Conclusions

Abrasion due to micro-cutting by the hard particle sharp edges or its roughness is one important forme to limits the durability of friction pairs.

The abrasive particle is considered to be an ovoid, characterised by two spherical segments and the distance between the spheres centre.

The ovoid particles number is a very important parameter for the wear phenomena of gears and it depends on the teeth number of the driving gear and the gear ratio.

We developed a theoretical model of the maximum dimensionless elastic load supported by the cylinder in contact with abrasive ovoid.

For higher penetration (interference) values, the deformations are plastic and we apply the Hencky slip- lines and thus, we obtained the Archard wear coefficient as a function on the cutting angle and the adhesion coefficient.

The exponential and normal probabilistic laws are exemplified to analyze the wear parameter.

The random wear coefficient is predicted and can be used to determine the durability of friction pairs.

R E F E R E N C E S

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