A NUMERICAL STUDY OF LAMINAR NATURAL CONVECTIVE HEAT TRANSFER INSIDE A CLOSED CAVITY WITH DIFFERENT ASPECT RATIO

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Two dimensional steady state laminar natural convection was studied numerically for differentially heated air filled closed cavity with adiabatic top and bottom walls. The temperature of the left heated wall and cooled right wall were assumed to be constant. The governing equations were iteratively solved using the control volume approach. In this paper, the effects of the Rayleigh number and the aspect ratio were examined. Flow and thermal fields were exhibited by means of streamlines and isotherms, respectively. Variations of the maximum stream function and the average Heat Transfer Coefficient were also shown.

The average Nusselt number was correlated to the Rayleigh number based on curve fitting for each aspect ratio. The investigation covers the range $10^4 \leq Ra \leq 10^7$ *was done at Prandlt number equal to 0.693. The result shows the average Nusselt number is the increasing function of Rayleigh number. As aspect ratio increases Nusselt number decreases along the hot wall of the cavity. As Rayleigh number increases Nusselt number increases. Result indicates that at constant aspect ratio, with increase in Rayleigh number heat transfer rate increases.*

Keywords: Natural convection, aspect ratio, heat transfer coefficient, finite volume method, closed cavity.

1. Introduction

Natural convection in enclosures is a topic of contemporary importance because enclosures filled with fluid are central components in a long list of engineering and geophysical systems with various geometries. The flow and heat transfer induced, for example, in the air space of a double-pane window system differs fundamentally from the external natural convection in which boundary layer considered. There have been numerous investigations conducted on natural convection in empty cavities under various configurations and boundary conditions [1-3]. Buoyancy induced flows are more complex because of the coupling between the flow and transport the first unified and comprehensive review of this subject made by Ostrach[4,5]. Lo et al. studied convection in cavities heated from left vertical wall and cooled from opposite vertical wall with both horizontal walls insulated using differential quadrature method. Numerical results are reported for several values of width-to-height aspect ratio of enclosure and Rayleigh number [6]. Basak et al have studied the effect of uniform

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and non uniform temperature at the bottom wall on natural convection in square cavity for Rayleigh numbers from 10^3 to 10^5using Galarkin finite element method [7].They reported that the uniform temperature gives more heat transfer than the non uniform temperature.

Natural convection is discussed in details for uniform heating at below or side of the square cavity in the references [8-15]. Corcione studied natural convection in a rectangular cavity heated from below and cooled from top as well as sides for variety of thermal boundary conditions. Numerical results are reported for several values of both aspect ratios of enclosure and Rayleigh Numbers [16]. The objective of this paper is to document the numerical data of steady state laminar natural convective heat transfer inside closed cavities filled with air having heated left wall and cooled right wall for the range of Ra from 10^4 to 10^7 for aspect ratios ranging from 1 to 3. The top and bottom walls of the cavities are adiabatic. The cavity illustrated in Fig.1 is chosen for simulating natural convective flow and heat transfer characteristics. It consists of dimensions L×H. The gravitational force is acting downwards. The aspect ratio of the cavity is defined as$= H/L$. The top and bottom walls are adiabatic.

Stream function

Stream function is defined as the scalar function of space and time, such that its partial derivatives w.r.t. any direction gives the velocity components u & v at right angles to that direction. It is denoted as ψ (psi).

$$u = \frac{\partial \psi}{\partial y} \ \& \ v = -\frac{\partial \psi}{\partial x}$$

2. Mathematical formulation

The cavity of length (L) & height (H) has a hot left wall with and cold right wall at constant temperatureT_h and T_c respectively. The top and bottom walls are adiabatic. The gravitational force is acting downwards. A buoyancy flow develops because of thermally induced density gradient. Heat is transferred from the hot wall to cold wall. The governing equations for natural convection flow are conservation of mass, momentum and energy can be written as:

Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

X-momentum: $$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x^2}\right) \quad (2)$$

Ymomentum$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial x^2}\right)+$
$g\beta(T-T_c)$ (3)

Energy Equation: $u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$ (4)

Rayleigh Number: $Ra=\frac{g\beta\Delta T L^3}{\alpha\gamma}$

The thermo physical properties of air are evaluated at the reference temperature that is the average of the two wall temperatures .In this study with Pr=0.693. The effects of the Rayleigh number which is varied from$10^4 to\ 10^7$,are of great interest in this study.

Boundary conditions:
Temperature:

At left wall T (0, y) =T_h
At right wall T (L, y) =T_c
At the top wall $\frac{\partial T}{\partial y}(x,H)=0;$
At the bottom wall $\frac{\partial T}{\partial y}(x,0)=0$;

Velocity:

$$u(x,0)=u(x,L)=u(0,y)=u(L,y)=0$$
$$v(x,0)=v(x,L)=v(0,y)$$
$$=v(L,y)=0$$

Here x and y are the distances measured along the horizontal and vertical directions, respectively; u and v are velocity components in the x-and y-directions, respectively,T denotes the temperature;p is the pressure and ρ is the density; T_h and T_c are the temperature at the hot and cold walls, respectively;L is the length of the cavity and H be the height of the cavity.

3. Numerical scheme

In the present investigation, the set of governing equations are integrated over the control volumes, which produces a set of algebraic equations. The SIMPLE algorithm is to used to solve the coupled system of governing equations. Discretization of the momentum and energy equations is performed by a second order upwind scheme and pressure interpolation is provided by PRESTO scheme. Convergence criterion considered as residuals is admitted10^{-3} for momentum and continuity equations and for the energy equation it is lower than10^{-6}. The calculations are carried out using the FLUENT 6.3 commercial code. The uniform grid system of 110×110 is adopted here.

4. Result and Discussion

Different cases with the vertical left hot wall of the closed cavity were studied. The effects of the Rayleigh number from $10^4 to\, 10^7$ and aspect ratio L/H=1, 1.5, 2, 3 were systematically investigated. Comparisons of the flow field and overall heat transfer rates among different aspect ratio were presented. When Ra greater than critical value, the buoyant force overcomes the fluid resistance and initiates natural convection currents, under certain boundary conditions which are observed to be in the form of hexagonal cells called Bénard cells. In the next phase of the work the study will be validated with experimental results.

4.1 Effects of Rayleigh number

4.1.1. Flow patterns and flow intensity

The flow fields are shown by means of streamlines shown below. Due to the buoyant effects caused by the temperature difference between the left hot wall and right cold wall, recirculating vortices are formed which are clearly demonstrated by the closed streamlines.

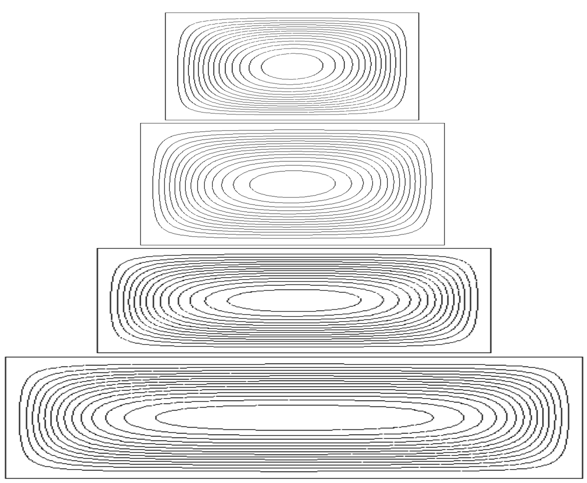
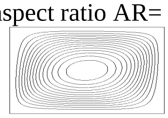


Fig 2 . Streamlines of aspect ratio AR= 1, 1.5, 2 , 3 at Ra=2×10^4



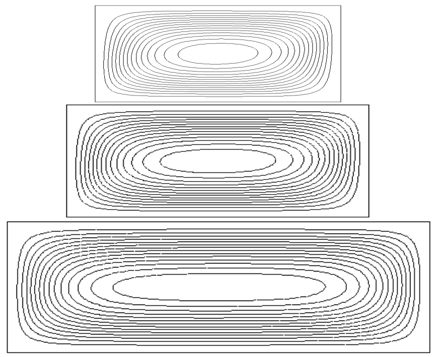


Fig 3. Streamlines of aspect ratio AR= 1, 1.5, 2,3 at Ra=3.5×10^5.

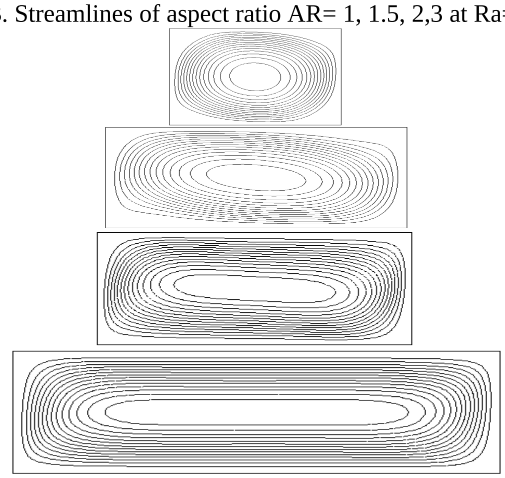


Fig 4. Streamlines of aspect ratio AR= 1, 1.5, 2, 3 at Ra=5×10^6.

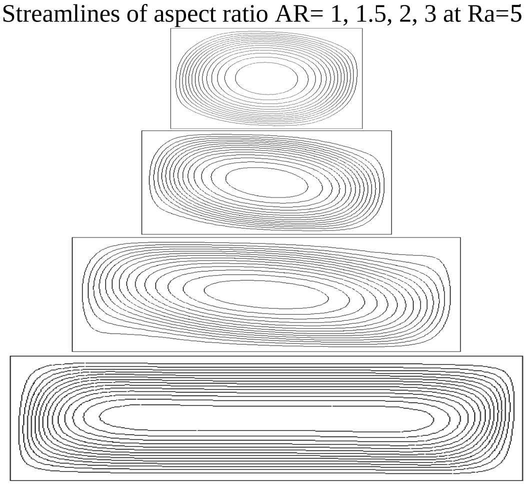


Fig 5. Streamlines of aspect ratio AR= 1, 1.5, 2, 3 at Ra=10^7.

In the above figures we observe that at different values of Ra as aspect ratio increases the central vortices changes its shape from circular to ellipse. As

Rayleigh number increases streamlines become more densely packed next to the walls. Also at constant values of aspect ratio as Ra increases the core of the vortices become larger.

On the other hand, the flow intensity is examined through the maximum value of the dimensionless stream function, , whose variation versus the Rayleigh number is shown in Fig.6. As expected, the maximum steam function is significantly increased with increased in Rayleigh number due to enhanced buoyancy-driven convection. Since the Rayleigh number is based on the length (L) of the closed cavity. Therefore, at constant Rayleigh number, the maximum stream function becomes greater as the aspect ratio L/H is decreased due to enlarged flow area. As indicated by Moukalled and Acharya [17], the increment of flow area also enhances the various frictions that tend to decay the flow intensity. As Rayleigh number increases the enhanced natural convection becomes dominant and the additional friction is negligible, leading to that the maximum stream functions at lower aspect ratio are apparently greater than those at higher aspect ratios.

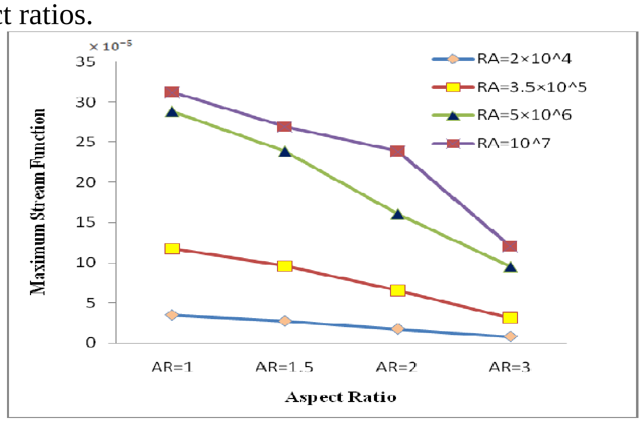


Fig 6.The maximum value of the Stream Function for different aspect ratio and Rayleigh numbers.

4.1.2. Heat Transfer:

The thermal fields are presented in the form of isotherms in figures shown below with same arrangement as in Fig.2, Fig3, Fig.4, and Fig.5. The isotherms pattern reveals that as Rayleigh number increases the packing of isotherms near the active walls become prominent implying rise in Nusselt number. The isotherms are orthogonal at the insulated walls ensure that the heat transfer rate becomes zero. As Rayleigh number increases heat transfer rate increases, velocity increases and the isotherms are no longer orthogonal, they bends. Here temperature of the left wall =910K and =790K.

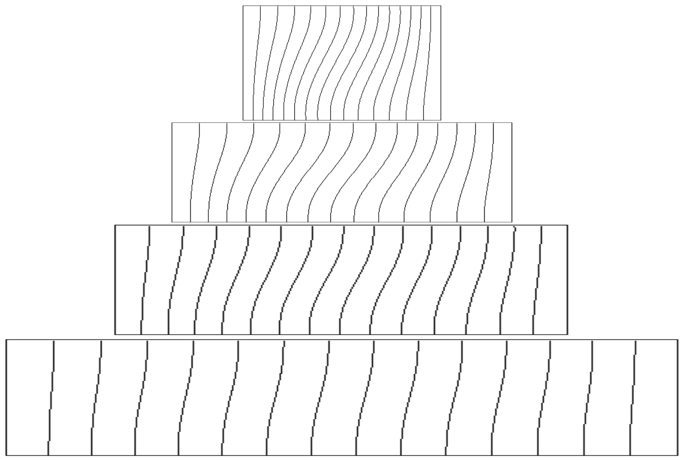


Fig 7. Isotherms of aspect ratio AR= 1, 1.5,2, 3 at Ra=2×10^4(T_h=910K and T_c=790K)

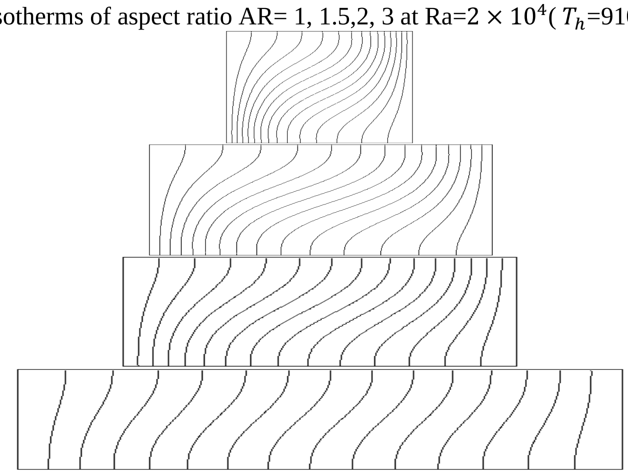


Fig 8. Isotherms of aspect ratio AR= 1, 1.5,2, 3 at Ra=3.5×10^5(T_h=910K and T_c=790K.)

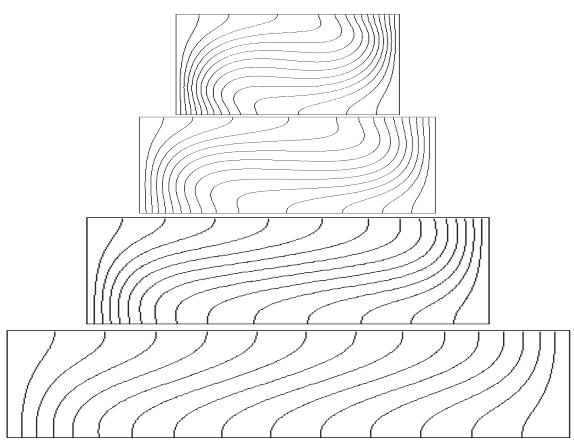


Fig 9. Isotherms of aspect ratio AR= 1, 1.5,2, 3 at Ra=5×10^6(T_h=910K and T_c=790K)

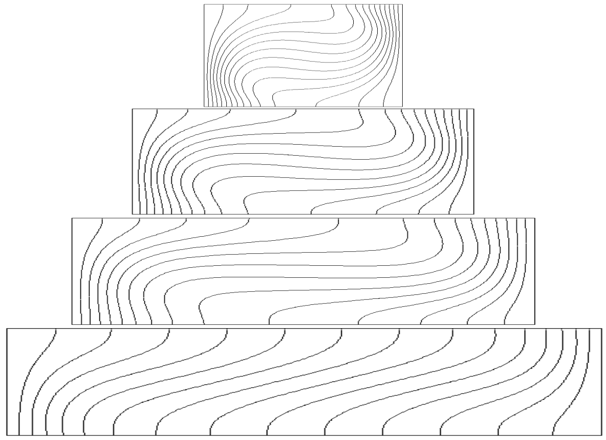


Fig 10. Isotherms of aspect ratio AR= 1, 1.5,2, 3 at Ra= (=910K and =790K)

Variation s of average heat transfer coefficient along the hot wall of the cavity for different aspect ratios and Rayleigh number are plotted in Fig.11. As Rayleigh number increases heat transfer rate along the hot wall of the cavity increases at constant aspect ratio but at constant Rayleigh number heat transfer rate along the same decreases as aspect ratio increases.

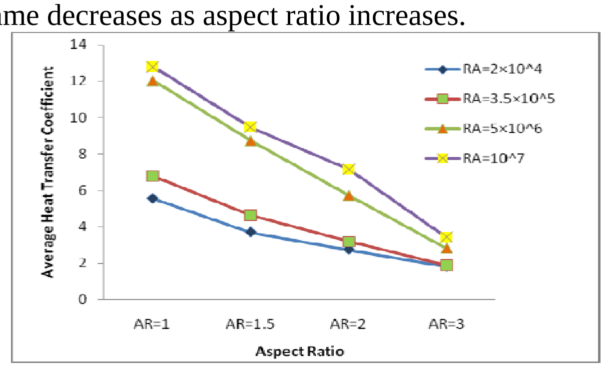


Fig11. Variation of average heat transfer coefficient along hot wall of cavity for different aspect ratio and Rayleigh number.

4.2 Correlation of average Nusselt number with Rayleigh number:

The variations of Average Nusselt numbers for different aspect ratios and Rayleigh numbers are plotted in the graph shown below in Fig.12.Thus , the equations were obtained that demonstrate the behavior of the Nusselt number as a function of Rayleigh number are given in theTable1.

Table1

Correlation of average Nusselt number with Rayleigh number

AR=1	AR=1.5	AR=2	AR=3

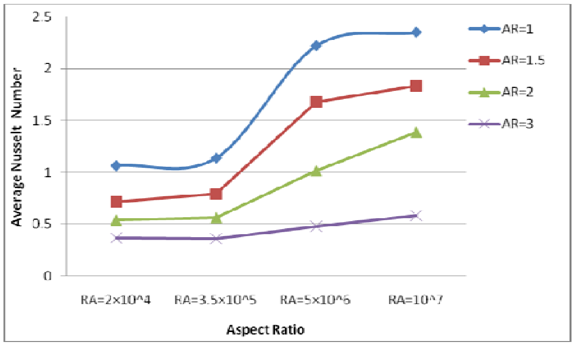


Fig12.Variation of average Nusselt number along hot wall of cavity for different aspect ratio and Rayleigh number

5. Conclusions

A numerical study on steady laminar natural convective heat transfer inside a closed cavity filled with air was carried out. The effects of Rayleigh number, aspect ratio on the flow and heat transfer were systematically studied. From the result and discussion we conclude that as Raleigh number increases streamlines become more densely packed next to the walls. Also at constant values of aspect ratio as Ra increases the core of the vortices become larger. As Rayleigh number increases the enhanced natural convection becomes dominant and the additional friction is negligible, leading to that the maximum stream functions at lower aspect ratios are apparently greater than those at higher aspect ratios. As Rayleigh number increases heat transfer rate increases, velocity increases and the isotherms are no longer orthogonal, they bends. Result indicates that at constant aspect ratio as Rayleigh Number increases Nussetl number increases along the hot wall of the cavity and heat transfer rate also increases but at constant Rayleigh number as Aspect ratio increases Nusselt number decreases along the same.

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