THE REALIZATION OF THE PLANE ANGLE UNIT BASED ON THE INM'PRIMARY STANDARD INSTALLATION

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Trasabilitatea în măsurarea unghiului plan este asigurată prin utilizarea poligoanelor optice, ca etaloane de transfer. Exactitatea punerii în practică a definiției unității de unghi plan prin această metodă depinde de precizia poziționării poligoanelor optice în timpul măsurării. Articolul prezintă metoda şi instalația cu două autocolimatoare fotoelectrice realizată la Institutul Național de Metrologie pentru etalonarea poligoanelor optice, evaluarea incertitudinilor parțiale şi rezultatele comparative la etalonarea unui poligon optic cu 8 unghiuri în cadrul proiectului COOMET 133/SK

Traceability in the plane angle measurements is carried out using the optical polygons, that are considered as transfer standards in the field. Accuracy of the plane angle unit reproduction is directly dependent of the accuracy of the optical polygons adjustment during their calibration. This paper presents the method and the installation with two photoelectric autocollimators used at National Institute of Metrology Bucharest, for calibration of optical polygon. The estimation of the measurement uncertainty components and the comparative results for the calibration of the 8–sided polygon during COOMET 133/SK inter-comparison are also pointed out.

Keywords: plane angle, uncertainty, optical polygons

Introduction

On the international scale, there is not an agreed international standard for angle plane unit. In order to realize the angle plane unit, every interested country has to define an appropriate measurement standard and to characterize it as a primary standard. The traceability of the measurement results to the International System of Units (SI) is carried out by international or bilateral comparisons, using optical polygons as transfer standards. At national level traceability in this field is carried out in accordance with the national hierarchy scheme that consists in the hierarchy of standards as primary, reference and working standards. The calibration of each standard and the validated measurement methods used at each

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level are also included in the hierarchy scheme. The uncertainties corresponding to each level represent the links through the traceability scheme.

The reproduction of plane angle units ***degree (^{0})***, ***minute (ʹ)*** and ***second (ʺ)*** is carried out either by uniform division and calibration of a circle division or through the determination of the ratio of two lengths according to a definite mathematical function.

This paper presents the measurement method and the installation with two photoelectric autocollimators used at National Institute of Metrology (INM), Bucharest, for the calibration of optical polygons. It is also presents the estimation of measurement uncertainty, by identifying the main uncertainty components caused by the inaccuracy of the optical polygons adjustment during their calibration. The comparative results for calibration of the 8–sided polygon by INM, by Physikalisch–Technische Bundesanstalt (PTB), by Central Office of Measures (GUM WAW) and by Slovak Institute of Metrology (SMU) are presented as well.

1. Measurement method and instrumentation for the standards calibration

The optical polygons are calibrated using the direct measurement method as it is described in [1,2,3]. The measurements consist in calibration of a single optical polygon, using a rotating table and two autocollimators. The main parts of the installation used to calibrate the optical polygons consists in:

- the base-plate
- the rotating table for the regulation and rotation of the polygons
- two photoelectric autocollimators for establishing the reference angle
- the recording devices.

1.1 Polygon mounting device and measuring instruments

The configuration and the measurement concept of the installation with two photoelectric autocollimators used at INM [2], for calibration of optical polygons are shown in Fig. 1.

The autocollimator Am, with two measurement axis, type ***electronic Autocollimator ELCOMAT 2000***, has the technical specifications as are indicated by the manufacturer (MOLLER-WEDEL, Germany):

- the resolution: 0.05"
- the accuracy: ± 0.1" over any 20" range
- the measurement range: 2000".

The fixed autocollimator A_f, with two measurement axis, is used as zero reference. It is a ***photoelectric Autocollimator type TA 53,*** with its technical performances mentioned by the manufacturer (Rank Taylor Hobson, UK):

- the resolution: 0.1"
- the accuracy: ± 0.1" over any 1" range.

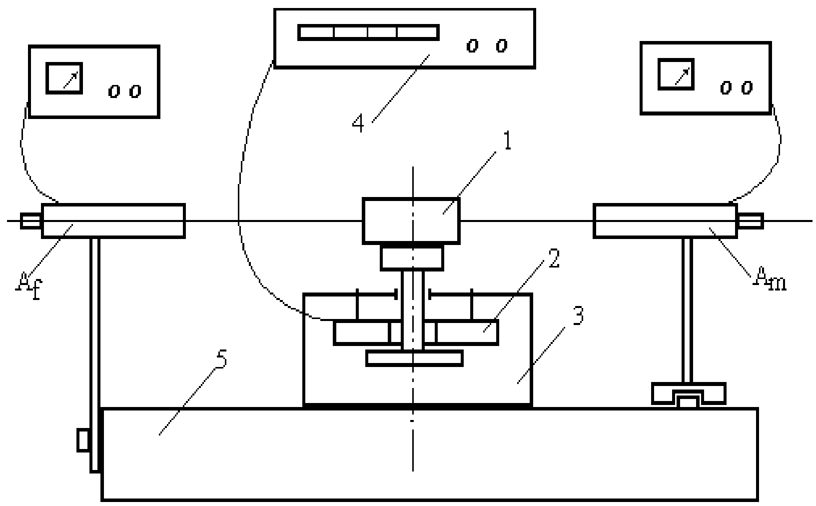

Fig. 1. Installation with two photoelectric autocollimators: 1 - polygon, 2 - rotation transducer, 3 - rotating table, 4 - display module, 5 - base-plate, A_f - fixed autocollimator, A_m - measurement autocollimator

Rotating table is used for the rotation of the optical polygons. The following technical performances specifications are indicated by the manufacturer (Carl Zeiss Jena, Germany):

- the stability of the rotation: 5"
- the radial wobble for polygon axis: < 10 µm.

1.2 Measurement procedure

The principle of "direct method" [1,2,3], consists in calibrating of a single polygon using a rotating table and two autocollimators. The graphical illustration of the measurement procedure is presented in Fig. 2.

A two - axial autocollimator telescope is required to obtain information of the axial run-out of the rotating table and pyramidal error of optical polygon.

The principle of calibration consists in the comparison of each angle of optical polygon (α_i) with the fixed angle between the view axes of the two autocollimators (β_1).

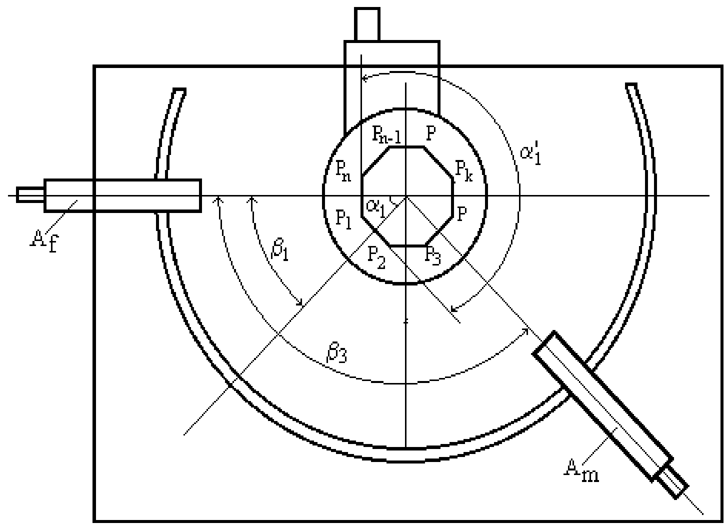

Fig. 2. The measurement procedure

The following mathematical models from (1) to (6) describe the above measurement procedure of the optical polygons calibration.

$$\sum_{i=1}^{n}\alpha_i - n\beta_1 = \sum_{i=1}^{n} d_i\,; \qquad \sum_{i=1}^{n}\alpha_i = 360^0 \tag{1}$$

$$\beta_1 = \frac{1}{n}\left(360^0 - \sum_{i=1}^{n} d_i\right) \tag{2}$$

$$\alpha_i = d_i + \frac{1}{n}\left(360^0 - \sum_{i=1}^{n} d_i\right) \tag{3}$$

$$x_i = \alpha_i - \frac{360^0}{n}; \qquad \Delta\beta_1 = \beta_1 - \frac{360^0}{n} \tag{4}$$

$$x_i - \Delta\beta_1 = d_i\;; \qquad \sum_{i=1}^{n} x_i = 0 \tag{5}$$

$$\Delta\beta_1 = \frac{1}{n}\sum_{i=1}^{n} d_i \qquad (6)$$

where:
d_i the indicated values of the autocollimator A_m
n: the number of sides of polygon
p the number of replicates

2 Evaluation of measurement uncertainty

The reproduction of plane angle unit, based on optical polygons, is directly dependent of the accuracy of the adjustment of the optical polygons during their calibration.

In industrial practice the summary correction $K_{1,i}$ is mostly used, which is defined as the correction to polygon normal particular direction related to first functional face. In this case the correction of the first face is naturally zero, which does not correspond to equally distributed probability on all functional faces. This inhomogenity is eliminated by the reduced summary corrections $K_{i,j}$.

If all quantities on which the result of a measurement depends are varied, its uncertainty can be evaluated by statistical means. However, because this is rarely possible in practice, the uncertainty of a measurement result is usually evaluated using a mathematical model of the measurement and the law of propagation of measurement uncertainty.

An estimate of the measurand, denoted by $K_{i,j}$, is obtained using input estimates for the values of the N quantities. Thus the output estimate $K_{i,j}$, which is the result of the measurement, is given by:

$$K_{ij} = \bar{x}_i + C_m + C_f + C_g + C_d + C_p + C_c \qquad (7)$$

where:

K_{ij} - correction for the angle between *i* and j sides of the polygon

$\bar{x}_i$ - deviation of the angle between *j* and *i* sides of the polygon, determined by relations (1…6);

C_m - corrections due to the ELCOMAT autocollimator

C_f - corrections due to the TA 53 autocollimator

C_g - corrections due to the change in the angle between the view axes of the two autocollimator

C_d - corrections due to flatness errors of the reflecting surfaces of the polygon under test

C_p - corrections due to the deviation from the perpendicular position of the reflecting surface of the polygon relative to the base surface (pyramid errors of the polygon)

C_c – corrections due to the deviation from centre of the polygon on the rotating table.

2.1 Type A evaluation of standard uncertainty

In most cases, the best available estimate of the expectation or expected value μ_q of quantity q that varies randomly, and for which n independent observations q_k have been obtained under the same conditions of measurement, is the arithmetic mean or average of the n observations. Thus, for an input quantity $X_{i,j}$, the arithmetic mean is used as the input estimate x_i in equation (7) to determine the measurement result K_{ij} [4, 5].

The individual observations differ in value because of random variations in the influence quantities or random effect. The experimental variance or the observations, which estimates the variance of the probability distribution of q, is given by s^2.

This estimate of variance and its positive square root s, termed the *experimental standard deviation*, characterize the variability of the observed value, or more specifically, their dispersion about their mean.

2.2 Type B evaluation of standard uncertainty

For an estimate x_i of an input quantity X_i that has not been obtained from repeated observations [4,5], the associated estimated variance $u^2(x_i)$ or the standard uncertainty $u(x_i)$ is evaluated by scientific judgement based on all or the available information on the possible variability of X_i. The pool of information may include:

- previous measurement data
- experience with or general knowledge of the behaviour and properties of relevant materials and instruments
- manufacturer's specifications
- data provided in calibration and other certificates
- uncertainties assigned to reference data taken from handbooks

2.3 The evaluation of the combined standard uncertainty

The estimated standard deviation associated with the output estimate of measurement result, termed *combined standard uncertainty* (u_c), is determined from the estimated standard deviation associated with each input estimate, termed *standard uncertainty* [4,5], by equation:

$$u_c = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial K}{\partial C_i}\right)^2 u^2(C_i)} \qquad (8)$$

Table 1 presents the estimated standard deviation associated with each input estimate and an example of calibration uncertainty evaluation for optical polygon.

Table 1

The evaluation of calibration uncertainty

Quantity	Estimated values	Standard uncertainty	Probability distribution	Sensitivity coefficient	Partial uncertainty
$\bar{x}_i$	- 0.25"	0.10"	normal	1	0.10"
C_m	0.00"	0.06"	rectangular	1	0.06"
C_f	0.00"	0.06"	rectangular	1	0.06"
C_g	0.00"	0.05"	rectangular	1	0.05"
C_d	0.00"	0.06"	rectangular	1	0.06"
C_p	0.00"	0.04"	rectangular	1	0.04"
C_c	0.00"	0.04"	rectangular	1	0.04"
K_{12}	0.25"			u_c=0.16"	

2.4 The evaluation of the expanded uncertainty

Although u_c can be universally used to express the uncertainty of a measurement result it is necessary to give a measure of uncertainty that defines an interval about the measurement result that may be expanded to encompass a large fraction of the distribution of values that could reasonably be attributed to the measured.

The additional measure of uncertainty is termed expanded uncertainty *U*.

$$U = k\,u_c = 0.32'' \text{ for } k = 2 \tag{9}$$

3 Results and discussion on INM inter-laboratory comparison

The optical polygons are the basic standards in the field of plane angle metrology used in all metrology institutes which are dealing with these kind of measurements. In order to compare the metrological performances of these standards, some inter-laboratory comparisons are usually organised. The comparative results obtained by INM and other three National Metrology Institutes (NMI 1…NMI 3) in the calibration exercise of the 8-sided optical polygon No 233368, manufactured by Hilger & Watts are presented in Table 2.

The reference value x_{ref} and its associated uncertainty u_{ref} considered as consensus value for all participating laboratories are calculated by pilot laboratory using the following relationships:

$$x_{ref} = \frac{\sum_{j=1}^{m} u_j^{-2} \cdot x_j}{\sum_{j=1}^{m} u_j^{-2}}, \quad u_{ref} = \left(\sum_{j=1}^{m} u_j^{-2} \right)^{-1/2} \tag{10}$$

Table 2

Comparative results

Face of polygon	NMI 1 ["]	NMI 2 ["]	NMI 3 ["]	INM ["]	Mean value ["]	s of mean value ["]	x_{ref} ["]	u_{ref} ["]
0	-0.400	-0.359	-0.340	-0.350	-0.362	0.026	-0.380	0.026
45	-0.120	-0.121	-0.100	-0.100	-0.110	0.012	-0.118	0.012
90	-0.210	-0.215	-0.220	-0.210	-0.214	0.005	-0.212	0.005
135	0.770	0.807	0.790	0.810	0.794	0.018	0.786	0.018
180	0.830	0.834	0.830	0.830	0.831	0.002	0.831	0.002
225	-0.350	-0.376	-0.390	-0.380	-0.374	0.017	-0.362	0.017
270	-0.150	-0.156	-0.160	-0.150	-0.154	0.005	-0.152	0.005
315	-0.370	-0.416	-0.400	-0.430	-0.404	0.026	-0.390	0.026

The "normalized error" so-called "E_n – criterion" is evaluated [6] in order to check the internal consistence between the result of a particular measurement and the reference value:

$$E_n = \frac{1}{k} \cdot \frac{x_i - x_{ref}}{\sqrt{u_i^2 - u_{ref}^2}} \tag{11}$$

The acceptance criteria is $|E_n| \le 1$.

Variance of values inside of a particular laboratory is small. The internal consistence by means of E_n – criterion was tested too. Particular values are presented in Table 3. Good agreement of all participating laboratories for the measurements of 8-sided polygon was reported.

Table 3

E_n – criterion

Face of polygon	E_n NMI 1	E_n NMI 2	E_n NMI 3	E_n INM
0	-0.316	0.182	0.122	0.098
45	-0.034	-0.024	0.054	0.058
90	0.034	-0.023	-0.024	0.007
135	-0.252	0.184	0.013	0.078
180	-0.023	0.025	-0.004	-0.005
225	0.198	-0.113	-0.084	-0.057
270	0.038	-0.029	-0.023	0.008
315	0.326	-0.217	-0.029	-0.128

Conclusions

Calibration measurement method presented in this paper was used and put in practice at the INM specialised laboratory. The standard installation of plan angle with two photoelectric autocollimators for calibration of optical polygons was realised as well as certified as primary national standard in this field.

Experimental results and the associated measurement uncertainty of national optical polygons are in good agreement with the reported results by other experienced national laboratories. As a consequence, the national standard and measurement capability in this field was recognised in the framework of mutual recognition arrangement at international level [7], and these kinds of calibration are included in the BIPM-database.

R E F E R E N C E S

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