A MATHEMATICAL MODEL FOR HUMIDIFYING THE ELECTRODES COATS

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Lucrarea prezintă un model matematic al procesului de umidificare a învelişului electrozilor folosind metoda experimentului activ care presupune utilizarea metodelor statistice în toate etapele experimentului:

- *înaintea desfăşurării experimentului, prin stabilirea numărului de experienţe şi a condiţiilor de realizare a acestora;*
- *în timpul desfăşurării experienţelor prin prelucrarea rezultatelor obţinute;*
- *după încheierea experimentului prin concluzii referitoare la realizarea unor experinţe viitoare, etc.*

În cadrul acestui mod de tratare a modelării matematice s-au parcurs etapele prezentate mai jos.

This research paper deals with a mathematical model for humidifying the electrodes coats by using the method of active experiment, which involves the use of statistical methods in all experiment phases as follows:

- *before the experiment by establishing the number of tests and the conditions that shall be performed in;*
- *during the tests by processing the outcomes;*
- *after the experiment by conclusions regarding future tests, etc;*

Within this way of treating the mathematical modelling, the below mentioned stages were completed.

Keywords: mathematical model, diffusible hydrogen, electrode, humidity

Introduction

Solving the problem starts with its definition and in this respect the purpose of this paper should be clearly and accurately represented. In order to solve the problem the function to be improved should have a physical meaning, should allow numerical expression and indicate extreme values [1, 2, 4,].

In this case the function target shall be $\tilde{y}$, the humidity of the electrodes coat, in percentages [%].

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Selection of the independent variables and their variation range is the next step. The following factors of influence on the coat humidity have been studied [3]:

z_1- air humidity varying in the range 0.6 to 0.9;

z_2, z_3- maintenance time, [h]; $z_2 \in [0;1]$ and $z_3 \in [1;8]$

1. Determination of the mathematical model and the experimental program

As the humidity is rapidly absorbed by the electrodes coat when the humidifying process starts, two forms for this function target were established as follows:

$\widetilde{y}_1$ - for maintenance time z_2 and humidity z_1;

$\widetilde{y}_2$ - for maintenance time z_3 and humidity z_1.

To define the mathematical model the independent variables should be codified and the variation levels be determined.

The codified values of the independent variables are:

x_1- for the independent variable z_1;

x_2- for the independent variable z_2;

x_3-for the independent variable z_3.

The variation levels of the codified values where the higher level is codified with $^{+1}$, the lower level with $^{-1}$ and the basic level with 0, are listed in Table 1.

Table 1.

The levels of the codified values

Variation Level	x_1	x_2	x_3
Higher (+1)	0.90	1	8
Average (basic) (0)	0.75	0.5	4.5
Lower (-1)	0.60	0	1

If we consider x_1 the codified value of factor z_1, it shall result from the formula:

$$x_i = \frac{z_i - z_0}{\Delta z_i} \tag{1}$$

where

z_0 - the basic level

Δz_i - the variation interval

To determine the mathematical model of the researched process more forms were tried and the one with the best results was the non-linear one of order two which most accurately showed the humidifying process in the electrodes coats [1].

The two forms (mathematical models) of the function target are:

$$\tilde{y}_1 = b_0 + b_1 x_1 + b_2 x_2 + b_{12} x_1 x_2 + b_{11}\left(x_1^2 - \frac{2}{3}\right) + b_{22}\left(x_2^2 - \frac{2}{3}\right) \tag{2}$$

$$\tilde{y}_2 = c_0 + c_1 x_1 + c_3 x_3 + c_{13} x_1 x_3 + c_{11}\left(x_1^2 - \frac{2}{3}\right) + c_{33}\left(x_2^2 - \frac{2}{3}\right) \tag{3}$$

The used experimental program is given by the formula:

$$N = n^k \tag{4}$$

where N= number of tests;
n = variation level;
k = number of independent variables.

In our case it means: $N = 3^2$ tests.

The matrix of the programmed experiment (for non-linear mathematical models) and the results obtained are shown in Table 2.

Table 2

The matrix of the programmed experiment

Test no.	x_0	x_1	x_2	x_3	x_1x_2	x_1x_3	$x_1^2 - \frac{2}{3}$	$x_2^2 - \frac{2}{3}$	$x_3^2 - \frac{2}{3}$	y value			
										$y_1$exp		$y_2$exp	
1	+1	+1	+1	+1	+1	+1	1/3	1/3	1/3	y_{11}	2.45	y_{21}	4.25
2	+1	+1	-1	-1	-1	-1	1/3	1/3	1/3	y_{12}	0.21	y_{22}	2.45
3	+1	-1	+1	+1	-1	-1	1/3	1/3	1/3	y_{13}	1.38	y_{23}	2.82
4	+1	-1	-1	-1	+1	+1	1/3	1/3	1/3	y_{14}	0.19	y_{24}	1.38
5	+1	+1	0	0	0	0	1/3	-2/3	-2/3	y_{15}	1.65	y_{25}	4.00
6	+1	-1	0	0	0	0	1/3	-2/3	-2/3	y_{16}	0.80	y_{26}	2.55
7	+1	0	+1	+1	0	0	-2/3	1/3	1/3	y_{17}	2.05	y_{27}	3.90
8	+1	0	-1	-1	0	0	-2/3	1/3	1/3	y_{18}	0.20	y_{28}	2.05
9	+1	0	0	0	0	0	-2/3	-2/3	-2/3	y_{19}	1.14	y_{29}	3.65

2. Calculation of the coefficients for non linear mathematical models

Calculation formulae of the models coefficients are the followings:

$$b_i = \frac{\sum_{u=1}^{N} x_{iu} y_{1u}}{\sum_{u=1}^{N} x_{iu}^2}\ ;\ c_i = \frac{\sum_{u=1}^{N} x_{iu} y_{2u}}{\sum_{u=1}^{N} x_{iu}^2} \tag{5}$$

$$b_{ij} = \frac{\sum_{u=1}^{N} x_{iu} y_{ju} y_{1u}}{\sum_{u=1}^{N} \left(x_{iu} x_{ju}\right)^2}\ ;\ c_{ij} = \frac{\sum_{u=1}^{N} x_{iu} y_{ju} y_{2u}}{\sum_{u=1}^{N} \left(x_{iu} x_{ju}\right)^2} \tag{6}$$

$$b_{ii} = \frac{\sum_{u=1}^{N} x_{iu} y_{1u}}{\sum_{u=1}^{N} \left(x_{iu}^{'}\right)^2}\ ;\ c_{ii} = \frac{\sum_{u=1}^{N} x_{iu}^{'} y_{2u}}{\sum_{u=1}^{N} \left(x_{iu}^{'}\right)^2} \tag{7}$$

$$x_{iu}^{'\,2} = x_{iu}^2 - \frac{2}{3} \tag{8}$$

Using these relations the following calculation formulae for the models coefficients result:

$b_0=1/9(y_{11}+y_{12}+y_{13}+ y_{14}+ y_{15}+ y_{16}+ y_{17}+ y_{18}+ y_{19})$;

$b_1=1/6(y_{11}+y_{12} + y_{15}- y_{13}- y_{14}- y_{16})$;

$b_2=1/6(y_{11}+y_{13}+y_{17}- y_{12}- y_{14}- y_{18})$;

$b_{12}=1/4(y_{11}+y_{14}- y_{12}- y_{13})$;

$b_{11}=1/6[(y_{11}+y_{12}+y_{13}+ y_{14}+ y_{15}+ y_{16})-2(y_{17}+ y_{18}+ y_{19})]$;

$b_{22}=1/6[(y_{11}+y_{12}+y_{13}+ y_{14}+ y_{17}+ y_{18})-2(y_{15}+ y_{16}+ y_{19})]$;

$c_0=1/9(y_{21}+y_{22}+y_{23}+ y_{24}+ y_{25}+ y_{26}+ y_{27}+ y_{28}+ y_{29})$;

$c_1=1/6(y_{21}+y_{22}+ y_{25}- y_{23}- y_{24}- y_{26})$;

$c_2=1/6(y_{21}+y_{23}+y_{27}- y_{22}- y_{24}- y_{28})$;

$c_{12}=1/4(y_{21}+y_{24}- y_{22}- y_{23})$;

$c_{11}=1/6[(y_{21}+y_{22}+y_{23}+ y_{24}+ y_{25}+ y_{26})-2(y_{27}+ y_{28}+ y_{29})]$;

$c_{33}=1/6[(y_{21}+y_{22}+y_{23}+ y_{24}+ y_{27}+ y_{28})-2(y_{25}+ y_{26}+ y_{29})]$;

Replacing the experimental values y_1u and y_2u the following values for the models coefficients are obtained:

b_0=1.188	b_1=0.323	b_2=0.880
b_{12}= 0.265	b_{11}= 0.016	b_{22}= -0.116
c_0=3,005	c_1= 0.658	c_2= 0.848
c_{12}=0.090	c_{11}= -0.292	c_{33}= -0.592

The following regression equations result:

$$\widetilde{y}_1 = 1.088 + 0.323x_1 + 0.88x_2 + 0.265x_1x_2 + $$
$$+ 0.016\left(x_1^2 - \frac{2}{3}\right) - 0.116\left(x_2^2 - \frac{2}{3}\right)$$
$$\widetilde{y}_2 = 3.005 + 0.658x_1 + 0.848x_3 + 0.09x_1x_3 - $$
$$- 0.292\left(x_1^2 - \frac{2}{3}\right) - 0.592\left(x_3^2 - \frac{2}{3}\right)$$

The results of calculation are:

$$\widetilde{y}_1 = 1.254 + 0.323x_1 + 0.88x_2 + 0.265x_1x_2 + 0.016x_1^2 - 0.116x_2^2;$$
$$\widetilde{y}_2 = 3.594 + 0.658x_1 + 0.848x_3 + 0.09x_1x_3 - 0.292x_1^2 - 0.592x_3^2.$$

Using formula (1), the codified values x_1, x_2, x_3 are replaced by the real variables z_1, z_2, z_3 and the following nonlinear models result:

$$\widetilde{y}_1 = 0.3675 - 0.678z_1 - 0.426z_2 + 3.533z_1z_2 + 0.71z_1^2 - 0.464z_2^2 \quad (9)$$
$$\widetilde{y}_2 = -8.479 + 23.082z_1 + 0.546z_3 + 0.171z_1z_3 - 12.977z_1^2 - 0.048z_3^2 \quad (10)$$

3 Checking the inconsistency of the non linear mathematical models

This verification aims in determining the possibility of using nonlinear models in studying the process or a model of higher order is needed.

The hypothesis about the model consistency can be verified on the basis of Fischer [1, 4] criteria whose calculated value is established by the formula:

$$F_c = \frac{S_{conc}^2}{S_0^2} \quad (11)$$

where:

S_{conc}^2 = variance caused by the calculated model

S_0^2 = variance of the results reproducibility

The variance caused by the non linear model is calculated by the formula:

$$S_{conc}^2 = \frac{\sum_{u=1}^{N}\left(\widetilde{y}_u - y_{u\exp}\right)}{N - k'} \quad (12)$$

where:

$\tilde{y}_u$ respectively $y_{u\exp}$ are the values calculated with the regression equation, respectively obtained experimentally under the test *u*;

$N-k'$ - number of degrees of freedom representing the difference between number *N* of tests and number k' of coefficients in the regression equation (including b_o).

Calculation of S_0^2 is made with formula:

$$S_0^2 = \frac{\sum_{u=1}^{N}(\tilde{y}_u - \bar{y})^2}{n-1} \tag{13}$$

where

u- test number;

n- number of parallel tests;

y-arithmetical average of the results obtained in the n parallel tests.

The calculated model is consistent when:

$$F_c < F_{0.05}; \upsilon_1 > \upsilon_2 \tag{14}$$

where:

υ_1 - number of degrees of freedom, calculated by

$S_{conc}^2(\upsilon_1 = N - k')$

υ_2 - number of degrees of freedom, calculated by

$S_0^2(\upsilon_2 = n-1)$

n- number of tests for the basic level

To calculate the variance of models reproducibility (experimental error) the Tables 3 and 4 are drawn up.

Table 3

Test no.	y_{1u}	$\bar{y}_{1u}$	$(y_{1u} - \bar{y}_{1u})$	$(y_{1u} - \bar{y}_{1u})^2$	$y_2 = n-1$
1	1.20	1.14	0.06	0.0036	2
2	1.11		-0.03	0.0009	
3	1.11		-0.03	0.0009	
$s_{0_1}^2 = \frac{0.0054}{2} = 0.0027$				0.0054	

Table 4

Test no.	y_{2u}	$\bar{y}_{2u}$	$(y_{2u} - \bar{y}_{2u})$	$(y_{2u} - \bar{y}_{2u})^2$	$y_2 = n-1$
1	3.75	3.65	0.10	0.01	2
2	3.62		-0.03	0.0009	
3	3.58		-0.07	0.0049	
$s_{0_2}^2 = \frac{0.0158}{2} = 0.0079$				0.0158	

To calculate the variance caused by nonlinear regression equations the Table 5 is drawn up.

The calculated values of Fischer criteria for the two nonlinear models are:

$$F_{c_1} = \frac{0.0171}{0.0027} = 6.33 ; \qquad F_{c_1} = \frac{0.0122}{0.0079} = 1.544 \qquad (15)$$

For $\alpha = 0.05$, the values of Fischer criteria for $\upsilon_1 = 3$ and $\upsilon_2 = 2$ are:

$$F_{0.05;3;2} = 19.6$$

As F_{c1} and $F_{c2} < F_{0,05;3;2}$, the hypothesis of the model consistency is verified and they can be considered useful for analysing the humidifying process of the electrodes coats. The result shows that the calculated nonlinear models y_1 and y_2 are proper and they express the real studied process with a good approximation.

Table 5

	$y_{1\exp}$	$\tilde{y}_{1\exp}$	$y_{1u} - \bar{y}_{1\exp}$	$(\tilde{y}_1 - y_{1\exp})^2$	$\tilde{y}_2$	$\tilde{y}_{2\exp}$	$y_{2u} - \bar{y}_{2\exp}$	$(\tilde{y}_2 - y_{2\exp})^2$	y_{11}	y_{12}
1	2.45	2.58	0.13	0.1069	4.13	4.25	0.06	0.0036	9-6=3	9-6=3
2	0.21	0.30	0.09	0.0081	2.44	2.45	-0.01	0.0001		
3	1.38	1.44	0.06	0.0036	2.82	2.82	0.00	0.0000		
4	0.19	0.21	0.02	0.0004	1.29	1.38	-0.09	0.0081		
5	1.65	1.60	-0.05	0.0025	3.96	4.00	-0.04	0.0016		
6	0.80	0.88	0.08	0.0064	2.64	2.55	0.09	0.0081		
7	2.05	2.02	-0.03	0.0009	3.85	3.90	-0.05	0.0025		
8	0.20	0.25	0.05	0.0025	2.15	2.05	0.1	0.01		
9	1.14	1.24	0.1	0.01	3.60	3.65	-0.05	0.0025		
s^2conc1=0.0513/3=0.0171				0.0513	s^2conc2=0.0365/3=0.0122			0.0365		

Conclusions

By analysing the nonlinear models in the relations (9) and (10) we can draw the following conclusions as to the studied physical process:

1) mathematical instruments can be used for studying humidification in electrodes coat under concrete conditions of air conditioning;
2) if a certain humidity level is implemented in the coat responsible for a critical level of diffusible hydrogen (which from the danger of cold cracking) and a relative air humidity it is possible to easily determinate the strength of the related electrode coat to humidification;
3) for the same above mentioned conditions of humidity and diffusible hydrogen we can calculate the humidity necessary for reaching the maximum strength to humidification and accordingly we can establish the conditions for handling electrodes coated at welding place. For example to reach in a coat a 2% humidity and a 1h resistance to humidification we haze to maintain in the electrodes a relative air humidity of about 72%;

4) deriving with respect to the time the two regression equations we get the humidification speeds for the two periods of time:

$$\frac{\partial \tilde{y}_1}{\partial z_2} = -0.426 + 3.533 z_1 - 0.928 z_2 \tag{16}$$

$$\frac{\partial \tilde{y}_2}{\partial z_3} = -0.546 + 0.171 z_1 - 0.096 z_2 \tag{17}$$

By analysing the two partial derivatives of the regression equation we can find the followings:

- the humidification speed is much higher in the first time internal because the pressure of the saturated steams is much higher than that of the steams inside capillary;
- the longer is the maintenance time the more the humidification speed lowers tending to zero in long maintenance times. That happens because the pressure of the saturated steams inside capillary equals the balance pressure corresponding to the absorption layer from expanded condition to intermediate liquid state (condensed). In such conditions the water absorption in the coat equals zero (it is proportional to ln p_1/p_2) and regardless the time the electrode is kept in this climate it no longer absorbs humidity.

5) it is possible to determine the influence of humidification factors on the coat humidity and its strength to humidification.
6) relations can replace, in certain cases, the humidity determinations which are very expensive both as to the required equipment and to the sensitivity of determinations and measurements.

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