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ANALYTICAL AND NUMERICAL STUDY OF THE STABILITY PHENOMENON OF A PUNCHING DIE

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In lucrarea de față este prezentat un studiu analitic destinat calculului forței critice de flambaj şi numeric, cu ajutorul metodei elementelor finite, în vederea determinării stării de deformație din organul activ de ştanțare dintr-o matriță combinată cu acțiune succesiv-simultană. Deoarece în timpul proceselor de deformare plastică la rece, elementele active sunt solicitate la sarcini ce variază periodic, fenomenul de oboseală a materialului poate grăbi distrugerea acestuia. Noutatea acestui articol rezidă în faptul că în nicio lucrare de specialitate nu există rezolvări explicite pentru barele cu secțiune variabilă în trepte şi cu condiții de rezemare altele decât cele corespunzătoare cazurilor fundamentale.

In this paper an analytical study is presented for the calculus of the critical buckling load as well as a numerically one, using the finite element method, for the determination of strain state for the active body of a combined punching die with a successive-simultaneous action. Since during the cold forming processes the active elements are subjected to periodically varying loads, the phenomenon of fatigue of the material may hasten its damage. The novelty of this article is that in any specialty course there are no explicit solutions for bars with stepped variable sections and with bearing conditions other than those used for basic cases.

Key-words: buckling, punching die, critical force, finite element method

1. Introduction

During manufacturing processes, the reliability study of the technological assets of molds and die bodies plays an important role in meeting contractual requirements as well as those related to the quality of products made by plastic deformation. Because of a poor design and lack of thorough checking, it is possible to affect the manufacturing process, through partial or total damage of the active punching body.

The inertia of a constructive design can lead to a much simpler solution in terms of analytical calculations and execution (i.e., a punch of constant section), but with an increased risk of loss of elastic stability in exploitation.

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For these reasons, the authors propose in this paper, an improved version of the constructive solution, leading to a better reliability of the manufacturing process.

2. Analytical calculus of critical buckling load

The character of the equilibrium is studied starting from Dirichlet principle: in stable equilibrium, the potential function has a minimum, whereas in an unstable condition, a maximum. In indifferent (neutral) equilibrium, the potential function, in all neighboring positions face to the analyzed one, is constant [1]. In the spirit of Dirichlet data definition, if one considers a conservative elastic system initially in equilibrium under the action of a set of forces, the system will leave the equilibrium state only if it is acted by a force which is external to the initially equilibrium system. Considering the total energy, E, induced into the system by a perturbing force, one can write the following balance equation under the conservation of energy law:

$$E = E_C + E_P = const \tag{1}$$

where E_C is the kinetic energy and E_P the potential energy of the system.

An increase in kinetic energy is accompanied by a decrease of potential energy and vice versa, in accordance with the law of energy conservation. If the system is initially in a equilibrium configuration with a minimum of potential energy, then the potential energy from the equation of conservation increases and in these conditions the kinetic energy due to the system motion must decrease. Thus, the displacement from the initial equilibrium state caused by a system disturbance with an external force will remain low leading to a stable equilibrium state.

In case of beams with variable sections (Figure 1) – representing in fact the present application – the determination of the buckling load is not a simple problem.

Starting from solutions based on established equations and boundary conditions (restraint conditions and continuity conditions) it will result the critical buckling load. This method is applicable only if the deflection occurs in the elastic domain [2], [3].

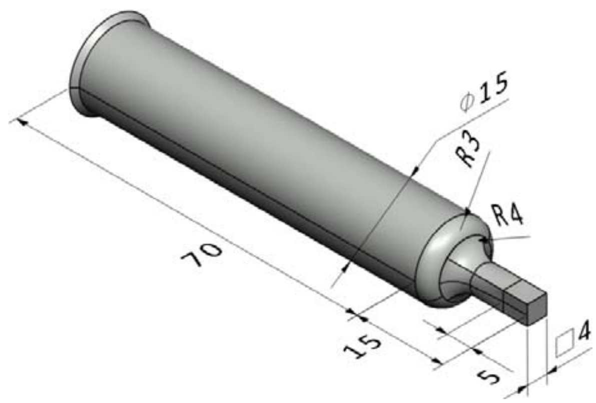

Fig. 1. Draft of the punching die

The differential equation of the curvilinear equilibrium shape is:

$$EI\frac{1}{\rho} = -Fw \tag{2}$$

where:

$$\frac{1}{\rho} \approx \frac{d^2w}{dx^2} \tag{3}$$

represents the curvature of the bar.

In Fig. 2 is presented a schematization of the bar, with the two different cross-sections. The bar is considered embedded at one extremity and simply supported at the other one. In this support a reaction force H will occur, the bending moments in sections defined by x_1 and x_2 being respectively [4]:

$$M_{x_1} = Fw_1 - Hx_1 \tag{4}$$

$$M_{x_2} = Fw_2 - H(12 + x_2) \tag{5}$$

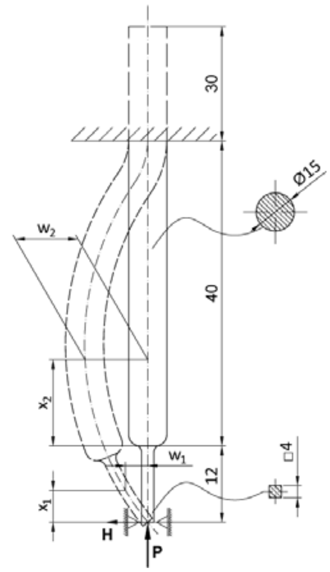

Fig. 2. Bar schematization

Starting from the differential equation of the deformed median fiber written for the two intervals, it will result:

$$EI_1 \frac{d^2 w_1}{dx_1^2} = Hx_1 - Fw_1 \text{ or: } \frac{d^2 w_1}{dx_1^2} = \frac{H}{EI_1} x_1 - \frac{F}{EI_1} w_1 \quad (6)$$

$$EI_2 \frac{d^2 w_2}{dx^2} = H(12 + x_2) - Fw_2 \text{ or: } \frac{d^2 w_2}{dx^2} + \frac{F}{EI_2} w_2 = \frac{H}{EI_2}(12 + x_2) \quad (7)$$

The two equations admit a solution form:

$$w_1 = A_1 \sin(\alpha_1 x_1) + B_1 \cos(\alpha_1 x_1) + \frac{H}{F} x_1 \quad (8)$$

$$w_2 = A_2 \sin(\alpha_2 x_2) + B_2 \cos(\alpha_2 x_2) + \frac{H}{F}(12 + x_2) \quad (9)$$

$$\text{where: } \alpha_1^2 = \frac{F}{EI_1},\ \alpha_2^2 = \frac{F}{EI_2} \text{ and } \frac{\alpha_1}{\alpha_2} = \sqrt{\frac{I_2}{I_1}} = 10{,}792 \quad (10)$$

$$\text{and } I_1 = \frac{a^4}{12} = 21{,}33\, mm^4,\ I_2 = \frac{\pi \cdot 15^4}{64} = 2485{,}04\ mm^4 \quad (11)$$

represent the moments of inertia for the two sections and E the Young modulus.

By imposing boundary conditions, respectively:

- at $x_1 = 0$ $w_1 = 0$, at $x_2 = 40$, $w_2 = 0$ and $\frac{dw_2}{dx_2} = 0$ as well as the condition of continuity:

- at $x_1 = 12$ and $x_2 = 0$, $w_1 = w_2$ and $\frac{dw_1}{dx_1} = \frac{dw_2}{dx_2}$, one could obtain:

$$B_1 = 0, \text{ respectively } A_2 \sin(40\alpha_2) + B_2 \cos(40\alpha_2) + 52\frac{H}{F} = 0 \quad (12)$$

$$B_2 = A_1 \sin(12\alpha_1) \quad (13)$$

From the condition of continuity in rotational angles $\varphi_1 = \varphi_2$, it results:

$$A_1\alpha_1 \cos(12\alpha_1) = A_2\alpha_2 \quad (14)$$

By imposing the condition $\varphi_2 = 0$ for $x_2 = 40$, one could obtain:

$$A_2\alpha_2 \cos(40\alpha_2) - B_2\alpha_2 \sin(40\alpha_2) + \frac{H}{F} = 0 \quad (15)$$

From equations (10), (11), (12) it results:

$$A_1 = \frac{1}{\alpha_2 \sin(12\alpha_1)}\left[\frac{H}{F}\sin(40\alpha_2) - 52\frac{H}{F}\cos(40\alpha_2)\right] \quad (16)$$

Introducing the value of A_1 obtained above in equation (11) it will result:

$$\frac{\alpha_1}{\alpha_2} ctg(12\alpha_1)\cdot\frac{H}{F}\cdot[\sin(40\alpha_2) - 52\cos(40\alpha_2)] = -\frac{H}{F}[52\alpha_2 \sin(40\alpha_2) + \cos(40\alpha_2)] \quad (17)$$

After simplification and taking into account that, $\alpha_1 = 10{,}7928\alpha_2$ the final form of the equation (14) will become:

$$10{,}7928ctg(129{,}51\alpha_2)\cdot[52\cos(40\alpha_2) - \sin(40\alpha_2)] - 52\alpha_2 \sin(40\alpha_2) - \cos(40\alpha_2) = 0 \quad (18)$$

This transcendental equation has several solutions, computed with Newton-Raphson method. The nearest value to 0 obtained was $\alpha_1 \cong 12{,}11\cdot 10^{-3}[rad]$.

For this value, the corresponding critical buckling force will be:

$$F_{cr} = EI_2\alpha_2^2 = 76531\,N \quad (19)$$

The necessary punching force is determined with the relation:

$$F_{nec} = k\cdot p\cdot t\cdot \tau_r = 1.25\cdot 16\cdot 1.5\cdot 0.8\cdot 340 = 8160\,N \quad (20)$$

where p = represents the perimeter of the punching surface, t – the material thickness, τ_r – the fracture shear stress ($\tau_r = 0{,}8\sigma_r$) and k – a coefficient depending on the anisotropy of physical and mechanical properties, thickness deviations of the band and the wear degree of the cutting edges. Since the material band is a low carbon steel the value of the ultimate tensile strength has been considered σ_r = 340 MPa. It is obvious that it will be a range of values for the punching force due to the modifications, which can occur in the degree of wear of the cutting edges and variation of the ultimate tensile stress.

Due to the fact that the obtained safety factor is: $c_f = \dfrac{F_{cr}}{F} = 9.37$, this value being higher than the allowable coefficient for such components, included in the range (4…8), it is obvious that the punching die will not buckle.

2. Numerical modeling of the buckling phenomenon

In order to check the reliability of the analytical solutions, the authors proposed a numerical model. The structure geometry was modeled using the CAD software CATIA, being then analyzed with the CAE ANSYS software program. The purpose of this modeling was to evaluate the stability behavior of the structure under static loading. In order to obtain the critical buckling load, the structure model was first subjected to compressive static load. The obtained results were transferred to the buckling analysis in order to determine the coefficient of stability. The loadings that act upon the structure as well as the restraints are presented in Figure 3.

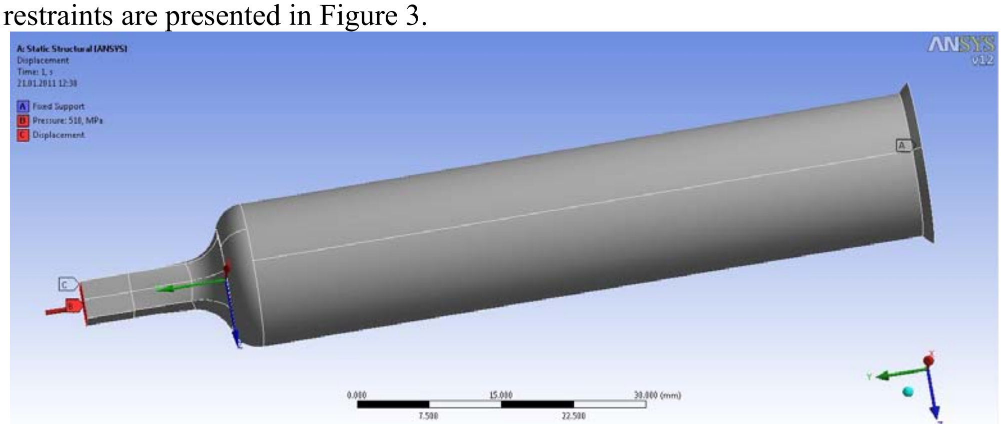

Fig. 3. Loads and restraints

SOLID186 is a higher order 3-D 20-node solid element that exhibits quadratic displacement behavior. The geometry of this element is presented in Figure 4.

Twenty nodes, having three degrees of freedom per node - translations in the nodal *x*, *y*, and *z* directions, define this element. The element supports plasticity, hyper-elasticity, creep, stress stiffening, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly incompressible elasto-plastic materials, and fully incompressible hyper-elastic materials.

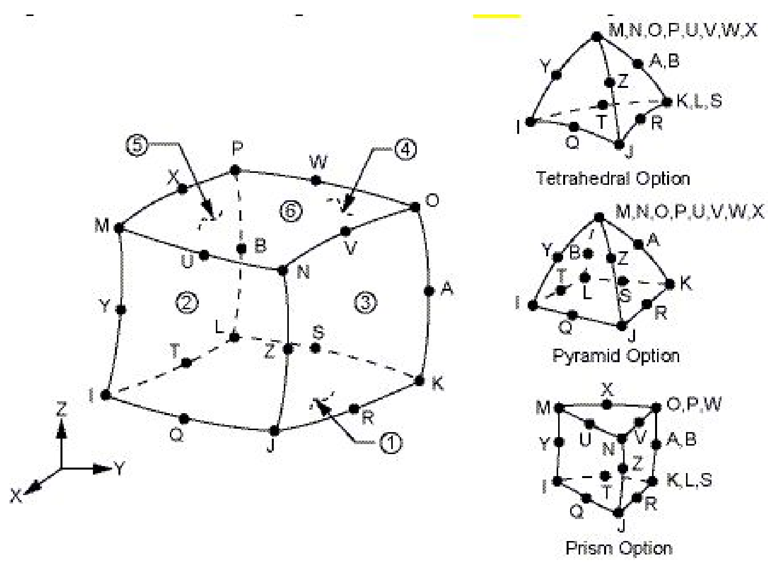

Fig. 4. Geometry of SOLID186 element

Fig. 5 presents the punching die meshing. The whole structure has 7475 nodes and 4650 elements.

Fig. 6 presents the buckling deformation form and the value of the numerically value of the load multiplier equal to 8.27, confirming the initial assumption.

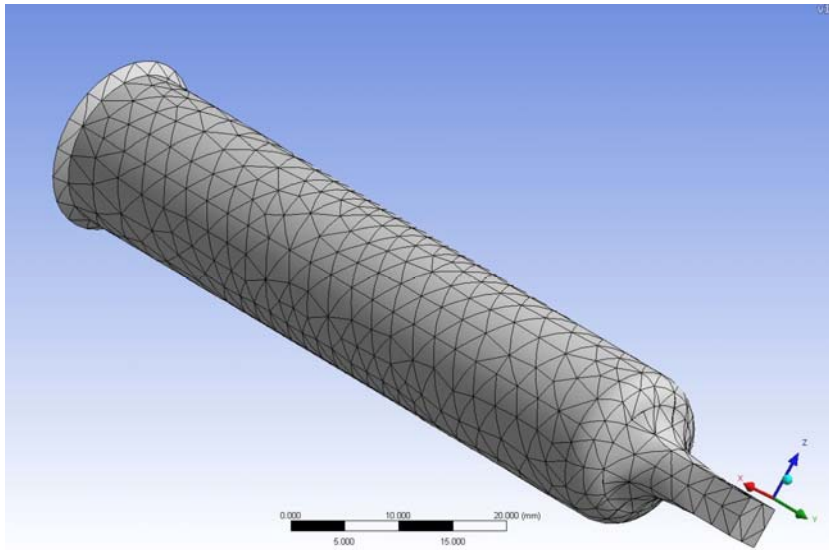

Fig. 5. Structural meshing

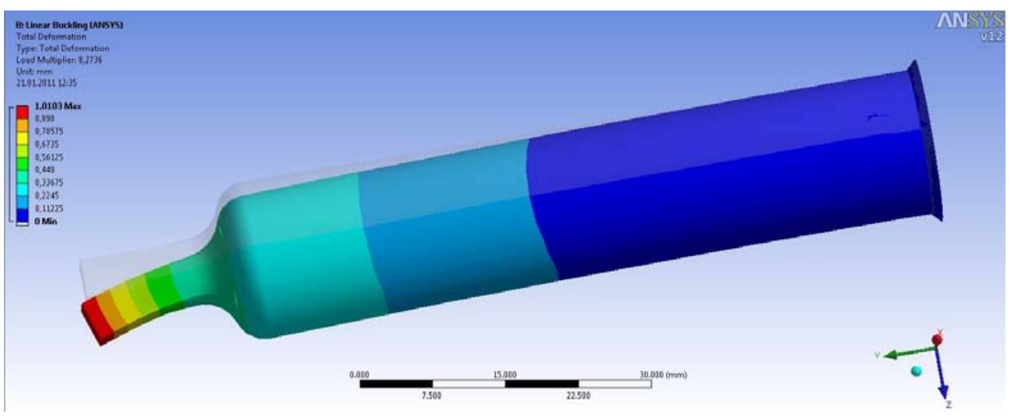

Fig. 6. Buckling deformation form

Since the numerical value obtained is quite close to the theoretical one (with an error ε = 11.73 %), one can conclude that the finite element analysis (modeling, meshing and restraint conditions) was appropriate. Figs. 7 and 8 present the equivalent plastic and elastic strains.

As one can see in Fig. 7 no plastic strain occurs during the structure buckling, confirming thus the analytical results, where it was assumed that the loss of stability occurs in elastic strain domain in limits of applicability of Hooke's law.

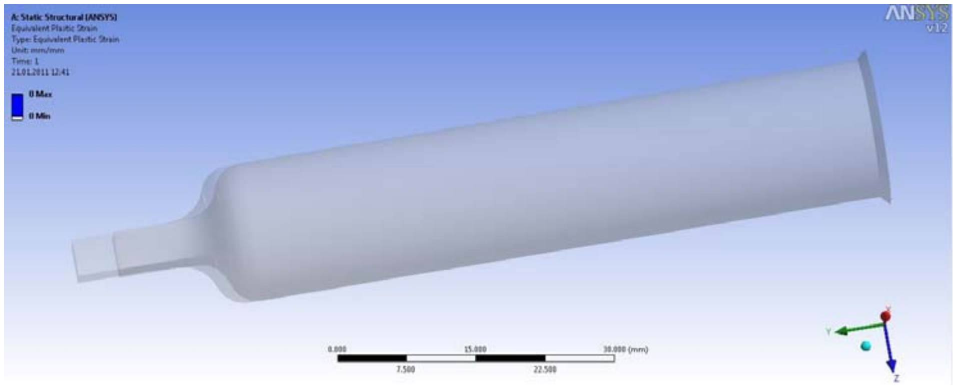

Fig.7. Equivalent plastic strain

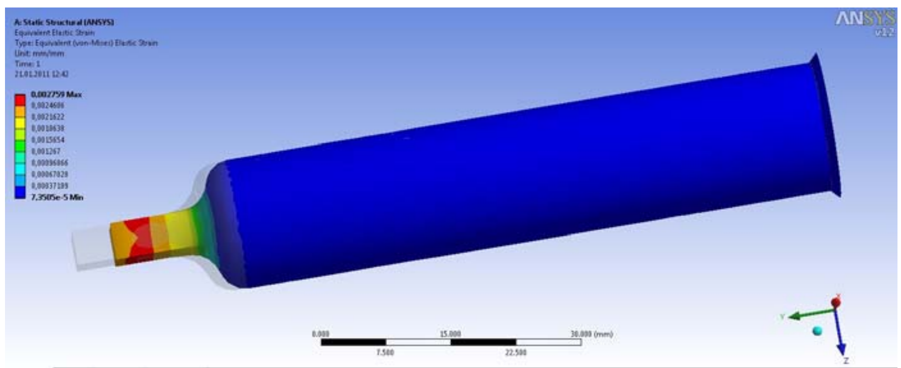

Fig .8. Equivalent elastic strain

3. Conclusions

In the present paper, the authors proposed two comparative methods – an analytical and numerical one – in order to check the stability phenomenon of a punching die. It is easy to prove that there is a real necessity to check punching dies with low slenderness because these active elements are predisposed to buckle under intense exploitation. On the other side, since stamped parts have strict designs, very often impossible to modify, the adaptation of the punching dies active ends is not an option whereas the rest of the punching body has the possibility to be adapted in order to respect safety and reliability standards.

From the example shown in this paper, it could be seen that both analytical and numerical results obtained, otherwise quite complex, are close. This confirms that for the analysis of complex structures (bars with more than two steps, bars with variable section, bars with intermediate supports etc.), where it is difficult to perform analytical calculations, the finite element modeling remains a quickly and useful method.

Although the material requirements are higher to manufacture stiff punching dies, the difference in cost is covered by the economical benefits of having reliable manufacturing processes.

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