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APPEARANCE OF ΔI=1 STAGGERING EFFECTS IN SIGNATURE PARTNERS OF ODD SUPERDEFORMED Tl AND Pb NUCLEI

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The transition energies of super deformed rotational bands in five pairs of signature partners of $^{191,193,195}Tl$ and $^{193,195}Pb$ odd-A nuclei are calculated using a proposed energy formula, depending on mixing of rotational and vibrational modes and perturbation term. The level spins of the super deformed band have been estimated with a fit of the Harris expansion to the measured dynamic moment of inertia values as a function of frequency. The model parameters are determined by using a simulated-fitting search program. An excellent agreement between the calculated transition energies and the observed ones supports the model well. Dynamic and kinematic moments of inertia have been calculated, and their dependence on rotational frequency is discussed. The $\Delta I = 1$ staggering splitting in the studied signature partner pairs has been examined through three staggering functions, depending on the dipole and quadrupole γ-ray transition, linking the two signature partner bands and quadrupole transition within each band. For these signature partner pairs, band head moments of inertia and the intrinsic structure of each pair have been found as almost identical and show a large amplitude staggering pattern.

Keywords: Energy Staggering; Signature Partners; Nuclear Superdeformed Bands

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1. Introduction

Superdeformation (SD) was first observed [1] in 152Dy (Z = 66, N = 86) nucleus whose strongly deformed prolate structure reflects the typical distribution of high j and low K of SD nuclei in A ≈ 150 mass region with quadrupole deformation β = 0.65, corresponding to elongated ellipsoid with an axis ratio close to 2:1:1.

Now more than 350 SD bands have been observed experimentally in several nuclear mass regions, ranging from A ≈ 40 to A ≈ 190 [2] . The first observation of superdeformation in A ≈ 190 was in ^{191}Hg[3] ; since then, more than 90 SD bands have been observed in 25 different nuclei identified in Au, Hg, Tl, Pd, Bi and Po nuclei. The A ≈ 190 mass region is a fascinating region because most A ≈ 190 SD bands in even-even and odd-A nuclei exhibit the same smooth increase in dynamic moment of inertia $J^{(2)}$ with an increasing rotational frequency ħω. The common cause of this smooth increase in $J^{(2)}$ is due to the gradual alignment of quasiparticles occupying high-N intruder orbital, originating from the $i_{13/2}$ proton or $j_{15/2}$ neutron subshells in the presence of pair correlations. Another interesting feature connected to the SD bands in A ≈ 190 regions is the measurement of the magnetic properties, such as magnetic dipole M1 decay. The branching ratios of M1 transition between signature partner SD bands and the cross-talk between them have been measured in a few A ≈ 190 high-K signature partner SD bands, such as ^{193}Hg [4] and ^{193}Pb [5] , and confirmed the quantum numbers of high-K neutron as well as intruder proton orbital near to the Fermi surface.

The detailed experimental investigation of SD bands reveals many fascinating phenomena, such as the identical bands [6-8] , the ΔI = 2 energy staggering [9-11] and the identical bands with ΔI = 2 staggering [12] . Different theoretical methods were applied for the study of these phenomena in SD bands, such as the cranked relativistic mean-field (CRMF) theory [13-15] , the cranked Nilsson Strutinsky approach based on Woods-Saxon potential [16, 17] , the self-consistent cranked Hartree-Fock-Bogaliubov approach based either on Skyrme[18, 19] or Gogny forces [20-22] , nonrelativistic Skyrme-Hartree-Fock formalism[23] and different phenomenological methods, such as Harris ω^2 expressions [24-27] , the Bohr-Mottelson formula [28-31], the a b and a b c formula [32] , the variable moment of inertia (VMI) model [33-36], the interacting boson model (IBM) [37-39] , the nuclear softness formula [24, 27, 40-42] and the particle rotor model (PRM) [43, 44] . A particularly striking feature in odd SD nuclei is the observation of ΔI = 1 staggering [45-48] .

This paper aims to investigate the ΔI = 1 energy staggering in signature partner of odd SD nuclei in ^{191,193,195}Tl and ^{193,195}Pb by using a model which contains pairs rotational, vibrational and perturbation terms.

2. Method of calculations

2.1 Formalism

I. Spin Assignment and Moments of Inertia

Spins of states in most superdeformed rotational bands (SDRB's) are not determined experimentally, this is due to the difficulty of establishing the deexcitation of SD band into known yrast states. Several methods were proposed to assign the spins [24, 27, 41] . The most familiar method is to use Harris formula [49] . A power series expansion in the square of rotational frequency was introduced by Harris as an extension of the cranking model to describe the excitation energies in the regions of strongly deformed nuclei. We restrict ourselves to two terms Harris formula

$$E(I) = \frac{1}{2}\theta_0\omega^2 + \frac{3}{4}\theta_1\omega^4 \tag{1}$$

with the cranking inertial parameters θ_0 and θ_1. The dynamical moment of inertia $J^{(2)}$ is defined as [50]

$$J^{(2)} = \frac{1}{\omega}\frac{dE}{d\omega} \qquad then \quad J^{(2)} = \theta_0 + 3\theta_1\omega^2 \tag{2}$$

The parameters θ_0 and θ_1 are obtained by fitting of $J^{(2)}$ versus ω^2.
Integrating equation (2) concerning ω leads to an expression for the intermediate nuclear spin I ($J^{(2)} = (\frac{dE}{d\hat{I}})^{-1}/\hbar$)

$$\hbar\hat{I} = \int J^{(2)}\, d\omega = \theta_0\omega + \theta_1\omega^3 + i_0 \tag{3}$$

The alignment i_o appears as a constant of integration in this approach and can be assumed to be zero since no alignment occurs at $\omega = 0$.
The corresponding expression for kinematic moment of inertia $J^{(1)}$ of the SD band can be derived as

$$\frac{J^{(1)}}{\hbar^2} = \frac{I}{\hbar\omega} = \theta_0 + \theta_1\omega^2 \tag{4}$$

The two moments of inertia are related as follows

$$J^{(2)} = J^{(1)} + \omega\frac{dJ^{(1)}}{d\omega} \tag{5}$$

In particular, for a rigid rotor, one has $J^{(1)} = J^{(2)}$.
Experimentally for the SD bands, γ-ray transition energy E_γ is the only spectroscopic quantity detected. Therefore, to compare the structure of SD bands, information about their E is commonly translated into values of rotational frequency ħω and dynamical moment of inertia $J^{(2)}$ as:

$$\hbar\omega(I) = \frac{1}{4}[E\gamma(I+2) + E\gamma(I)] \tag{6}$$

$$\frac{J^{(1)}}{\hbar^2} = \frac{4}{\Delta E\gamma} \tag{7}$$

Where $\Delta E\gamma$ is the spacing between consecutive SD-band transitions. In other words, $J^{(2)}$ can be extracted from the energy difference between consecutive

transitions in the band; $J^{(2)}$ does not depend on the knowledge of the spin I but only on the γ-ray transition energies.
The experimentally γ-ray transition energies themselves may extract kinematic moment of inertia J(1):

$$J^{(1)} = \frac{2I-1}{E_{\gamma2}(I)} \quad (8)$$

$$E_{\gamma2}(I) = E(I) - E(I-2) \quad (9)$$

II. Proposed Nuclear Model

The level energies of a well-deformed nucleus are given by the pure rotator formula E(I)=AI(I+1) with the inertial parameter $A = \frac{\hbar^2}{2J}$ where J represents the moment of inertia of the band. On the other band, the level energies of the collective vibrational states for harmonic vibrator are given by E(I) = BI with B = ħω where ħω represent the vibrational frequency $\hbar\omega = E(2_1^+)/2$. The complex nature of the collective spectra of deformed nuclei leads some authors[51-53] to mix the rotational and vibrational collective modes with a phenomenological formula for band energies including $\alpha I(I+1)$ and βI.
Our proposed model is based on the assumption that the energy E(I) of the superdeformed rotational bands as a function of the unknown spin I can be expressed in terms of the following rotational, vibrational and perturbation terms:

$$E(I) = E_{rot} + E_{vib} + E_{pert} = \alpha I(I+1) + \beta I + \gamma I^3 \quad (10)$$

The perturbation third term is introduced to improve the agreement between theory and experimental data. Such a term is based on the assumption that, on rotation, the moment of inertia of the nucleus increases as the quadratic function of the angular velocity of rotation.
For SD bands, gamma-ray transition energies $E_\gamma(I)$ are the only spectroscopic information universally available; thus, $E_\gamma(I)$ within the band has the following form:

$$E_{\gamma2}(I) = E(I) - E(I-2) = 2\{I[2\alpha + 3\gamma(I-2)] - (\alpha - \beta - 4\gamma)\} \quad (11)$$

Now the spins of the levels Io, Io+2, Io+4,…. are known by using the Harris approach. As a first-hand estimation for the model parameters α, β and γ, we use the experimental first three consecutive transition energies $E_\gamma(I_0)$, $E_\gamma(I_0+2)$ and $E_\gamma(I_0+4)$; therefore, by using the bandhead spin Io extracted from Harris approach, the trial values for α, β and γ are as follows:

$$\alpha = \frac{1}{16}[-(I_0+2)E\gamma(I_0+4) + (2I_0+6)E\gamma(I_0+2) - (I_0+4)E\gamma(I_0)] \quad (12)$$

$$\beta = \frac{1}{16}\{\left[I_0(I_0+1) - \frac{10}{3}\right]E\gamma(I_0+4) - \left[2I_0(I_0+3) - \frac{26}{3}\right]E\gamma(I_0+2) + [I_0(I_0+5) + \frac{8}{3}]E\gamma(I_0)\} \quad (13)$$

$$\gamma = \frac{1}{48}[E\gamma(I_0+4) - 2E\gamma(I_0+2) + E\gamma \quad (14)$$

2.2 Energy Staggering in SD Signature Partners

To explore more clearly the $\Delta I = 1$ energy staggering in SD signature partner pairs of odd-A nuclei, we consider three staggering functions: the first function depends on the dipole transition energies $E\gamma_1$ $(I \rightarrow I - 1)$, linking the two signature partners and the quadrupole transition energies $E\gamma_2$ $(I \rightarrow I - 2)$ within each band:

$$Y(I) = \frac{2I-1}{I}\frac{E(I)-E(I-1)}{E(I)-E(I-2)} - 1 \qquad (15)$$

The staggering function Y(I) vanishes for a strongly coupled rotational bands. If we plot Y(I) against I, such a plot exhibits significant deviation from the linear I(I + 1) dependence and effectively splits into two different curves, leading to the discovery of a staggering pattern. The second staggering function S(I) represents the difference between the average transition energies $E\gamma$ $(I+1 \rightarrow I - 1)$, $E\gamma$ $(I-1 \rightarrow I - 3)$ in one band and the transition $E\gamma$ $(I \rightarrow I - 2)$ in the signature partner

$$S(I) = \frac{1}{2}[E_{\gamma 2}\,(I-1) - 2E_{\gamma 2}\,(I) + E_{\gamma 2}\,(I+1)] \qquad (16)$$

The third staggering function EGOS(I) is defined as the transition energy of Gamma-ray Over Spin

$$EGOS(I) = \frac{E_{\gamma 1}(I)}{2I} \quad with \quad E_{\gamma 1}\,(I) = E(I) - E(I-1) \qquad (17)$$

3. Results and discussion

The experimental data are given in the form of a series of interband gamma-ray transition energies. The data set include five signature partner pairs in Tl and Pb nuclei namely: ^{191}Tl(SD1,SD2), ^{193}Tl(SD1,SD2), ^{195}Tl(SD1,SD2), ^{193}Pb(SD5,SD6) and ^{195}Pb(SD3,SD4). The dynamical moment of inertia is independent of the unknown spins of the levels. Information about γ-ray transition energies E_γ in SD bands are translated into values of the dynamical moments of inertia $J^{(2)}$ equation (2). To assign the bandhead spin for each band, the two parameters θ_0 and θ_1 of the $J^{(2)}_{cal}$ values in a Harris approach have been calculated by fitting $J^{(2)}_{cal}$ to the experimental ones $J^{(2)}_{exp}$ extracted from the experimental energies. The quality of the fit is indicated by the common quantity

$$x\left(J^{(2)}\right) = \left\{\frac{1}{N_J}\sum_i\left(\frac{J^{(2)}_{exp}\,(i) - J^{(2)}_{cal}}{\delta J^{(2)}_{exp}\,(\mathrm{i})}\right)^2\right\}^{1/2} \qquad (18)$$

where N_J is the total number of data points entering into the fitting procedure and $\delta J^{(2)}_{exp}$ (i) are the experimental errors in $J^{(2)}_{exp}$. The adapted parameters θ_0 and θ_1 have been used to determine the spin with the help of equation (18). The resulting best parameters θ_0 and θ_1 and the values of the band head spin I_o are listed in the Table(1). The SD bands are identified by the lowest observed. The presently assigned spins for our selected estimated values in previous works [31, 41, 48]. After determining the level spins of our selected SD bands, the model parameters α, β and γ can be adjusted to obtain a minimum root mean square derivation of the calculated transition energies $E_\gamma^{cal}(I_i)$ from the measured transition energies $E_\gamma^{exp}(I_i)$:

$$x(E_\gamma) = \left\{\frac{1}{N_J}\sum_i\left(\frac{E_\gamma^{exp}(I_i)-E_\gamma^{cal}(I_i)}{\delta E_\gamma^{exp}(I_i)}\right)^2\right\}^{1/2} \qquad (19)$$

N is the number of data points, and $\delta E_\gamma^{exp}(I_i)$ is the experimental error in the γ-ray energies. The experimental data for the transition energies $E_\gamma^{exp}(I_i)$ are taken from[2] . The resulting best model parameters α, β and γ are listed in Table 1.
Fig.1 shows the dynamic $J^{(2)}$ and kinematic $J^{(1)}$ moments of inertia for our various studied superdeformed rotational bands compared with the experimental ones. It can be seen that the $J^{(2)}$ values exhibit a substantial increase as a function of rotational frequency ħω. Fig. 2 compares the calculated E_γ with the experimental data where perfect agreement has been obtained. For the investigation of the ΔI = 1 energy staggering effect between signature partner pairs, the staggering function Y(I) has been calculated for each signature partner pairs in terms of dipole transitions $E_{\gamma1}$ (I → I - 1), linking the partner pairs and the quadrupole transitions $E_{\gamma2}$ (I → I - 2) within each band. The calculated values are listed in Table 2 and plotted against spin I in Fig. 3. We notice that all signature partner pairs exhibit significant amplitude staggering. Also, the results for the other two ΔI = 1 staggering function S(I) and EGOS(I) are listed in Tables 3 and 4 and Fig. 3. It is interesting to note that the band head moments of inertia of each signature partner's SD band are almost identical.

Table 1

The calculated optimized best model parameters α, β and γ adopted from the fitting procedure and the suggested band head spin proposition Io for our selected SD signature partners; the experimental lowest transition energy Eγ (Io + 2 → Io) for each SD band is given [2]. The last column gives the relative root mean square deviation χ (Eγ).

SD band	θ_0 ħ^2 MeV^{-1}	θ_1 ħ^4 MeV^{-3}	α (KeV/ ħ^2)	β (10^{-3}KeV/ ħ)	γ (10^{-3}KeV)	I_0 (ħ)	$E_\gamma(I_0+2\rightarrow I_0)$	χ (KeV)
^{191}Tl (SD1)	92.756	59.949	5.53695	10.3880	-10.3880	13.5	276.5	0.2064
(SD2)	92.818	66.917	5.916961	16.91916	-16.91916	14.5	296.3	0. 0939
^{193}Tl (SD1)	95.681	73.603	5.344554	9.64450	-9.64450	10.5	247.3	0.1787
(SD2)	95.696	66.593	5.323105	8.19733	-8.19733	9.5	227.3	0.1228
^{195}Tl (SD1)	95.127	60.699	5.368280	8.80633	-8.80633	5.5	146.2	0.2684
(SD2)	94.830	76.072	5.398193	10.66366	-10.66366	6.5	167.5	0.3328
^{193}Pb (SD5)	92.655	94.253	5.460628	9.91466	-9.91466	8.5	213.2	0.0800
(SD6)	92.363	104.183	5.482892	10.70816	-10.70816	9.5	234.6	0.1716
^{195}Pb (SD3)	90.818	104.243	5.521662	11.20283	-11.20283	7.5	198.2	0.2930
(SD4)	91.881	102.048	5.526286	11.18283	-11.18283	8.5	213.6	0.2884

Table2

The calculated ΔI = 1 staggering function Y(I) for the studied five signature partner pairs; the calculated excitation energies are also given.

^{191}Tl(SD1,SD2)			^{193}Tl(SD1,SD2)			^{195}Tl(SD1,SD2)		
I(ħ)	E(I)(KeV)	Y(I) (KeV)	I(ħ)	E(I)(KeV)	Y(I) (KeV)	I(ħ)	E(I)(KeV)	Y(I) (KeV)
11.5	782.829		8.5	427.161		5.5	191.032	
12.5	835.882		9.5	525.025		6.5	261.540	
13.5	1061.516	0.55929	10.5	636.227	0.01314	7.5	339.456	-0.02008
14.5	1130.608	-0. 5473	11.5	754.063	-0.01577	8.5	431.364	0.01876
15.5	1380.959	0.51686	12.5	885.568	0.01262	9.5	529.260	-0.02293
16.5	1467.096	-0. 50353	13.5	1023.326	-0.01467	10.5	642.227	0.02045
17.5	1740.557	0.47747	14.5	1174.645	0.01081	11.5	759.968	-0.02369
18.5	1844.341	-0. 46465	15.5	1332.354	-0.01224	12.5	893.553	0.02052
19.5	2139.696	0.44201	16.5	1502.907	0.00763	13.5	1031.097	-0.02297
20.5	2261.328	-0. 43084	17.5	1680.675	-0.00844	14.5	1184.743	0.01890
21.5	2577.746	0.41106	18.5	1869.793	0.00307	15.5	1342.151	-0.02055
22.5	2717.019	-0. 40232	19.5	2067.805	-0.00325	16.5	1515.187	0.01555
23.5	3054.063	0.38510	20.5	2274.723	-0.00293	17.5	1692.622	-0.01637
24.5	3210.355	-0. 379312	21.5	2493.246	0.00338	18.5	1884.263	0.01042
25.5	3567.980	0.36446	22.5	2717.108	-0.01042	19.5	2081.990	-0.01041
26.5	3740.251	-0. 36206	23.5	2956.488	0.01150	20.5	2291.332	0.00344
27.5	4118.815	0.34951	24.5	3196.339	-0.01944	21.5	2509.723	-0.00258
28.5	4305.599	-0. 35081	25.5	3457.006	0.02116	22.5	2735.740	-0.00544
29.5	4705.862	0.34053	26.5	3711.789	-0.03006	23.5	2975.271	0.00713
30.5	4905.260	-0. 34586	27.5	3994.255	0.03240	24.5	3216.813	-0.01631
31.5	5328.390	0.33781	28.5	4262.813	-0.04233	25.5	3478.070	0.01883
32.5	5538.063	-0. 34751	29.5	4567.676	0.04529	26.5	3733.862	-0.02923
33.5	5985.643	0.34164	30.5	4848.743	-0.05634	27.5	4017.538	0.03256
34.5	6202.818	-0.35607	31.5	5176.687	0.05987	28.5	4286.173	-0.04430
35.5	6676.834	0.35227	32.5	5468.888	-0.07213	29.5	4593.075	0.05072
36.5	6898.251	-0. 37194	33.5	5820.684	0.07623	30.5	4873.013	-0.06385
37.5	7401.146	0.37009	34.5	6122.532	-0.08979	31.5	5204.056	0.06644
			35.5	6499.040	0.09442	32.5	5493.620	-0.08119
			36.5	6808.927	-0. 10942	33.5	5849.836	0.08674
			37.5	7211.119	0.11456	34.5	6147.206	-0.10322
			38.5	7527.285	-0. 13118	35.5	6529.744	0.10941
			39.5	7956.197	0.13674	36.5	6832.951	-0.12779

Table2

(continued)

^{193}Pb(SD5,SD6)			^{195}Pb (SD3,SD4)		
I(ħ)	E(I)(KeV)	Y(I)) KeV)	I(ħ)	E(I)(KeV)	Y(I) (KeV)
8.5	436.429		7.5	349.111	
9.5	540.068		8.5	441.593	
10.5	650.015	-0.01949	9.5	343.819	-0.00522
11.5	774.916	0.01742	10.5	657.381	0. 00241
12.5	904.735	-0.02146	11.5	780.155	-0.00619
13.5	1050.591	0.01898	12.5	914.530	0. 00331
14.5	1200.036	-0.02274	13.5	1087.498	-0.00720
15.5	1366.488	0.01984	14.5	1212.412	0.00423
16.5	1435.353	-0.02332	15.5	1375.213	-0.00823
17.5	1721.987	0.01998	16.5	1550.387	0.00519
18.5	1910.105	-0.02317	17.5	1732.652	-0.00929
19.5	2116.456	0.01939	18.5	1927.802	0.00618
20.5	2323.702	-0.02227	19.5	2129.152	-0.01040
21.5	2549.245	0.01803	20.5	2343.987	0.00722
22.5	2775.533	-0.02061	21.5	2564.033	-0.01155
23.5	3019.688	0.01589	22.5	2798.254	0.00828
24.5	3264.974	-0.01814	23.5	3036.599	-0.01273
25.5	3527.103	0.01293	24.5	3289.900	0.00939
26.5	3791.380	-0.01486	25.5	3546.135	-0.01396
27.5	4070.782	0.00913	26.5	3818.199	0.01052
28.5	4354.092	-0.01072	27.5	4091.903	-0.01523
29.5	4650.001	0.00443	28.5	4382.407	0.01171
30.5	4952.417	-0.00569	29.5	4673.146	-0.01655
31.5	5263.996	-0.00118	30.5	4981.751	0.01292
32.5	5585.647	-0.00273	31.5	5289.077	-0.01791
33.5	5882.011	-0.00776	32.5	5715.437	0.01419
34.5	6253.05	0.00717	33.5	5938.886	-0.01933
35.5	6563.229	-0.01532			
36.5	6953.854	0.01500			

Table 3

Similar to Table 2 except for the staggering function S(I)

^{191}Tl(SD1,SD2)		^{193}Tl(SD1,SD2)		^{195}Tl(SD1,SD2)		^{193}Pb (SD5,SD6)		^{195}Pb (SD3,SD4)	
I(ħ)	S (I) (KeV)	I(ħ)	S (I) (KeV)	I(ħ)	S (I) (KeV)	I(ħ)	S (I) (KeV)	I(ħ)	S (I) (KeV)
14.5	4.339	13.5	0.1655	8.5	-0.7025	11.5	-0.695	10.5	-0.266
15.5	-3.836	14.5	-0.1905	9.5	0.5365	12.5	0.5415	11.5	0.132
16.5	3.0325	15.5	-0.054	10.5	-0.607	13.5	-0.6645	12.5	-0.3085
17.5	-2.7315	16.5	0.0685	11.5	0.385	14.5	0.485	13.5	0.172
18.5	2.1235	17.5	-0.3585	12.5	-0.406	15.5	-0.5885	14.5	-0.353
19.5	-2.023	18.5	0.4125	13.5	0.1285	16.5	0.382	15.5	0.214
20.5	1.608	19.5	-0.7475	14.5	-0.0985	17.5	-0.4645	16.5	-0.399
21.5	-1.712	20.5	0.84	15.5	-0.2365	18.5	0.231	17.5	0.2565
22.5	1.4925	21.5	-1.221	16.5	0.3175	19.5	-0.2945	18.5	-0.4455
23.5	-1.803	22.5	1.354	17.5	-0.7105	20.5	0.033	19.5	0.2995
24.5	1.782	23.5	-1.783	18.5	0.844	21.5	-0.0755	20.5	-0.4936
25.5	-2.3025	24.5	1.4565	19.5	-1.2965	22.5	-0.2145	21.5	0.3445
26.5	2.4805	25.5	-1.434	20.5	1.4815	23.5	0.192	22.5	-0.5435
27.5	-3.213	26.5	2.149	21.5	-1.994	24.5	-0.512	23.5	0.3905
28.5	3.593	27.5	-3.1775	22.5	2.233	25.5	0.51	24.5	-0.595
29.5	-4.5425	28.5	3.434	23.5	-2.8075	26.5	-0.86	25.5	0.437
30.5	5.1265	29.5	-4.0135	24.5	3.0995	27.5	0.8795	26.5	-0.648
31.5	-6.296	30.5	4.3125	25.5	-3.738	28.5	-1.2625	27.5	0.4855
32.5	7.0875	31.5	-4.9445	26.5	4.0855	29.5	1.2995	28.5	-0.702
33.5	-8.474	32.5	5.2855	27.5	-4.788	30.5	-1.718	29.5	0.534
34.5	9.467	33.5	-5.9725	28.5	5.19	31.5	1.7835	30.5	-0.7585
35.5	-11.097	34.5	6.3585	29.5	-5.96	32.5	-2.227	31.5	0.5845
36.5	12.3185	35.5	-7.1025	30.5	6.4185	33.5	2.3015	32.5	-0.816
37.5	-14.183	36.5	7.5325	31.5	-7.2575	34.5	-2.7845		
		37.5	-8.336	32.5	7.7735	35.5	2.8845		
		38.5	8.8215	33.5	-8.6035	36.5	-3.4055		
		39.5	-9.702	34.5	9.258				
		40.5	10.2205	35.5	-10.242				

Similar to Table 2 except for the staggering function EGOS(I)

^{191}Tl(SD1,SD2)		^{193}Tl(SD1,SD2)		^{195}Tl(SD1,SD2)		^{193}Pb (SD5,SD6)		^{195} Pb (SD3,SD4)	
I (ħ)	E_{γ}/I (EGOS)	I(ħ)	EGOS(I)	I(ħ)	EGOS(I)	I(ħ)	EGOS(I)	I(ħ)	EGOS(I)
12.5	4.24424	9.5	10.30147	6.5	10.84738	9.5	10.90936	8.5	10.88023
13.5	16.71362	10.5	10.59828	7.5	10.3888	10.5	10.47114	9.5	10.76063
14.5	4.76496	11.5	10.24660	8.5	10.81270	11.5	10.86095	10.5	10.81542
15.5	16.15167	12.5	10.5204	9.5	10.30484	12.5	10.38552	11.5	10.6760
16.5	5.22042	13.5	10.20429	10.5	10.75876	13.5	10.80414	12.5	10.7500
17.5	15.62634	14.5	10.43575	11.5	10.23834	14.5	10.30655	13.5	10.59022
18.5	5.60994	15.5	10.17477	12.5	10.6868	15.5	10.73883	14.5	10.68372
19.5	15.14641	16.5	10.33654	13.5	10.18844	16.5	10.23424	15.5	10.50329
20.5	5.93326	17.5	10.15817	14.5	10.59627	17.5	10.6648	16.5	10.61660
21.5	14.71711	18.5	10.22259	15.5	10.15535	18.5	10.16854	17.5	10.41514
22.5	6.18991	19.5	10.15446	16.5	10.48703	19.5	10.58210	18.5	10.54859
23.5	14.34229	20.5	10.09356	17.5	10.13914	20.5	10.10956	19.5	10.32564
24.5	6.37926	21.5	10.16386	18.5	10.35897	21.5	10.49037	20.5	10.47975
25.5	14.02450	22.5	9.94942	19.5	10.13984	22.5	10.05724	21.5	10.23469
26.5	6.50079	23.5	10.18638	20.5	10.21180	23.5	10.38957	22.5	10.40982
27.5	13.76596	24.5	9.78983	21.5	10.15772	24.5	10.01167	23.5	10.14234
28.5	6.55382	25.5	10.22223	22.5	10.0452	25.5	10.27956	24.5	10.33881
29.5	13.56823	26.5	9.61445	23.5	10.19280	26.5	9.99271	25.5	10.04843
30.5	6.53763	27.5	10.27149	24.5	9.85885	27.5	10.16007	26.5	10.26656
31.5	13.43269	28.5	9. 42308	25.5	10.24537	28.5	9.94070	27.5	9.95287
32.5	6.45157	29.5	10.33433	26.5	9.65252	29.5	10.03081	28.5	10.19312
33.5	13.36059	30.5	9.21531	27.5	10.31549	30.5	9.91527	29.5	9.85555
34.5	6.29492	31.5	10.41092	28.5	9.42578	31.5	9.89139	30.5	10.11819
35.5	13.35256	32.5	8.9908	29.5	10.42633	32.5	9.89695	31.5	9.75638
33.5	13.36059	33.5	10.50137	30.5	9.15616	33.5	9.74220	32.5	10.04184
34.5	6.29492	34.5	8.74921	31.5	10.50930	34.5	9.88518	33.5	9.65519
35.5	13.35256	35.5	10.60585	32.5	8.90966	35.5	9.58250		
36.5	6.06621	36.5	8.49005	33.5	10.63313	36.5	9.88013		
37.5	13.41053	37.5	10.72512	34.5	8.61942	37.5	9.41210		
38.5	5.76441	38.5	8.21210	35.5	10.77571				
		39.5	10.85853	36.5	8.30704				
		40.5	7.91646	37.5	10.93674				

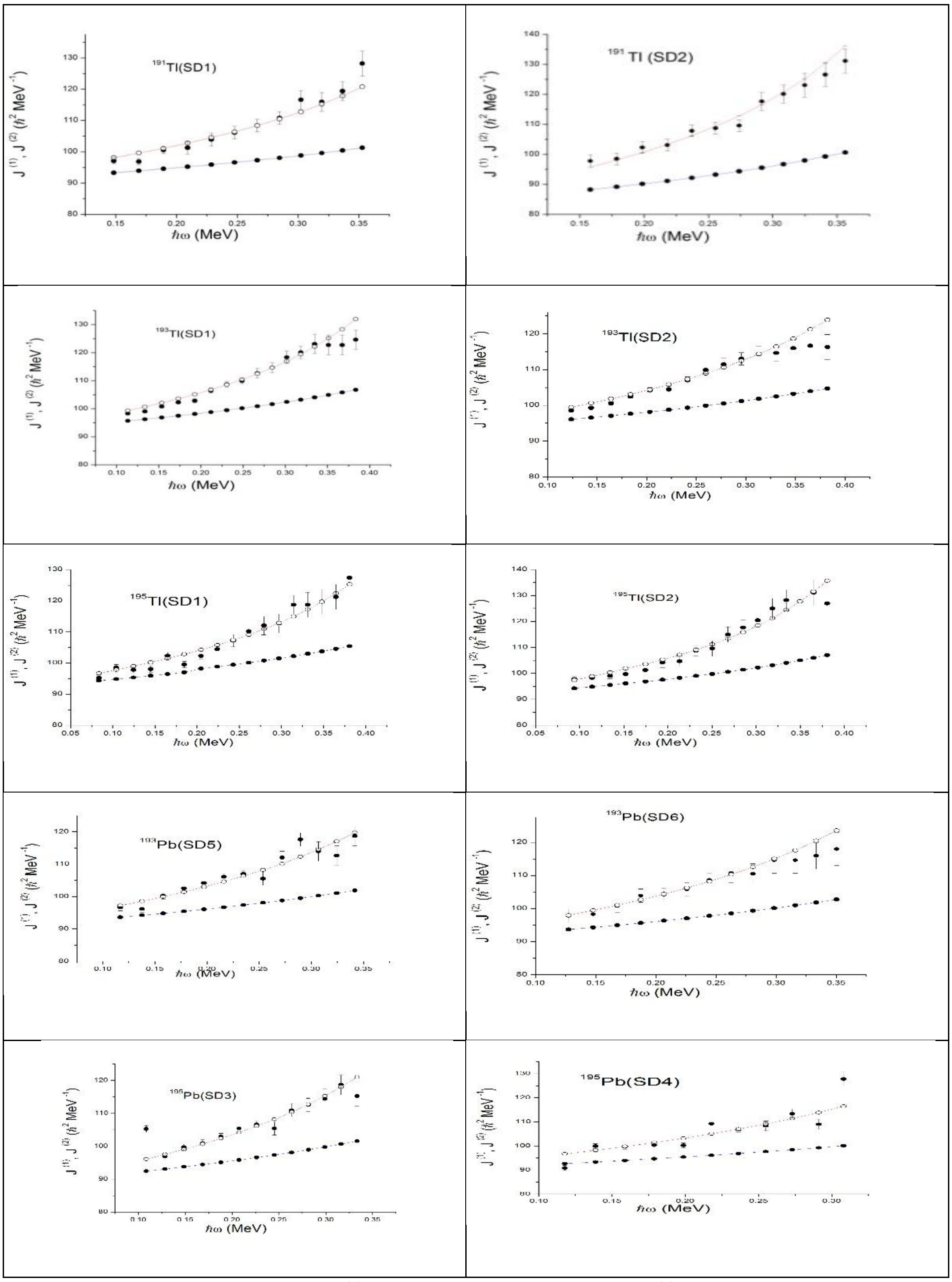

Fig. 1. The calculated kinematic $J^{(1)}$ (closed circles) and dynamic $J^{(2)}$ (open circles) moments of inertia as a function of rotational frequency ħω for all the ten studied SD bands. The experimental $J^{(2)}$ are labeled by a closed circle with error bars.

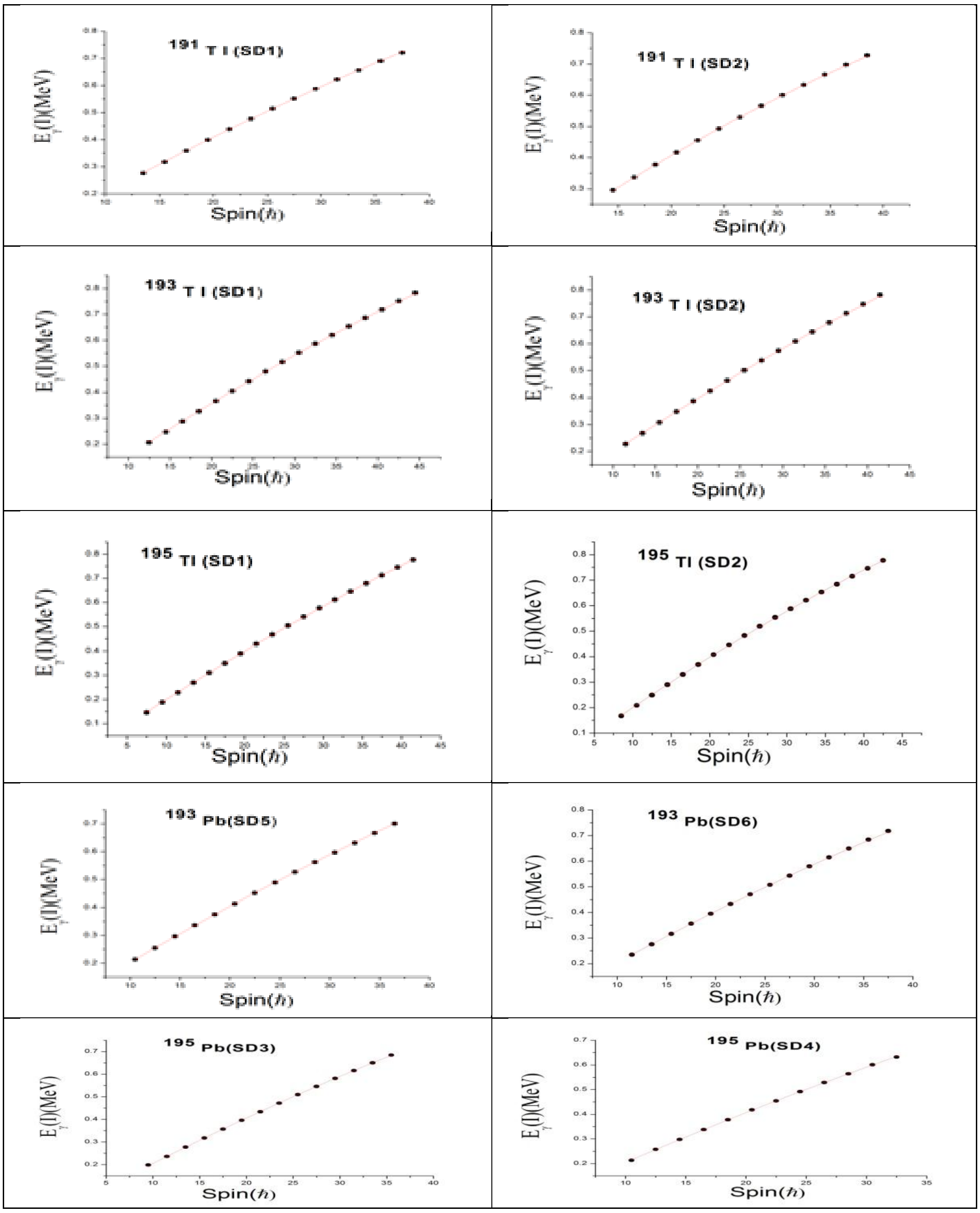

Fig. 2.The calculated gamma-ray transition energies Eɤ(I) pairs of signature partners against spin I for the five SD bands in ^{191}Tl(SD1, SD2), ^{193}Tl(SD1, SD2), ^{195}Tl(SD1, SD2), ^{193}Pb(SD5, SD6) and ^{195}Pb(SD3, SD4) are compared to the experimental values [2]. The solid curves indicate the theoretical values while closed circles indicate the experimental values.

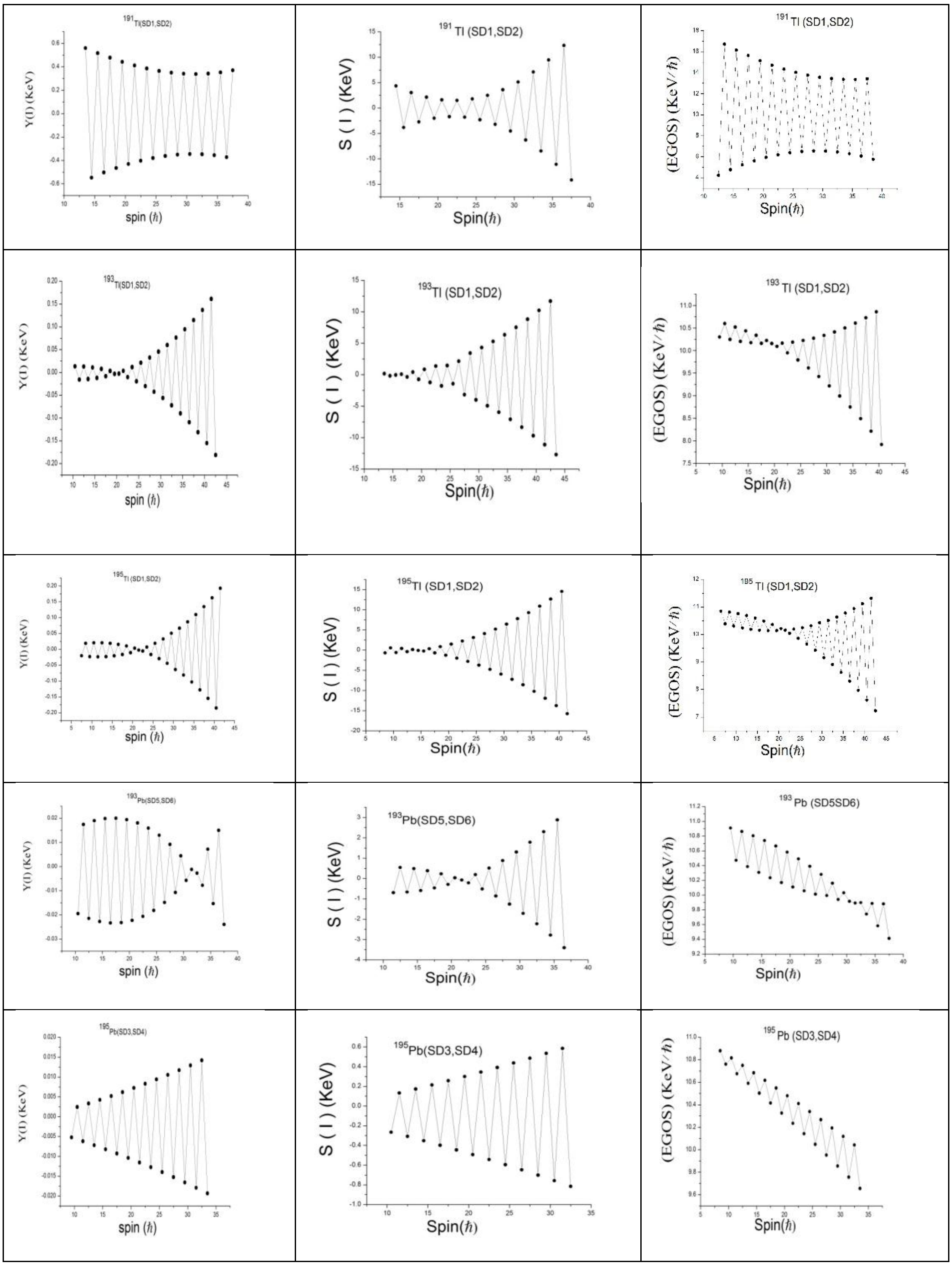

Fig. 3. The calculated staggering function Y(I) , S(I) and EGOS(I)) versus nuclear spin I for the studied signature partner SD bands observed in Tl and Pb nuclei.

4. Conclusion

The band head spins were suggested using the Harris expansion. The γ-ray transition energies within an SD band were calculated using a proposed energy formula, depending on the influence of the rotational, vibrational modes and perturbation terms. The calculation results agree with the experimental data very well. The evolution of moments of inertia with rotational frequency was examined. Three different proposal functions Y(I), S(I) and EGOS were tested to exhibit the ΔI = 1 energy staggering in some chosen signature partners of odd mass Tl, Pb SD nuclei in A ≈ 190 mass regions.

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