OPTIMAL CONTROL ON A MATHEMATICAL MODEL OF MALARIA

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In this paper, a malaria mathematical model is formulated by incorporating four control strategies: insecticide-treated bednets control, infected humans treatment control, sterile mosquitoes technique control and use of control on pregnant women and newborn births. It also explains the various stages of the disease jointly in humans and mosquitoes as well as the treatment of both asymptomatic and infectious humans. Preventive measures are developed to control the spread of disease. Forward-backward fourth-order Runge-Kutta method (Sweep method) is used to see the spread of disease and how to eradicate the disease. This is based on the fact that these measures are deployed adequately using control tools and without control tools respectively. On the other hand, their achievement depends on the appropriate and planned organization and dissemination.

Keywords: Transmission; Sterile Mosquitoes; Insecticides; Asymptomatic; Preventive Measures; Sweep Method.

1. Introduction

Malaria is an ancient disease and according to the record, malaria occurred from sixth century BC in Hindu. From 1570 BC in Egyptian Papyri, slabs of clay from 2000 BC in Mesopotamia and from about 2700 BC in a Chinese document [1]. Malaria can be seen largely in hot and sultry (tropical) regions such as the Pacific Islands, Indian subcontinent, South America, Central America, Sub-Saharan African and Southeast Asia [2]. Malaria continues to be a public health problem and a life-threatening disease transmitted by female anopheles mosquitoes, according to the World Health Organization report [3]. Globally, 212 million new cases and 429000 deaths were recorded. In Africa, millions of people still lack access to

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preventive tools such as insecticides, insecticide-treated bednets, proper treatment using effective drugs and others. The very essential result of the transmission dynamics of the Malaria model and optimal control have come out in the last decades. For instance; Khamis *et al.* [4]; Olaniyi et al. [5]; Bakare and Abolarin [6]; Joshi *et al.* [7]; Munzir et al. [8]; Otieno *et al.* [9]; Panja and Mondal [10]; Romero-Leiton *et al*. [11].

All of the above studies reveal an important result for malaria model dynamics by considering the different situations. But we have identified that till now no study has been done on malaria model which include nonlinear forces of infections with the application of optimal control which are; insecticide-treated bednets control, infected human's treatment control, sterile mosquitoes technique control and use of control on pregnant women and newborn births as control strategies. Because of the above, we developed a deterministic mathematical model of malaria by extending the model developed by Osman *et al*. [12] incorporating the above controls. We also include nonlinear forces of infection, the disease-induced death rate on the exposed compartment for the human population and relapse.

2. Materials and Methods

The model under consideration comprises of four stages for the human (host) population and three stages for the mosquito (vector) population. These are Susceptible humans (S_h), Exposed humans (E_h), Infectious humans (I_h), Recovered humans (R_h) and Susceptible mosquitoes (S_m), Exposed mosquitoes (E_m), Infectious mosquitoes (I_m) respectively. This shows the movement of human and mosquito from one stage to another at different rates. $N_h\Lambda_h$ is the rate at which humans enters into the susceptible population (recruitment rate), β_h is the force of infection in humans, υ is the developing rate of exposed humans (the rate at which humans move from exposed to infectious class), ω is the recovery rate of human from the disease, γ_2 is the relapse rate of humans that is, the rate at which humans with low immunity return from recovered class back to infectious class, ψ is the rate of newborn birth with infection of humans, μ is the natural death rate of humans, δ_h is the disease-induced death rate of humans, γ_1 is the rate of loss of immunity in humans, $N_m\Lambda_m$ is the recruitment rate of mosquitoes, β_m is the force of infection in mosquitoes, α is the developing rate of exposed mosquitoes that is, the rate at which mosquitoes move from exposed class to infectious class, η is the natural death rate of mosquitoes, δ_m is the disease-induced death rate of mosquitoes, σ_m is the interaction rate between human and mosquitoes and σ_h is the interaction rate between human and mosquitoes. The

total population of human is one (1) that is, $N_h = S_h + E_h + I_h + R_h = 1$ and the total population of mosquitoes is also one (1) that is, $N_m = S_m + E_m + I_m + R_m = 1$.
The reduction in the reproduction rate of a mosquito through insecticide-treated bednets and reduction rate in fecundity due to mating with released sterile male's mosquito population are by factors $(1-u_1)$ and $(1-u_3)$ respectively. There is an increase in the death rate of a mosquito population at a proportional rate u_1 and u_2. Also, the reduction of the mortality rate of pregnant women and newborn births is by a factor of $(1-u_4)$, a is the constant rate due to treatment and b is the constant rate due to the use of insecticide-treated bednets. The description of the parameters and values are given in table 1 below.

Table 1

Parameters and values of the model

Parameter	Value	Source	Parameter	Value	Source
Λ_h	1.2	Osman *et al.* [12]	Λ_m	0.7	Osman *et al.* [12]
υ	0.05	Osman *et al.* [12]	η	0.00083	Assumed
ω	0.0035	Osman *et al.* [12]	α	0.083	Osman *et al.* [12]
γ_1	0.00017	Osman *et al.* [12]	ξ	0.12	Olaniyi and Obabiyi [13]
γ_2	0.04	Mwamtobe *et al.* [18]	σ_h	0.1	Olaniyi and Obabiyi [13]
μ	0.01146	Osman *et al.* [12]	σ_m	0.09	Olaniyi and Obabiyi [13]
δ_h	0.068	Osman *et al.* [12]	ψ	0.003	Osman *et al.* [12]
δ_m	0.001	Assumed	ε_h	1.0	Assumed
			ε_m	1.0	Assumed

2.1. Differential Equations of the Model

The following normalized differential equations govern the model.

$$\frac{dS_h(t)}{dt} = \Lambda_h - \left[(1-u_1)+(1-u_4)\right]\frac{\xi\sigma_h I_m(t) S_h(t)}{1+\varepsilon_h I_m(t)} - \mu S_h(t) + \gamma_1 R_h(t)$$

$$\frac{dE_h(t)}{dt} = \left[(1-u_1)+(1-u_4)\right]\frac{\xi\sigma_h I_m(t) S_h(t)}{1+\varepsilon_h I_m(t)} - (\upsilon+\mu+\delta_h) E_h(t)$$

$$\frac{dI_h(t)}{dt} = \upsilon E_h(t) - (\omega + au_2 + \mu + \delta_h) I_h(t) + \psi I_h(t) + \gamma_2 R_h(t)$$

$$\frac{dR_h(t)}{dt} = (\omega + au_2) I_h(t) - (\mu + \gamma_1 + \gamma_2) R_h(t) \qquad (1)$$

$$\frac{dS_m(t)}{dt} = \Lambda_m - \left[(1-u_1)+(1-u_3)\right]\frac{\xi\sigma_m I_h(t) S_m(t)}{1+\varepsilon_m I_h(t)} - (bu_1+\eta) S_m(t)$$

$$\frac{dE_m(t)}{dt} = \left[(1-u_1)+(1-u_3)\right]\frac{\xi\sigma_m I_h(t) S_m(t)}{1+\varepsilon_m I_h(t)} - (bu_1+\alpha+\eta)E_m(t)$$
$$\frac{dI_m(t)}{dt} = \alpha E_m(t) - (bu_1+\eta+\delta_m)I_m(t)$$

Where $\{u_1, u_2, u_3, u_4\} \in [0, 1]$ that is, when $u_1 = u_2 = u_3 = u_4 = 0$, it means none of the controls are effective but when $u_1 = u_2 = u_3 = u_4 = 1$, it means all the controls are effective.

2.2. Autonomous Equations of the Model

In the absence of the four-time dependent control functions from the non-autonomous model (1) and by setting the control variables to zero that is, $u_1(t) = u_2(t) = u_3(t) = u_4(t) = 0$. Then, model (1) is given as

$$\begin{aligned}
\frac{dS_h(t)}{dt} &= \Lambda_h - \frac{\xi\sigma_h I_m(t) S_h(t)}{1+\varepsilon_h I_m} - \mu S_h(t) + \gamma_1 R_h(t) \\
\frac{dE_h(t)}{dt} &= \frac{\xi\sigma_h I_m(t) S_h(t)}{1+\varepsilon_h I_m} - (\mu+\delta_h+\upsilon)E_h(t) \\
\frac{dI_h(t)}{dt} &= \upsilon E_h(t) - (\mu+\delta_h+\omega)I_h(t) + \psi I_h(t) + \gamma_2 R_h(t) \\
\frac{dR_h(t)}{dt} &= \omega I_h(t) - (\mu+\gamma_1+\gamma_2)R_h(t) \\
\frac{dS_m(t)}{dt} &= \Lambda_m - \frac{\xi\sigma_m I_h(t) S_m(t)}{1+\varepsilon_m I_h} - \eta S_m(t) \\
\frac{dE_m(t)}{dt} &= \frac{\xi\sigma_m I_h(t) S_m(t)}{1+\varepsilon_m I_h} - (\alpha+\eta)E_m(t) \\
\frac{dI_m(t)}{dt} &= \alpha E_m(t) - (\eta+\delta_m)I_m(t)
\end{aligned} \tag{2}$$

3. Optimal Control

Optimal control is one of the tools used in mathematical biology to eradicate, reduce or minimize the infected and death rates of humans in the population. In formulating an optimal control problem on malaria, we propose a model to minimize the number of exposed humans to malaria, the number of infected humans and the total population of mosquitoes. As a way of eradicating or controlling malaria in our society, control measures have to be introduced such as, treated insecticide bednets (ITNs), insecticide spray against mosquitoes (ISAM), sterile insect technique (SIT) e.g. male mosquitoes and awareness approach (AA) e.g. Social media network, television broadcast, house-to-house awareness, e.t.c.

To find the solution to model (1), we considered the following steps: To

(i) describe the optimal control.
(ii) show the existence of optimal control.
(iii) show the uniqueness of optimal control.
(iv) solve the optimal control numerically.
(v) show the graphical solution with the effects of control variables on the model.

3.1. Description of the Optimal Control

The objective function of the system (1) is used to minimize the total number of exposed humans, infected humans and mosquitoes using the control variables $u_1(t), u_2(t), u_3(t)$ and $u_4(t)$. It is very important to show that all the control variables are non-negative. The objective function is defined as

$$J = \int_0^{t_f} \left(B_1 E_h(t) + B_2 I_h(t) + B_3 N_m(t) + \frac{1}{2}\left(P_1 u_1^2(t) + P_2 u_2^2(t) + P_3 u_3^2(t) + P_4 u_4^2(t) \right) \right) dt \quad (3)$$

Subject to the system (1), where $B_1, B_2, B_3, P_1, P_2, P_3$ and P_4 are positive weight constants. The quadratic costs $P_1 u_1^2(t), P_2 u_2^2(t), P_3 u_3^2(t)$ and $P_4 u_4^2(t)$ are the cost associated with the use of insecticide-treated bednets, treatment of infectious human, use of sterile mosquito and treatment to protect pregnant women and newborn births respectively. This quadratic cost and objective function are chosen in line with the literature on epidemic controls by Lashari *et al.* [14], Sharomi and Malik [15] and Momoh and Fügenschuh [16]. We intend to find an optimal control $u_1^*(t), u_2^*(t), u_3^*(t)$ and $u_4^*(t)$ such that

$$J\left(u_1^*, u_2^*, u_3^*, u_4^*\right) = \min\left\{\left(u_1, u_2, u_3, u_4\right) : u_1, u_2, u_3, u_4 \in \Omega\right\} \quad (4)$$

Where $\Omega = \left\{u_i : 0 \le u_i(t) \le 1, \text{ Lebesgue measurable } t = \left[0, t_f\right] \text{ for } i = 1,2,3,4\right\}$ is the control set.

3.2. Existence of the Optimal Control

To show the existence of the optimal control with the initial conditions $t = 0$, we state and prove theorems 1 and 2 below. This will also help us to analyze the properties of the system (1) with positive initial conditions $\forall\, t > 0$ since the model describes human and mosquito populations. Using the optimal control in the system (1) to see the existence of optimal control with the necessary conditions satisfying the Pontryagin's Maximum Principle. Pontryagin *et al.* [17]. We applied Pontryagin's Maximum Principle to convert equations (1), (3) and (4) into a problem of minimizing point-wise Lagrange, *L*, with respect to u_1, u_2, u_3, u_4 and to find the minimal value of the Lagrangian. This could be achieved according to Mwantobe *et al.* [18] by considering Hamiltonian, *H*.

$$
\begin{aligned}
H = {} & B_1E_h(t)+B_2I_h(t)+B_3N_m(t)+\frac{1}{2}\left(P_1u_1^2(t)+P_2u_2^2(t)+P_3u_3^2(t)+P_4u_4^2(t)\right) \\
& +\lambda_1\left\{\Lambda_h-\left[(1-u_1)+(1-u_4)\right]\frac{\xi\sigma_h I_m(t)S_h(t)}{1+\varepsilon_h I_m(t)}-\mu S_h(t)+\gamma_1 R_h(t)\right\} \\
& +\lambda_2\left\{\left[(1-u_1)+(1-u_4)\right]\frac{\xi\sigma_h I_m(t)S_h(t)}{1+\varepsilon_h I_m(t)}-(\upsilon+\mu+\delta_h)E_h(t)\right\} \\
& +\lambda_3\left\{\upsilon E_h(t)-(\omega+au_2+\mu+\delta_h)I_h(t)+\psi I_h(t)+\gamma_2 R_h(t)\right\} \\
& +\lambda_4\left\{(\omega+au_2)I_h(t)-(\mu+\gamma_1+\gamma_2)R_h(t)\right\} \\
& +\lambda_5\left\{\Lambda_m-\left[(1-u_1)+(1-u_3)\right]\frac{\xi\sigma_m I_h(t)S_m(t)}{1+\varepsilon_m I_h(t)}-(bu_1+\eta)S_m(t)\right\} \\
& +\lambda_6\left\{\left[(1-u_1)+(1-u_3)\right]\frac{\xi\sigma_m I_h(t)S_m(t)}{1+\varepsilon_m I_h(t)}-(bu_1+\alpha+\eta)E_m(t)\right\} \\
& +\lambda_7\left\{\alpha E_m(t)-(bu_1+\eta+\delta_m)I_m(t)\right\}
\end{aligned} \tag{5}
$$

The existence of optimal control of the system (1) will be considered by applying the following theorems in Lashari *et al.* [14], Lashari *et al.* [19], Lenhart and Workman [21]

Theorem 1: *There exists an optimal control* $u_i^* \in \Omega$ *for* $i = 1,2,3,4$ *such that* $J\left(u_1^*,u_2^*,u_3^*,u_4^*\right)=\min\left\{(u_1,u_2,u_3,u_4): u_1,u_2,u_3,u_4\in\Omega\right\}$, *subject to the control system (1) with initial conditions at t = 0.*

Proof: The state and control variables of the system (1) are positive values and the control set Ω is closed and convex. Therefore the integrand of the objective function J in which it was expressed in the system (1) is a convex function of (u_1,u_2,u_3,u_4) on the control set Ω. Since the state solutions are bounded, then Lipschitz property of the state system with respect to the state variables is satisfied. It can also be seen that ∃ positive numbers η_1, η_2 and a constant $\varepsilon > 1$ such that, $J(u_1,u_2,u_3,u_4) \geq \eta_1\left(|u_1|^2,|u_2|^2,|u_3|^2,|u_4|^2\right)^{\varepsilon/2}-\eta_2$ (6)

Therefore, the state variables are bounded and the existence of optimal control of the system (1) is concluded.

3.3. The uniqueness of Optimal Control

Pontryagin’s Maximum Principle is used to reveal the necessary conditions for this optimal control. This is as a result of the fact that minimizing the cost-functional in equation (3) subject to the system (1) is the existence of optimal control. According to Lashari et al. [14], If $\left(x,u\right)$ is an optimal solution of an optimal control problem then ∃ a non-trivial vector function λ = (λ1, λ2, λ3, λ4,.., λn) that satisfies the following equations

$$\frac{dx}{dt}=\frac{\partial H\left(t,x,u,\lambda\right)}{\partial\lambda};0=\frac{\partial H\left(t,x,u,\lambda\right)}{\partial u};\lambda'=\frac{\partial H\left(t,x,u,\lambda\right)}{\partial x} \tag{7}$$

Therefore, we can now apply necessary conditions to the Hamiltonian, *H*, in equation (5). Olaniyi et al [5] and Lashari et al. [19]

Theorem 2: *Let* $S_h^*, E_h^*, I_h^*, R_h^*, S_m^*, E_m^*$ *and* I_m^* *be optimal state solutions with associated optimal control variables* $\left(u_1^*,u_2^*,u_3^*,u_4^*\right)$ *for the optimal control problem in (1) and (3). Then there exist the co-states* λ_i *which verify (8) with the transversality conditions* $\lambda_i\left(t_f\right)=0$ *in (9) for* $i=1,2,3,\ldots,7$ *and in (11) the control variables* $\left(u_1^*,u_2^*,u_3^*,u_4^*\right)$.

Proof: Consider the system of differential equations in (8) governing the adjoint variables $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7$. This is obtained by differentiating the Hamiltonian *H* in equation (5) with respect to S_h, E_h, I_h, R_h, S_m, I_m and I_m. According to Fleming and Rishel [20], these are the state variables, by applying the first and third equations in equation (7) into equation (5)

$$\begin{aligned}
\frac{d\lambda_1}{dt}&=-\frac{\partial H}{\partial S_h}=\left[\left(1-u_1\right)+\left(1-u_4\right)\right]\left(\lambda_1-\lambda_2\right)\frac{\xi\sigma_h I_m}{1+\varepsilon_h I_m}+\mu\lambda_1\\
\frac{d\lambda_2}{dt}&=-\frac{\partial H}{\partial E_h}=\upsilon\left(\lambda_2-\lambda_3\right)+\left(\mu+\delta_h\right)\lambda_2-B_1\\
\frac{d\lambda_3}{dt}&=-\frac{\partial H}{\partial I_h}=\left(\omega+au_2\right)\left(\lambda_3-\lambda_4\right)+\left[\left(1-u_1\right)+\left(1-u_3\right)\right]\left(\lambda_5-\lambda_6\right)\frac{\xi\sigma_m S_m}{\left(1+\varepsilon_m I_h\right)^2}\\
&\qquad+\left(\mu+\delta_h-\psi\right)\lambda_3-B_2\\
\frac{d\lambda_4}{dt}&=-\frac{\partial H}{\partial R_h}=\mu\lambda_4-\gamma_1\left(\lambda_1-\lambda_4\right)-\gamma_2\left(\lambda_3-\lambda_4\right)\\
\frac{d\lambda_5}{dt}&=-\frac{\partial H}{\partial S_m}=\left[\left(1-u_1\right)+\left(1-u_3\right)\right]\left(\lambda_5-\lambda_6\right)\frac{\xi\sigma_m I_h}{1+\varepsilon_m I_h}+\left(bu_1+\eta\right)\lambda_5-B_3\\
\frac{d\lambda_6}{dt}&=-\frac{\partial H}{\partial E_m}=\alpha\left(\lambda_6-\lambda_7\right)+\left(bu_1+\eta\right)\lambda_6-B_3\\
\frac{d\lambda_7}{dt}&=-\frac{\partial H}{\partial I_m}=\left(\left(1-u_1\right)+\left(1-u_4\right)\right)\left(\lambda_1-\lambda_2\right)\frac{\xi\sigma_h S_h}{\left(1+\varepsilon_h I_m\right)^2}+\left(bu_1+\eta+\delta_m\right)\lambda_7-B_3
\end{aligned} \tag{8}$$

With the transversality conditions

$$\lambda_1\left(t_f\right)=\lambda_2\left(t_f\right)=\lambda_3\left(t_f\right)=\lambda_4\left(t_f\right)=\lambda_5\left(t_f\right)=\lambda_6\left(t_f\right)=\lambda_7\left(t_f\right)=0 \tag{9}$$

To evaluate the optimal control of the control variable set, where $u_i=\left(0,1\right)$. Let $S_h=S_h^*,\ E_h=E_h^*,\ I_h=I_h^*, R_h=R_h^*, S_m=S_m^*, E_m=E_m^*$ and $I_m=I_m^*$, and applying the second equation in equation (7), and differentiating the Hamiltonian, H, in equation (5) with respect to the control variables u_1, u_2, u_3 and u_4

$$
\begin{aligned}
\frac{\partial H}{\partial u_1} &= P_1 u_1^* - \frac{\xi \sigma_h I_m^* S_h^*}{1+\varepsilon_h I_m^*}\left(\lambda_2 - \lambda_1\right) - \frac{\xi \sigma_m I_h^* S_m^*}{1+\varepsilon_m I_h^*}\left(\lambda_6 - \lambda_5\right) - b\left(\lambda_5 + \lambda_6 + \lambda_7\right) = 0 \\
\frac{\partial H}{\partial u_2} &= P_2 u_2^* - a\left(\lambda_3 - \lambda_4\right) = 0 \\
\frac{\partial H}{\partial u_3} &= P_3 u_3^* - \frac{\xi \sigma_m I_h^* S_m^*}{1+\varepsilon_m I_h^*}\left(\lambda_6 - \lambda_5\right) = 0 \\
\frac{\partial H}{\partial u_4} &= P_4 u_4^* - \frac{\xi \sigma_h I_m^* S_h^*}{1+\varepsilon_h I_m^*}\left(\lambda_2 - \lambda_1\right)
\end{aligned}
\tag{10}
$$

By applying the optimal control to the control variable set, $u_i^* = (0,1)$ for $i = 1, 2, 3, 4$ into equation (10)

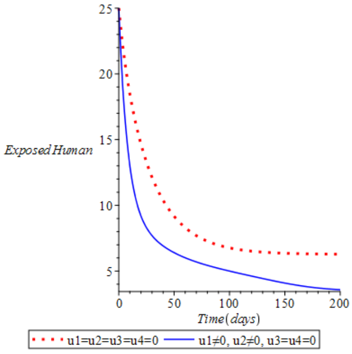
$$
\begin{aligned}
u_1^* &= \max\left\{0, \min\left(1, \frac{\beta_h^* S_h^* \left(\lambda_2 - \lambda_1\right) - \beta_m^* S_m^* \left(\lambda_6 - \lambda_5\right) + b\left(\lambda_5 S_m^* + \lambda_6 E_m^* + \lambda_7 I_m^*\right)}{P_1}\right)\right\} \\
u_2^* &= \max\left\{0, \min\left(1, \frac{a\left(\lambda_3 - \lambda_4\right) I_h^*}{P_2}\right)\right\} \\
u_3^* &= \max\left\{0, \min\left(1, \frac{\beta_m^* S_m^* \left(\lambda_6 - \lambda_5\right)}{P_3}\right)\right\} \\
u_4^* &= \max\left\{0, \min\left(1, \frac{\beta_h^* S_h^* \left(\lambda_2 - \lambda_1\right)}{P_4}\right)\right\}
\end{aligned}
\tag{11}
$$

where $\beta_h^* = \frac{\xi \sigma_h I_m^*}{1+\varepsilon_h I_m^*}$ and $\beta_m^* = \frac{\xi \sigma_m I_h^*}{1+\varepsilon_m I_h^*}$. This shows that the uniqueness of the optimal control of the model has been achieved for small t_f based on prior boundedness of the state variables as well as adjoint variables. This is made possible through the use of Lipschitz property of the ordinary differential equations.

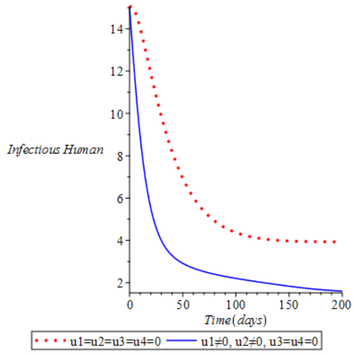
3.4. Numerical Simulation of the Optimal Control

The optimality system consists of state system in system (1), optimal control set in equation (11), adjoint system in equation (8), boundary conditions in equation (9) and initial conditions $S_h(0) = 100, E_h(0) = 25, I_h(0) = 15, R_h(0) = 5,\ S_m(0) = 1000, E_m(0) = 20$ and $I_m(0) = 30$ according to Olaniyi *et al.* [5]. Using this optimality system, the state variables and optimal control can be calculated. It shows that the second equation in system (7) applied on Hamiltonian equation (5) is positive which means that the optimal problem is minimal at controls u_1^*, u_2^*, u_3^* and u_4^*. We carried out the numerical simulation with Maple 18 by using the forward-backward fourth-order Runge-Kutta method and the result is shown in the graphs below.

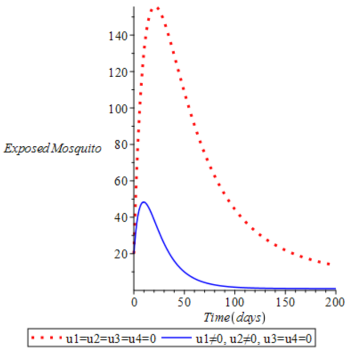
3.5. Graphical Solution of the Optimal Control



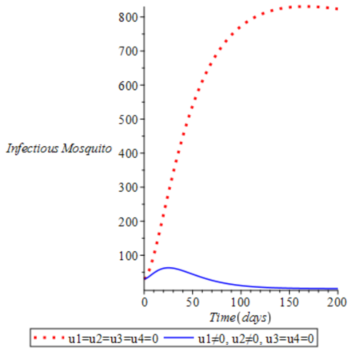
(a): Optimal Control graph of Exposed Human against Time



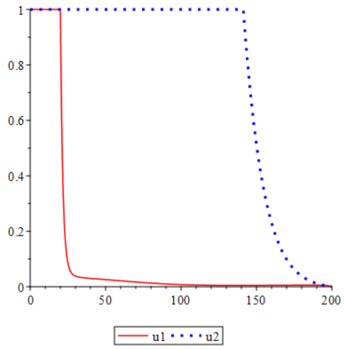
(b): Optimal Control graph of Infectious Human against Time



(c): Optimal Control graph of Exposed Mosquito against Time

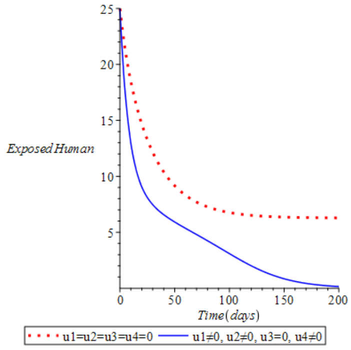


(d): Optimal Control graph of Infectious Mosquito against Time

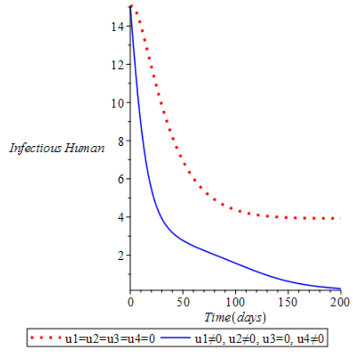


(e): Control Profiles graph of u_1 and u_2

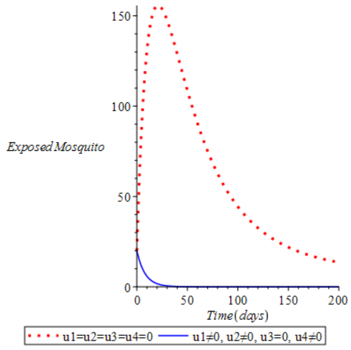
Fig. 1: Simulation of the model showing the effects of insecticide-treated bednets (u_1) and infected human treatment (u_2) on malaria transmission



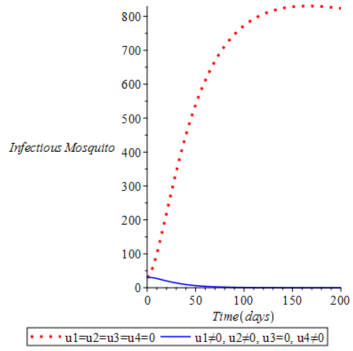
(a): Optimal Control graph of Exposed Human against Time



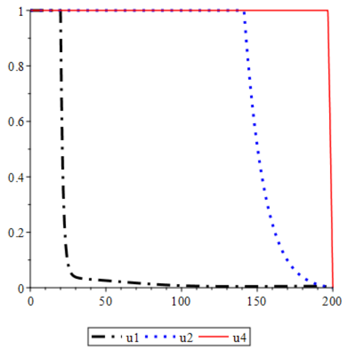
(b): Optimal Control graph of Infectious Human against Time



(c): Optimal Control graph of Exposed Mosquito against Time

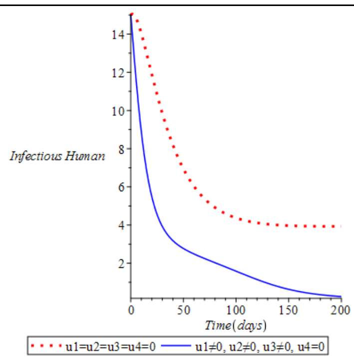


(d): Optimal Control graph of Infectious Mosquito against Time

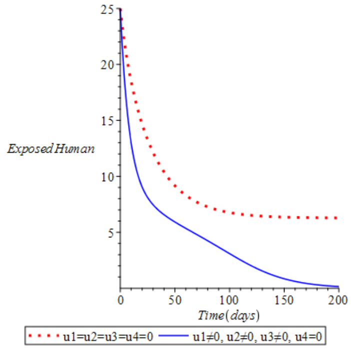


(e): Control Profiles graph of u_1, u_2 and u_4

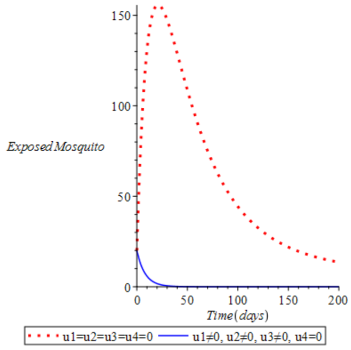
Fig. 2: Simulation of the model showing the effects of insecticide-treated bednets (u_1), infected humans treatment (u_2) and pregnant women & newborn births (u_4)



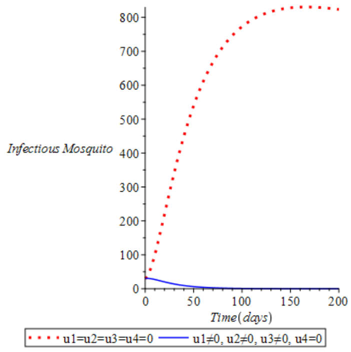
(a): Optimal Control graph of Exposed Human against Time



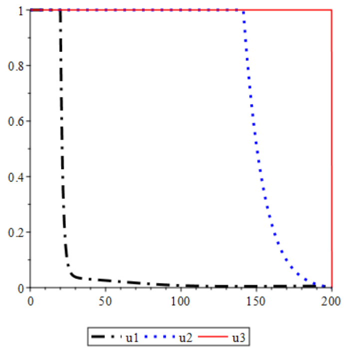
(b): Optimal Control graph of Infectious Human against Time



(c): Optimal Control graph of Exposed Mosquito against Time



(d): Optimal Control graph of Infectious Mosquito against Time



(e): Control Profiles graph of u_1, u_2 and u_3

Fig. 3**:** Simulation of the model showing the effects of insecticide-treated bednets (u_1), infected humans treatment (u_2) and sterile mosquitoes technique (u_3)

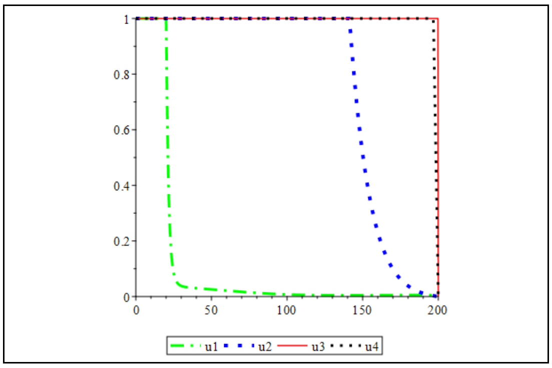


Fig. 4: Control Profiles graph of of u_1, u_2, u_3 and u_4

4. Results and Discussion

The results from the numerical simulation can be classified in the following categories:

4.1. Category A: Insecticide Treated Bednets and Infected Humans Treatment

In this category, figure 1 illustrates the impact of the insecticide-treated bednets control (u_1) and infected humans treatment control (u_2) in eradicating malaria in the population. It is verified that by applying the controls, the exposed and infectious humans, as well as exposed and infectious mosquitoes, decrease more rapidly to extinction compared to when there is no control. The control profile in figure 1(e) shows that u_1 and u_2 are at their upper bound until time *t = 20 days* and time *t = 141 days* respectively before decreasing to the lower bound.

4.2. Category B: Insecticide Treated Bednets, Infected Humans Treatment and Pregnant women & newborn births

In this category, figure 2 shows the collective impact of Insecticide-treated bednets control (u_1), infected humans treatment control (u_2) and pregnant women & newborn births control (u_4) on malaria spread for both human and mosquito populations. It is verified that by applying the control, the exposed and infectious humans, as well as exposed and infectious mosquitoes, diminish more rapidly compared to when control. The control profile in figure 2(e) shows that u_1, u_2 and u_4 are at their upper bound until time *t = 20 days, t =141 days* and *t =196 days* respectively before decreasing to the lower bound.

4.3. Category C: Insecticide Treated Bednets, Infected Humans Treatment and Sterile mosquitoes technique

In this category, figure 3 shows the collective impact of Insecticide-treated bednets control (u_1), infected humans treatment control (u_2) and Sterile mosquitoes technique control (u_3) on malaria transmission for both human and mosquito populations. It is verified that by applying the control, exposed and infectious humans, as well as exposed and infectious mosquitoes decrease more rapidly to extinction compared to without control. The control profile in figure 3(e) illustrates that u_1, u_2 and u_3 are at their upper bound until time *t = 20 days*, *t =141 days* and *t =200 days* respectively before decreasing to the lower bound.

The control profile in figure 4 illustrates that the Insecticide mosquitoes bednets (u_1), infected humans treatment control (u_2), Sterile mosquitoes technique control (u_3) and Pregnant women & newborn births (u_4) are at their upper bound until time *t = 20 days, t = 141 days, t = 200 days* and *t = 196 days* respectively before decreasing to the lower bound.

5. Conclusion

In this paper, we derived and analyzed a deterministic mathematical model on malaria with seven compartments, the optimal control analysis is achieved from the formulated model in line with the arrangement of the four control measures and the analysis gave credence to Pontryagin's Maximum Principle (PMP) together with numerical simulations. We conclude that if the controls are well managed and implemented, then it would curb or limit the transmission of malaria between vector-host related populations.

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