SLIDING MODE STRATEGY FOR CLOSED LOOP CONTROLLED TWO-LEVEL PWM INVERTER

Dan OLARU[1], Dan FLORICĂU[2]

Lucrarea îşi propune să determine o nouă strategie pentru comanda în buclă închisă a invertorului PWM cu două nivele, cuplat la un filtru LC trece jos. Soluţia prezentată simplifică metoda "sliding mode" clasică luând în consideraţie condiţiile specifice de funcţionare a circuitului. Criteriile de stabilitate şi convergenţă sunt determinate printr-o metodă de clasificare şi sunt validate prin simulări numerice. Comparată cu metoda de modulaţie cu undă purtătoare, această strategie oferă avantaje din punctul de vedere al calităţii aproximaţiei semnalului de referinţă şi al caracteristicilor de regim tranzitoriu, în special în cazul unui semnal de referinţă oarecare.

The paper purpose is to define a new closed loop control strategy for a two level PWM inverter coupled with a LC low-pass filter. The presented solution simplifies the classical "sliding mode" method taking into account the specific operating conditions of the circuit. The stability and convergence criteria are determined by a classifying method and are validated by numerical simulations. Compared with the carrier wave modulation method, this strategy gives advantages concerning the reference signal approximation and the transient regime characteristics more especially for a random reference signal.

Keywords: sliding mode, PWM modulation, phase space representation

1. Introduction

The closed loop control of a PWM inverter configuration, followed by a LC low-pass filter, gives some stability and convergence problems because the delaying effects of the reactive components. This is more important when we deal with instantaneous parameters and a phase synchronism is imposed. The classical solutions using PI or PLL controller techniques have poor performances concerning the step time response. The presented method tries to give a better solution for the error minimization, stability problems and transient regime reduction. The sliding mode techniques give better performances from all the points of view: stability, approximation error and low time response. The price that must be paid for these consists in a more sophisticated mathematical model

[1] Lecturer, Dept. of Electrical Engineering, University POLITEHNICA of Bucharest, Romania, e-mail: dan.olaru@upb.ro
[2] Prof., Dept. of Electrical Engineering, University POLITEHNICA of Bucharest, Romania

and advanced criteria for a good operation. However, the technical implementation is very simple and robust.

2. The sliding mode concept

This method is applied for the control of dynamical system, described by some time dependent parameters. The control function has only two constant values, which are switched during the time evolution. The method uses the "state space" representation, having as coordinates the system parameters and/or their derivatives. The time variation action is implicit and it isn't used as an independent variable. The state of the system is univocally represented by a generic point which evolutes following a curved trace. In the state space is defined a "switching surface" that is specific to the problem under consideration. The switching moments of the control function are determined by the intersection between the evolution trace and the switching surface. The surface splits the state space into two regions corresponding for the two values of the control function. The control function must act in a specific way, able to determine the evolution trace, so that the state of the system riches a defined "target point". In the classical case the switching surface passes through the coordinates' origin and the target point coincides with of point of origin.

The evolution process may be defined like an approximation problem, when a given reference value becomes the target point. The approximation process is stable if the variations of the output parameter are bounded for a certain reference value. The process is convergent if the error between the output parameter and the reference value may be continuously reduced, for a certain operating conditions.

The sliding mode concept may be extended to a dynamical formulation, when the reference value is variable. This is a more complicated problem because the switching surface characteristics become themselves variables in a dynamical way.

The sliding mode problem is solved when the switching surface is completely defined, satisfying the stability and convergence criteria.

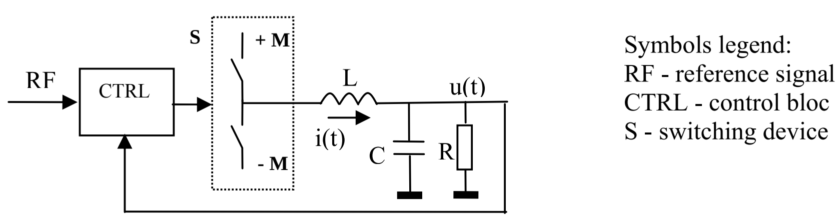

Fig. 1. The functional scheme

3. The mathematical model

The simplified control configuration of a two level PWM inverter followed by a LC low-pass filter, operating on a resistive load is given in Fig.1. This may be described by a first order non-linear differential equation system (1,2):

$$\frac{du(t)}{dt} = \frac{1}{C} \cdot i(t) - \frac{1}{R \cdot C} \cdot u(t) \tag{1}$$

$$\frac{di}{dt} = -\frac{1}{L} \cdot u(t) + \frac{1}{L} \cdot E(t,u,i) \tag{2}$$

The model parameters are the capacitor voltage u(t) and the inductor current i(t). The control function E(t,u.i) has only two values (-M,+M) depending on the state of the system, determined by the inverter power supply. For numerical solving purposes a discrete formulation is done, defined by the vector recursive equation (3), using the Mathcad language representation (with normal symbols):

$$u^{\langle k+1 \rangle} := u^{\langle k \rangle} + h \cdot \left[\begin{bmatrix} -\frac{1}{R \cdot C} & \frac{1}{C} \\ -\frac{1}{L} & 0 \end{bmatrix} \cdot \begin{bmatrix} \left(u^{\langle k \rangle}\right)_0 \\ \left(u^{\langle k \rangle}\right)_1 \end{bmatrix} + \begin{pmatrix} 0 \\ \frac{1}{L} \cdot E(k,u) \end{pmatrix} \right] \tag{3}$$

where: *h* - the calculation time step;

$\boldsymbol{u}^{<k>}$, $\boldsymbol{u}^{<k+1>}$ - the vector at the moment t_k, and t_k+h respectively;

E(k,u) - the switching function depending on *k* and ***u***.

The two components $(\boldsymbol{u}^{<k>})_0$ and $(\boldsymbol{u}^{<k>})_1$ denote respectively the capacitor voltage and the inductor current at the t_k moment, as $u(t_k)$ and $i(t_k)$. For the switching function *E(k,u)* the conditional expression (4) is used:

$$E(k,u) := \mathrm{if}\left[\left(u^{\langle k \rangle}\right)_1 + K \cdot \left[\left(u^{\langle k \rangle}\right)_0 - RF\left(t_k\right) \right] > 0, -M, M \right] \tag{4}$$

The function *E(k,u)* becomes -M or +M, function of the positive or negative sign of the test expression (5):

$$\left(u^{\langle k \rangle}\right)_1 + K \cdot \left[\left(u^{\langle k \rangle}\right)_0 - RF\left(t_k\right) \right] \tag{5}$$

The expression (5) must include the information about the position of the generic point in the state space relative to the switching surface. For our model the state space has only two dimensions (state plane), defined as the $(\boldsymbol{u}^{<k>})_0$ and $(\boldsymbol{u}^{<k>})_1$ variables. In consequence the switching surface becomes a switching curve. In this context the solution of the sliding mode process is based on a linear approximation having a "switching line".

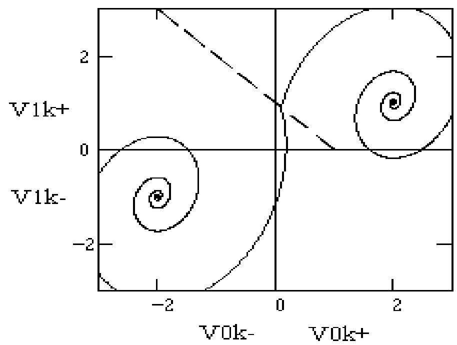

Fig. 2. The damped oscillatory behavior

4. The switching line determination

In general, the switching curve is non-linear and must be determined by an analytical reasoning. In reference [1] an optimal control problem is solved by a method where the branches of the switching curve represent solutions of the differential problem in the state space. In this case the approximation process becomes optimal from the time duration point of view. For a certain initial state, after the first intersection, the evolution trace follows the switching curve until the trace reaches the target point.

In this paper is considered a linear approximation of the switching curve. Firstly it is considered that the switching line passes through the origin and the reference value is zero (the target point is the origin of the coordinates). The only parameter defining the sliding mode is the slope of the line, denoted by *K*. Thus, choosing *K* the stability and convergence criteria may be satisfied. For the *K* value estimation a series of numerical experiments were organized in order to determine the sub-domains corresponding to the switching line. For this purpose the particularities of the circuit were taken under consideration. Thus, if the control function *E(t,u,i)* has a constant value M or -M the output of the circuit evolutes in an damped oscillatory manner towards this value as in Fig.2. The variables V0k- ,V0k+, V1k- V1k+ correspond respectively to $(\mathbf{u}^{<k>})_0$ and $(\mathbf{u}^{<k>})_1$ for the +/-M value. It can be seen that depending of the control function value the evolution trace becomes closer or more distant relative to the target point, corresponding to the convergence or to divergence of the process. The problem is to determine the convergence sub-domains, the divergence sub-domains and their invariance. Thus, the switching line may be completely defined.

On the other hand, there is no difference, from the convergence point of view, if a generic point represents an intermediary state of the circuit or the initial one. Thus, the convergence characteristic of a sub-domain during the process may be tested using different initial state points, covering the entire state space. Taking into account the continuity proprieties of the problem the set of considered initial points may be discrete and enough rare.

For the numerical experiments there were considered certain sets of points, circularly positioned at different radius from the origin. Several representative cases, for the four quadrants, were represented in Fig.3,4,5,6, together with the evolution traces corresponding on the two values of the control function. The conclusions show that the switching line must have a negative slope. The switching line divides the plane in two regions. The up sub-domain corresponds to a positive value of expression (5) and the down one to a negative value. For a convergent process (negative feedback) the control function must take, respectively, the -M and +M value.

Now, consider non-zero value of the reference signal. The first coordinate (the output parameter) of the target point must be identified with this value, fact that is equivalent with a translation of the origin of the coordinate system.

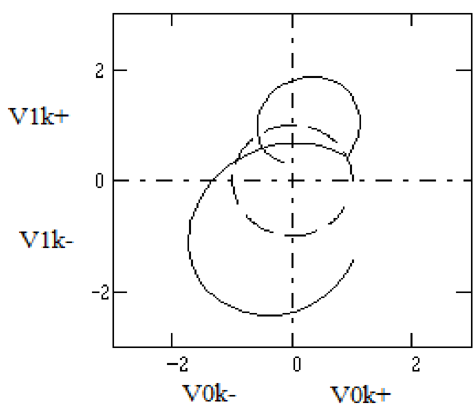

Fig. 3. The first quadrant behavior.

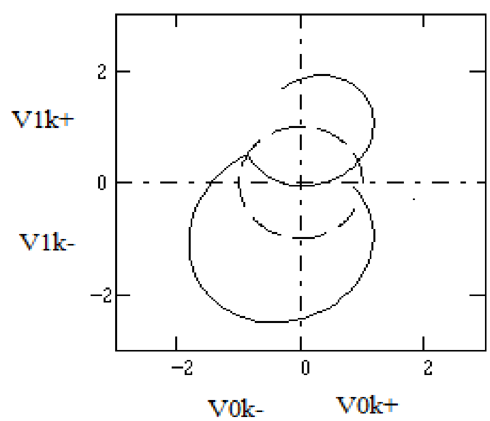

Fig. 4. The second quadrant behavior.

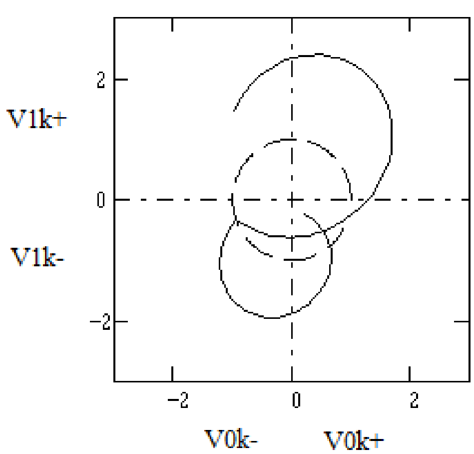

Fig. 5. The third quadrant behavior.

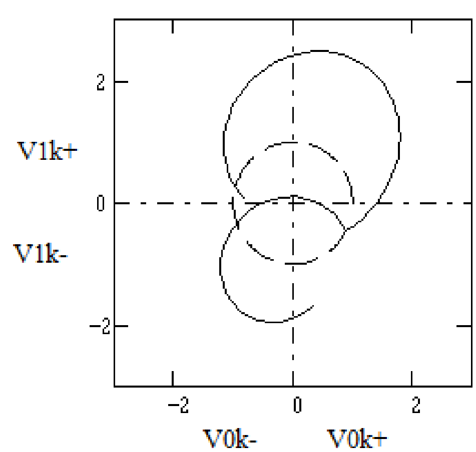

Fig. 6. The fourth quadrant behavior.

The convergence process is not affected. If it takes place for all possible initial states of the system, a translation transformation is not important.

When the reference signal has a continuous variation, the approximation process of the target point becomes a following process. At each time, the sliding mode tries to approximate the instantaneous position of the target point.

5. Closed loop operation

The control function must ensure the stability and the convergence of the system by as close as possible to the optimal approximation process of the reference signal. During a dynamical regime (continuous variation of the reference) the optimality characteristic determines implicitly the approximation error and the time response of the system. When the process is near optimal the trace reaches the target point in a very short time. Thus, the transient regime is shorter for every variation step of the reference signal and implicitly, the delay of the following process becomes lower.

When a control function is based on a switching curve method, the stable behavior of the system may have two modes. The first mode gives a trace which evolutes on the both sides of the target point, intersecting alternatively the both branches relative to the origin and the sense of variation is the same after each intersection with the switching curve. In the second mode the trace changes the sense of variation after each intersection and remains near the one of the branches until the target point is reached (true sliding mode).

The dynamical approximation process in the state space may be observed in Fig.7, for a model circuit with R=10, L=1mH, C=100μF. Here V0k,V1k and RFk correspond respectively to $(\mathbf{u}^{<k>})_0$, $(\mathbf{u}^{<k>})_1$ and $RF(t_k)$. The reference signal is sinusoidal. In order to test the stability of the process a non-zero initial condition is used. For a pure sinusoidal variation the trace becomes elliptic.

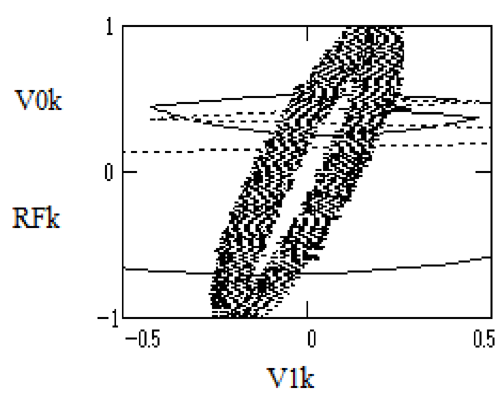

Fig.7. The dynamical approximation process in the state space

When the system is controlled by a switching function, the first stage of evolution follows the first mode and after the reference variation is "captured" the trace is conform to the second mode. Thus, during the variation, a cloud of points is obtained. These correspond to the harmonic content of the second parameter (inductor current).

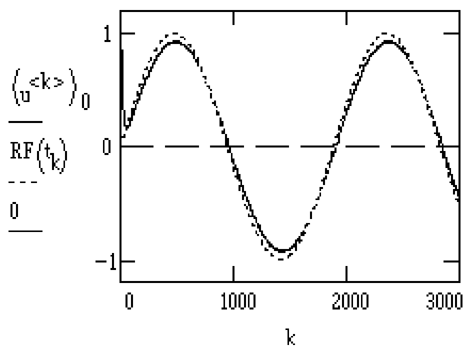

Fig.8. The evolution of reference and output signal.

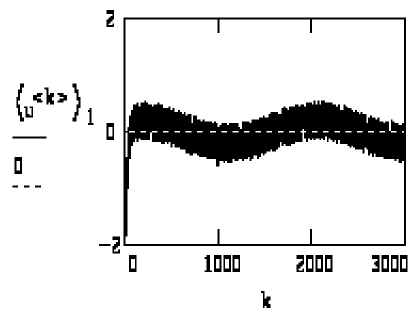

Fig.9. The L current signal evolution.

The time dependences of the two parameters showed in Fig.8. and Fig.9. are relevant for this point of view. Giving different values to the switching line slope parameter K, there are obtained as a compromise, different performances indexes for the approximation error and the time response.

It is interesting to observe that when the target point is defined, a zero value is considered for the second parameter (inductance current). This is not really a restriction because the convergence speeds of the two parameters are different. Thus, the second parameter may have its own quasi-independent variation relative to the first (output voltage). By the nature of the approximation process itself, at each step the control function tries to reduce the value of the inductance current. Physically, this leads to a reduction of the harmonic content.

6. Conclusions

The paper presents a new control strategy for a two level PWM inverter in a closed loop configuration. This is based on the state space representation and the switching curve method. Although the mathematical model is more complex, and some analytical reasoning are necessary, the technical implementation is very simple. The elimination of the inertial behavior devices like PI or PLL controllers is advantageous. This enables better stability, convergence and approximation error performances. By the switching curve method the control function acts "in advance" eliminating the delay between the reference and output signal. Although only a linear approximation was used for the switching curve, the numerical simulations show a good behavior. Our topological justification is complete and simple, having a useful geometrical interpretation, compared with others more complex or contextual ([6] or [9]).

The paper analyses the possibility of a command strategy, based on a linear switching criterion, for a closed loop topology. In [1] is given a rigorous optimal method for a general differential equation context. But the obtained criterion is useless for technical purposes because the non-linear formulation based on the derivative of the output parameter. The authors have demonstrated that a linear criterion with a single parameter may be used, maintaining the stability and error characteristics close to the optimum ones. The purposed method uses as second parameter in the phase's plane the inductive current, instead of the output parameter derivative. In cap.4 is given an original method for the determination of the stability domain of the K parameter using topological and geometrical representations. Thus, the variations limits of K values may be fixed, for a given combination of the R, L, and C values.

Having the stability, the optimal value for K may be determined by numerical simulation analysis for different kind of initial conditions and different types of discontinuities of the reference signal. The final results are showed in cap.5. Also, the vector recursive representation from cap.3 is very compact and enables a real time implementation of the error analysis and K parameter corrections.

The study considers, for simplicity, only the resistive load R. The reactive behavior belongs to the low pass filter. But the load, itself, may have its reactive component. The goal of the paper is first a theoretical one, concerning the determination criteria of the K parameter for a non-important physical configuration. The implementation is possible for any configuration containing a two level PWM inverter.

R E F E R E N C E S

[1] *A. Feldbaum*, Principles theoretiques des systemes asservis optimaux, Ed.Mir 1973

[2] *V. Zoubov,* Theorie de la comande, Ed.Mir 1978

[3] *V.I.Utkin*, Slidingmodes and their applications in variable structure systems, Mir Publishers, Moscow 1978

[4] *V.Utkin, J.Guldner, J.Shi*, Sliding Mode Control in Electromechanical Systems, Taylor & Francis 1999

[5] *J.Miret, L.Cruz*, A Simple Sliding Mode Control of an Active Power Filter, Proceedings of IEEE Power Electronics Specialists Conference, 2004

[6] *M.Nayeripour, T.Niknam,* Nonlinear Sliding Mode Control Design for Shunt Active Power Filter with the Minimization of Load Current, World Applied Sciences Journal 4 (1), 2008

[7] *D.Olaru, D.Floricau,* Qualitative aspects on closed loop power switching system control - EMES'09 International Conference, Oradea 2009

[8] *D.Olaru, D.Floricău,* Optimal method for controlled switching circuits, Sci.Bul.UPB, **vol.71**, Nr.3, 2009

(9) *H.Tumbelaka, L.Borle, C.Nayar, S.Lee,* A Grid Current-controlling Shunt Active Power Filter - Journal of Power Electronics, **vol. 9,** no. 3, 2009.