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CORRELATION OF FLUCTUATIONS IN THE FREQUENCY DISTRIBUTION WINGS OF TIME SERIES. CASE STUDY: LEU-USD AND LEU-EUR EXCHANGE RATES

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We propose a new method for the study of the correlations of fluctuations in time-series, using the multifractal analysis. The investigation is performed on the partial data sets obtained from the original series, for positive and negative fluctuations. The method offers the possibility of identifying correlations either in each sub-series or between them. The application of the method to financial time-series allows estimation on the predictability of evolution. As case study, the exchange rates Leu-USD and Leu-EUR are considered.

Keywords: frequency distribution, correlation analysis, financial time series.

1. Introduction

Since the publication of the first book on Econophysics [1], this relatively new area of study, established by the cooperation between economists, mathematicians and physicists, registered considerable development, manifested in a large number of relevant papers [2-5] and recent books [6,7]. It applies ideas, methods and models of statistical physics and complexity theory to analyze data from economical phenomena. Initially, it was not seen as a tool to predict future prices of stocks, exchange rates or commodities. However, recently, the preoccupation to extend the methods of Econophysics to predictive estimation acquired momentum, particularly stimulated by the observation that financial time series can be treated as fractal structures, and the development of multifractal analysis of fluctuations [8,9]. The present work is about the correlation of fluctuations in the two wings of the frequency distribution (histogram) of financial time series and illustrates the analysis with exchange rates of the Romanian currency Leu versus US Dollar (USD) and Leu versus Euro (EUR), respectively. The fluctuations in the dynamical output may be characterized by two components: magnitude (absolute value) and sign (direction) of the competing

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forces. These two quantities reflect the underlying interactions in a system, and the resulting force of these interactions at each moment determines the magnitude and direction of the fluctuations [10]. The proposed method is based on Multifractal Detrended Fluctuation Analysis (MFDFA) [11,12] and Multifractal Fluctuation Cross-Correlation Analysis (MFCCA) [13,14]. It bears some similarity to the treatments of [15,16] and can be applied to the analysis of any type of time series, particularly to the analysis of financial and economical ones.

2. Data

The exchange rates of Leu-EUR and Leu-USD were taken from the site Forex Trading and Exchange Rates Services (OANDA) [17] and correspond to the interval June 1999-November 2012, i.e. 4910 values.

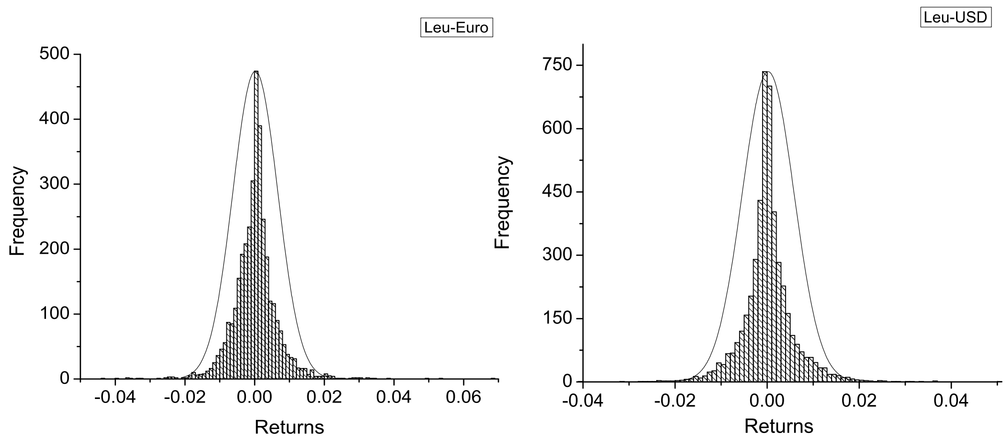

Fig.1 Absolute frequency distribution versus returns

The statistics of the returns of the two series are given in Fig.1. In order to emphasize the fat-tailed character of the distribution, the best fitting normal distribution is shown with continuous line.

The overall series is separated into two subsets containing the positive and negative wing of the corresponding distribution. As illustration, the new time-series of negative and positive values of Leu-Euro returns are presented in the left side of Fig.2. In the course of analysis an important role is played by the shuffled series. The graph on the right side of Fig.2 presents an example of shuffling of the corresponding series on the left side of the figure.

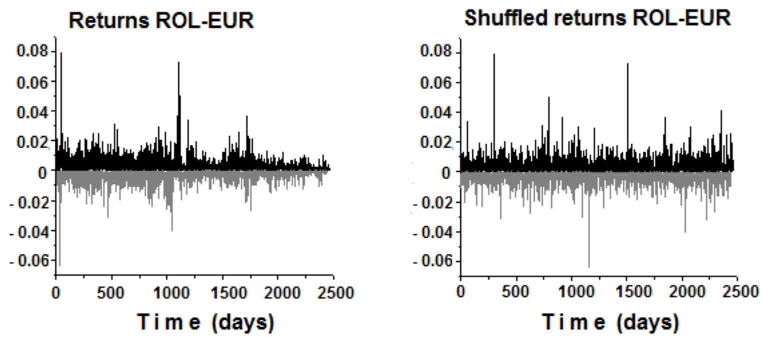

Fig.2 Time series for positive (black) and negative returns (gray) Leu-EUR (left) and shuffled series (right)

3. Method

The proposed method, based on MFDFA [11,12] and MFCCA [13,14], mainly involves the computation of the generalized Hurst exponent for various values of the *q*-order fluctuation functions.

In our analysis we follow the treatment presented in [14,18]. We denote the positive sub-series with *x*, the negative sub-series with *y* and the overall series with *z*. Each series can be characterized by a *q*-order fluctuation function:

$$F(q,s)=\left\{\frac{1}{2N_s}\sum_{\nu=1}^{2N_s}\left[F^2(s,\nu)\right]^{q/2}\right\}^{1/q} \tag{1}$$

computed as an average on the ν segments with length *s*. For the *x* series we have

$$F_x^2(s,\nu)=\frac{1}{s}\sum_{k=1}^{s}\left[X_\nu(k)-\hat{X}_\nu(k)\right]^2,\quad \nu=1,2,\ldots N_s \tag{2}$$

where the “profile” given by:

$$X(j)=\sum_{i=1}^{j}(x_i-\langle x\rangle);\quad j=1,2,\ldots,N \tag{3}$$

with $\langle x\rangle$ the mean of the respective series and where $\hat{X}_\nu(k),\ k=1,\cdots,s$ is the best polynomial fit of the signal $X_\nu(k)$ on each considered segment ν. Similar processing is performed on the *y* and *z* series.

For a statistically self-similar series, the dependence of the fluctuation function on the segment (window) *s* is expected to be power law type [8]:

$$F(q,s)\approx s^{h(q)}. \tag{4}$$

$h(q)$ is known as generalized Hurst exponent for the respective series and is computed from the log-log plot of the fluctuation function versus *s* as the slope in the scaling region. The main Hurst exponent is computed for *q*=2, *h*(2).

Additionally, a multifractal cross-correlation fluctuation function is constructed for two nonstationary series. In the case of our *x* and *y* sub-series:

$$F_{xy}^2(s,\nu)=\frac{1}{s}\sum_{k=1}^{s}\left|X_\nu(k)-\hat{X}_\nu(k)\right|\left|Y_\nu(k)-\hat{Y}_\nu(k)\right|,\quad \nu=1,2,\ldots N_s \tag{5}$$

The average value of this function for each *q*, is expected to have power law dependence on the length of the segments *s*:

$$F_{xy}(q,s)=\left\{\frac{1}{2N_s}\sum_{\upsilon=1}^{2N_s}\left[F_{xy}^2(s,\nu)\right]^{q/2}\right\}^{1/q}\approx s^{h_{xy}(q)} \tag{6}$$

where $h_{xy}(q)$ is the cross-correlation generalized Hurst exponent.

We also compute the multifractal (singularity) spectrum $f(\alpha)$ [19-22]:

$$f(\alpha)=q\alpha-\tau(q), \tag{7}$$

using the partition function exponent $\tau(q)$:

$$\tau(q)=qh(q)-1. \tag{8}$$

The singularity strength (Hölder exponent) is defined as:

$$\alpha=\frac{d\tau(q)}{dq}. \tag{9}$$

The generalized Hurst exponent represents the measure of correlations present in the fluctuations of the series. If $h(2)>0.5$ the correlations in the time series are persistent, *i. e.* an increment has higher probability of being followed by another increment. If $h(2)<0.5$ the correlations in the time series are anti-persistent, *i. e.* an increment has more chances of being followed by a decrement and vice-versa. If $h(2)=0.5$ only short range correlations or no correlations exist, as in the case of white Gaussian noise. For each order, the generalized Hurst exponent $h(q)$ is a measure for the correlation of the fluctuations related to *q, i.e.* small fluctuations for $q<0$ and large fluctuations for $q>0$. The meaning of $h(q)$ is the same as that of $h(2)$. The singularity spectrum (the right side of Figs. 3-5) gives information on the distribution of the dimension $f(\alpha)$ of subsets of the series characterized by various values of the singularity strength.

4. Results and discussions

Both Leu-USD and Leu-EUR subseries show multifractality, exhibiting monotonously decreasing $h(q)$ with q. Usually, for multifractal series, the intervals $q>0$ are characterized by smaller scaling exponents $h(q)$ than the intervals $q<0$.

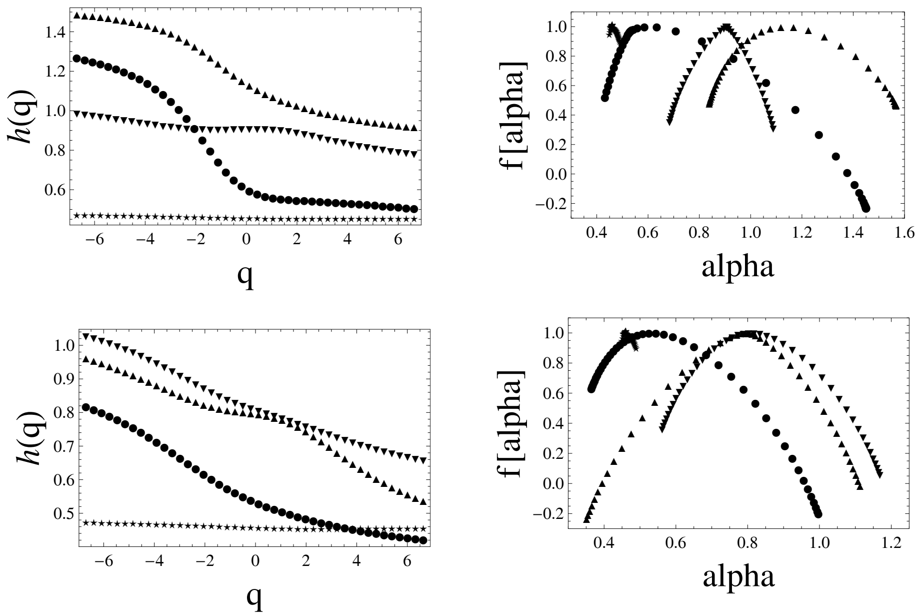

Fig.3 Multifractal analysis for Leu-USD (up) and Leu-EUR (down). Pointing up triangle - positive returns, pointing down triangle - negative returns, full circle - entire series, and star - Gaussian white noise as reference

While for the sub-series the large fluctuations are significantly correlated ($h(2) > 0.5$), for the whole series these are weakly correlated (they approach the Gaussian reference). The sub-series of negative returns of Leu-USD is close to monofractality. The interval of the values of the singularity strength encompassed by each singularity spectrum is an important characteristic of the fractality of the series (Figs.3-5 right).

The multifractality present in the time series can originate in either the difference of long range correlations for small and large fluctuations or in the broad probability mass distribution (p.m.d) for the data in the series. For uncorrelated series the multifractal character is of the second type, while for a series whose p. m. d. is regular with finite moments, multifractality is mostly first type. Discerning between the two types can be obtains by performing a shuffling of the series - randomly rearranging of the samples. This procedure clearly destroys the correlations while the p.m.d is not affected. If the shuffled series shows multifractality, this can only be an effect of the widths of the distribution.

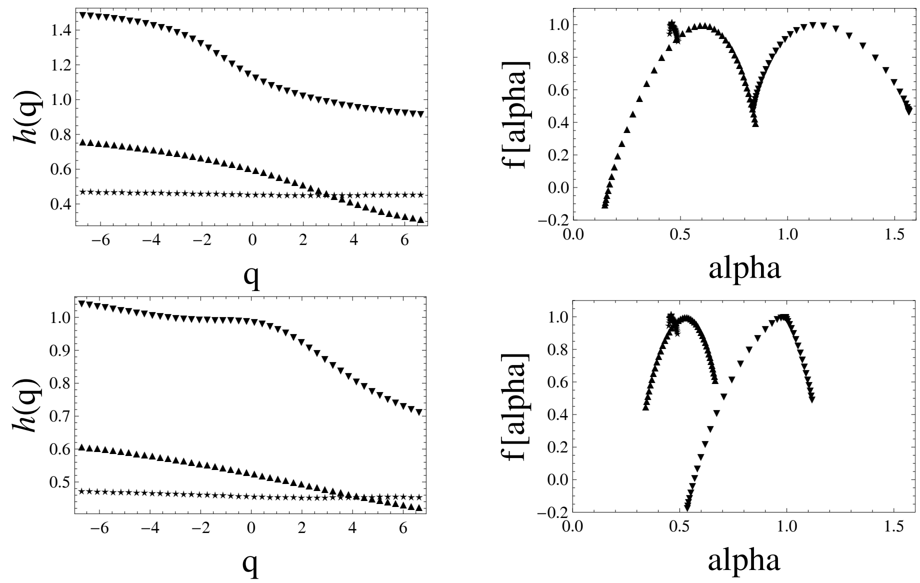

Fig.4 Multifractal analysis for Leu-USD for positive returns (up) and for negative returns (down). Pointing down triangle-original, pointing up triangle-shuffled, and star-Gaussian white noise

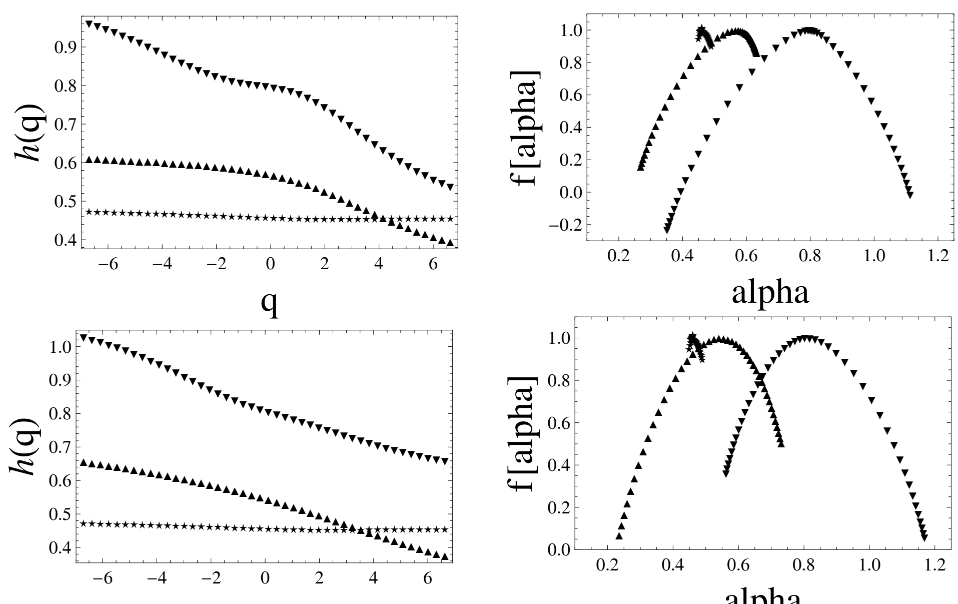

Fig.5 Multifractal analysis for LEU-EUR for positive returns (up) and for negative returns (down). Pointing down triangle - original, pointing up triangle - shuffled and star - Gaussian white noise

The difference between the generalized Hurst exponent of the original series and of the shuffled series gives the correct measure of the correlations.

Figs. 4 and 5 present the results of multifractal analysis performed on the subsets of the original. The $h(q)$ curves for the shuffled series are obtained as an average of 10 successive shufflings of the particular series.

A general remark is the asymmetry in the shape of $h(q)$ for the two sub-series also visible in the structure of the singularity spectrum. Also, we observe that the multifractality in the Leu-USD series is higher than for the Leu-Euro series, demonstrating higher correlations (Fig.3). With reference to Figs.4 and 5, we notice that in both sub-series most of the multifractality originates in the correlations although the multifractality originating in the spread of the distribution is important.

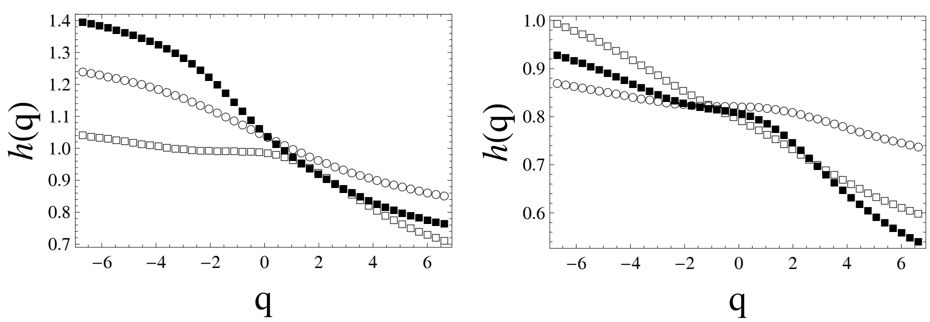

Fig.6 Multifractal cross correlation analysis for partial series: Leu-USD (left) and Leu-EUR (right); white squares-negative returns, black squares-positive returns; circle-cross correlation

The long-range cross correlation between the sub-series ($h_{xy}(2) > 0.5$) as observed from Fig.6 associated to the fact that the latter are auto-correlated series ($h_x(2) > 0.5, h_y(2) > 0.5$), imply that while each, separately, has long memory of its own previous values, additionally has a long memory of previous values of the other series.

4. Conclusion

A new method for indentifying the multifractal auto- and cross-correlations in the fluctuations of the time-series is proposed. The investigation is performed on the partial data sets obtained from the original series of returns, for positive and negative fluctuations.

Our results in the identification of correlations in exchange rate time-series are in reasonable agreement with the fact that competing economic and financial processes are essentially correlated. The application of the method to financial time-series allows estimation on the predictability of evolution. As an example, the curves in Fig.3 show that the series Leu-USD is considerable more predictable than the Leu-EURO series.

The multifractal cross-correlation between the wings of the original distributions shows that beside the existence in each series of long memory of its own previous values, additionally, a long memory of previous values of the other series is also observable.

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