# OPTIMAL MATERIAL SELECTION FOR UNIDIRECTIONAL LAMINAS USING DESIGN OF EXPERIMENT

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*În lucrarea prezentă se propune o metodă rapidă şi eficientă de selecţie a materialelor constituente pentru a obţine un strat unidirecţional compozit armat cu fibre. Bazându-se pe noţiuni teoretice şi folosind metoda experimentului programat se dezvoltă o metodă de optimizare a selecţiei acestor materiale. Folosind date ale producătorilor de materiale constituente ale compozitelor, se propune alegerea şi folosirea eficientă a acestora pentru a obţine un strat cu caracteristici precizate. Metoda poate fi aplicată cu succes atât pentru estimarea proprietăţilor optime ale materialelor pentru obţinerea unei singure proprietăţi a stratului, cât şi pentru determinarea configuraţiei optime pentru a obţine un set de proprietăţi în stratul compozit.*

*The current paper proposes a quick and effective method to select the optimal constituent materials to create a unidirectional fiber – reinforced lamina. Using consecrated theoretical design criteria, applying design of experiment method a new method was created and applied for exemplification. Using producer datasheets the scope of the method is to choose and use the best material selection to obtain a layer with specified material properties. The method not only resumes to determine optimal material selection for just one property and can be used to select the appropriate materials to ensure a set of given properties to the lamina.*

**Keywords:** material selection, lamina, composite, material selection

## 1. Introduction

Choosing adequate materials for a composite proves to be a difficult task, requiring extensive data and experience. The large amount of materials available, with a wide range of properties, makes the simple task of choosing the right materials an expanded study of material properties and interactions.

Composite materials due to their specific and unusual properties are needed for applications found in aerospace, underwater, bioengineering and transportation industries [1].

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The composite is considered any multiphase material that exhibits, with a large extent, the properties of constituent phases in such manner that a better combination of properties is accomplished [1]. Better properties are obtained by the combination of two or more distinct materials, sometimes property trade-offs are made for many composites [1].

Many composites are composed of two phases, the matrix and the dispersed phase. The properties of composites depend on the geometry of the dispersed phase, the properties of the constituent phases, and the relative amount of the phases [1].

Thus, there are many possible combinations and many variables to account for.

## 2. Theoretical background

**Design of experiment (DOE)** is a procedure for planning experiments in such manner that results are easily used to obtain valid and objective conclusions [2].

During an experiment a process variable is intentionally varied to observe its influence over one or several response variables.

Design of experiment needs to clearly define experiment objective and selecting adequate process variables to either choose a better alternative, to find influencing factors of a response, modeling a response, reaching a target, to maximize or minimize an response, and so on [2].

To obtain valid results from the DOE one should:

a. choose the right objectives
b. choose the right process variables
c. choose the right DOE
d. compare predicted and experimental data
e. analyze and compile the results

**The desirability function** (referred to as D – function in this paper) is an optimization method with multiple responses, proposed by Harrington [3]. It is a transposition of the response variable on a 0 to 1 scale.

The graphical aspect of the D – function varies, but regardless of its aspect, the "0" value indicates a totally undesired response, while "1" indicates the most favorable situation.

The D – function combines multiple responses quick and easy.

Using response estimates as input data, the target values and acceptability limits, the individual desirability is combined using the geometric mean. Depending in large amount on the result/product limit specifications, misuse of such data heavily affects the optimum response.

For multiple response optimizations, transformed responses ($r_i$) are used and combined, as previously mentioned, using geometric mean as follows:

$$D = \sqrt[n]{\prod_{i=1}^{n} r_i} \tag{1}$$

Using the geometric mean, and inherently the product of the transformed responses, in case of just one undesirable response, $r_i = 0$, the composed desirability will become null, thus, optimizing multiple responses simplifies to optimizing one response.

Let $Y_i$ be a response variable. We apply $r_i(Y_i(x)) \rightarrow [0.0;1.0]$.

To maximize a response the following definition is used:

$$r_i = \begin{cases} 0, \cdot if \cdot Y_i(x) \le Y_{\min i} \\ \left( \dfrac{Y_i - Y_{\min i}}{Y_{\max i} - Y_{\min i}} \right)^w ; \cdot if \cdot Y_{\min i} \le Y_i(x) \le Y_{\max j} \\ 1; \cdot if \cdot Y_i \ge Y_{\max i} \end{cases} \tag{2}$$

To minimize a response:

$$r_i = \begin{cases} 1, \cdot if \cdot Y_i(x) \le Y_{\min i} \\ \left( \dfrac{Y_i - Y_{\min i}}{Y_{\max i} - Y_{\min i}} \right)^w ; \cdot if \cdot Y_{\min i} \le Y_i(x) \le Y_{\max j} \\ 0; \cdot if \cdot Y_i \ge Y_{\max i} \end{cases} \tag{3}$$

For a „target is best" response:

$$r_i = \begin{cases} 0, \cdot if \cdot Y_i(x) \le Y_{\min i} \\ \left( \dfrac{Y_i - Y_{\min i}}{Y_{\max i} - Y_{\min i}} \right)^w ; \cdot if \cdot Y_{\min i} \le Y_i(x) \le Y_{\text{target}} \\ \left( \dfrac{Y_i - Y_{\min i}}{Y_{\max i} - Y_{\min i}} \right)^v ; \cdot if \cdot Y_{\text{target}} \le Y_i(x) \le Y_{\max i} \\ 1; \cdot if \cdot Y_i \ge Y_{\max i} \end{cases} \tag{4}$$

The exponents "*w*" and "*v*" determine the importance of achieving the target value.

If $w$=$v$=1 the D – function shows a linear increase to $Y_{target}$, if $w$<1 and $v$<1 the function becomes convex, while if $w$>1 and $v$>1 the function is concave.

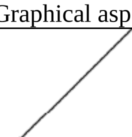
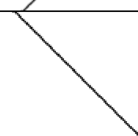
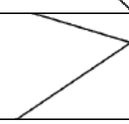
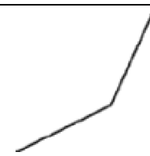
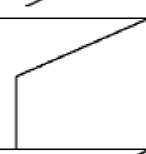
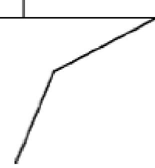
The desirability approach needs to conduct experiments and fit the models for all responses, obtain the individual desirability for each response and maximize the global desirability keeping respect of the controllable factors.

A problem rises when weighing the responses. The D – function assigns the same weight to each response. To bypass this inconvenient, one could repeatedly assign to the $Y$ variable the value of the response having a larger weight.

Several graphical representations of the D – function and the scope are illustrated in Table 1.

*Table 1*

**Several responses expected and graphical aspect of the D - function**

| Response | Graphical aspect |
|---|---|
| Maximize<br>Higher is better |  |
| Minimize<br>Lower is better |  |
| Target<br>Target is best |  |
| Maximum plateau<br>Improving up to a point, further improvements are unimportant |  |
| Maximum diminishing return<br>Improvements up to a point with slow improvements beyond |  |
| Minimum plateau<br>No improvements up to a point |  |
| Minimum diminishing return<br>Improvements up to a point followed by improvements, but not as pronounced |  |
| Desirability increase | → |

### 3. Worked out example

For ease of computing and graphical representations computer software was preferred to exemplify the method. The steps are common to all software which posses the DOE facility.

There are a large number of computer programs available that include the DOE method, including: Design Expert, Minitab, JMP and several macros are available for MS-Excel. The algorithm is for this method is easy to implement in most computing software.

The steps and parameterizations are described as programmed in the software.

The objective: find the optimal constituent material properties for a unidirectional epoxy – carbon fiber reinforced lamina.

The response variable was selected as the longitudinal tensile strength of the lamina, which, according to mechanics of materials approach, is determined using:

$$\sigma_1^T = \sigma_f V_f + \varepsilon_f E_m (1 - V_f) \tag{5}$$

And:

$$\varepsilon_f = \frac{\sigma_f}{E_f} \tag{6}$$

Where:

$\sigma_1^T$ (Found as Rm in the program) longitudinal tensile strength, in [GPa]

$\sigma_f$ (Found as Sf in the program) ultimate tensile strength of fibers, in [GPa],

$E_m$ (Found as Em in the program) Young's modulus of matrix, in [GPa],

$\varepsilon_f$ (Found as ef in the program) ultimate failure strain of fiber, in [%],

$V_f$ (Found as Vf in the program) the volume of fibers, in [%].

The influence factors are found in equation (5) and their parameterization is as follows:

*Table 2*

**Influence factors and response parameterization**

| Variable | Type | Value | | Unit |
|---|---|---|---|---|
| | | Minimum | Maximum | |
| Rm | Response | 3.2 | 3.8 | GPa |
| Vf | Influence factor | 0 | 1 | % |
| Sf | Influence factor | 0.30 [4] | 6.20 [4] | GPa |
| ef | Influence factor | 0.001667[5] | 0.020875[4] | % |
| Em | Influence factor | 4.00[5] | 6.00[5] | GPa |

According to the table above we try to find the optimal constituent materials to produce a lamina which will have a longitudinal tensile strength between 3.2 and 3.8 GPa.

The limit values for the ultimate tensile strength of fibers and Young's modulus for the matrix were found in [4] and [5] and used for the model.

The ultimate failure strain of the fiber was determined according to equation (6) using values mentioned in [4].

The limits for the fiber volume are self explanatory.

The restraints applied to the model: $V_f < 1$ and $V_f > 0.5$. These restraints improve the computation time and conform to practices to use a small amount of matrix, without jeopardizing the structural integrity of the layer.

Second order interactions were taken into consideration, whenever possible.

The model requires minimum 15 runs, but, for better accuracy, a number of 20 runs were imposed by the user.

The generated values are presented as a table with the last column, the experimental values, free for input.

These values are used to compute a theoretical longitudinal tensile strength and assigned as the response factor.

Running the automated script of the software the outputs are displayed.

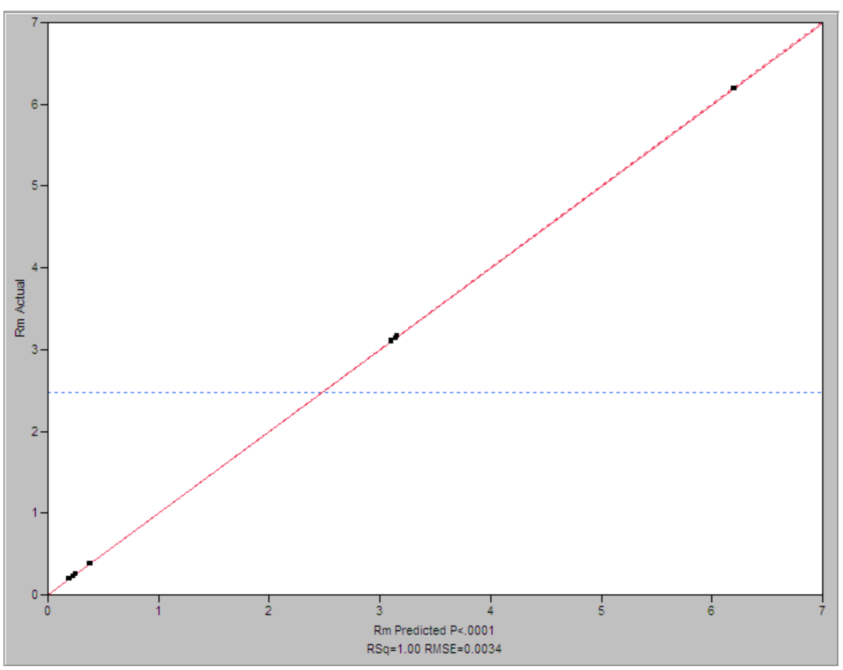



Fig. 1 Predicted vs. actual values

The plot predicted vs. actual values shows a good fit by the model. Since theoretical prediction of longitudinal tensile strength was used, the errors are negligible.

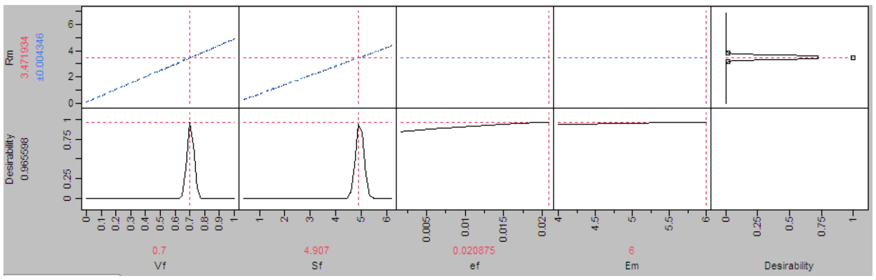


Fig. 2 Properties estimates according to the model

Fig. 2, in the first row, indicates the variation of the longitudinal tensile strength (Rm) in respect with the volume of fibers (Vf), the fiber strength (Sf), the ultimate tensile strain of the fiber (ef) and the Young's modulus of the matrix.

The second row in Fig. 2 depicts the desirability of choice; a volume of 70% fibers is most desirable, associated with fiber strength of 4.9Gpa.

The ultimate failure strain and the Young's modulus of the matrix show little importance in the current situation; variation of these parameters slightly modifies the overall desirability.

The type of output depicted in Fig. 2 is the most common type found for most of the software available.

The ultimate failure strain of the fiber shows a more significant influence than the elastic modulus of the matrix. Varying this value leads to a variation of approximately 0.1 in desirability.

*Table 3*

**Model optimized values**

| Property | Value | Unit |
|---|---|---|
| Rm | 3.471934 | GPa |
| Vf | 70 | % |
| Sf | 4.907 | GPa |
| ef | 0.020875 | % |
| Em | 6 | GPa |
| Desirability | 0.965598 | - |

To obtain a lamina with a mechanical resistance of 3.472GPa one should use a fiber volume of 70%, a fiber with a tensile strength of 4.9 GPa and a ultimate strain to failure of 0.020875 embedded in a resin with a Young's modulus of 6 GPa. The desirability of this selection of materials is 0.965598.

To verify the model the values proposed are used in equation (5). Replacing the model predicted values yields a longitudinal tensile strength of 3.472GPa, validating the model.

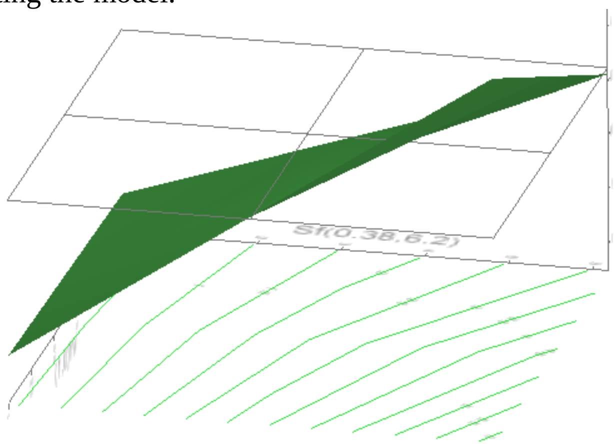


Fig. 4 Contour plots projected from the response surface of Rm as function of Vf and Sf.

The response surface was plotted in Vf – Sf coordinates ef and Em kept at constant values, those specified in Table 3.

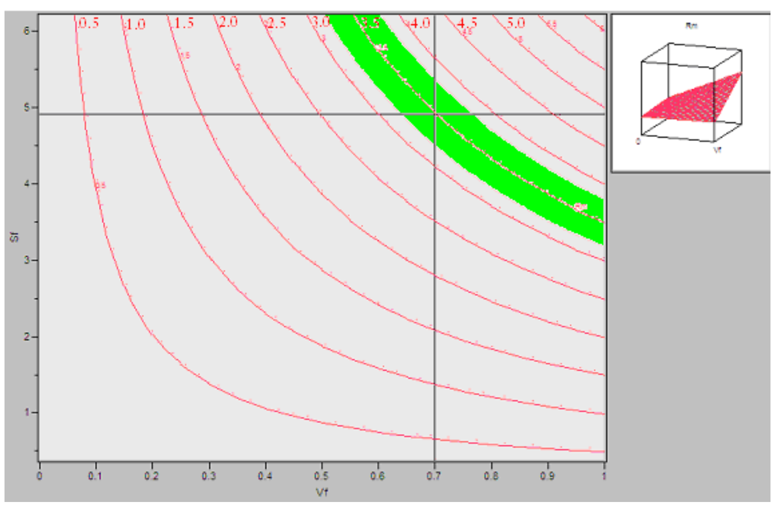



Fig. 5 Contour plots of Rm in (Vf, Sf) coordinates.
The highlighted area represents the range of the targeted values.

To obtain a tensile strength in the specified range, keeping in mind that the ultimate strain at failure of the fiber and the Young’s modulus of the matrix are kept constant, one could vary the fiber strength and the volume in the highlighted domain.

Variation of the ultimate strain at failure of the fiber and the Young's modulus of the matrix will generate similar plots, shifting the tensile strength curves either at higher or lower values.

If there are two or more properties imposed for the lamina, a similar procedure is performed.

If we consider, besides the longitudinal tensile strength, the longitudinal elastic modulus of the lamina to be found in specific range, then a new run of the method is needed.

Let us consider the targeted values from Table 4.

*Table 4*

**Lower and upper limit for the targeted range**

| Propriety | Lower limit | Upper limit | Unit |
|---|---|---|---|
| Rm | 3.2 | 3.6 | GPa |
| E1 | 400 | 500 | GPa |

From the material resistance perspective, the longitudinal elastic modulus is determined according to:

$$E_1 = E_f V_f + E_m (1 - V_f) \tag{7}$$

$E_f$ (Found as Efib in the program) - Young's modulus of the fiber, in [GPa].

The same procedure is to be followed: defining the response factors: longitudinal tensile strength and longitudinal elastic limit and specify the range of interest (table 4). A new influence fiber is needed, the Young's modulus of the fiber, besides the ones already used in the first run. The range for Efib is set as 170-965GPa, these values found in [4], [5]. The same constraints and number of iterations are used.

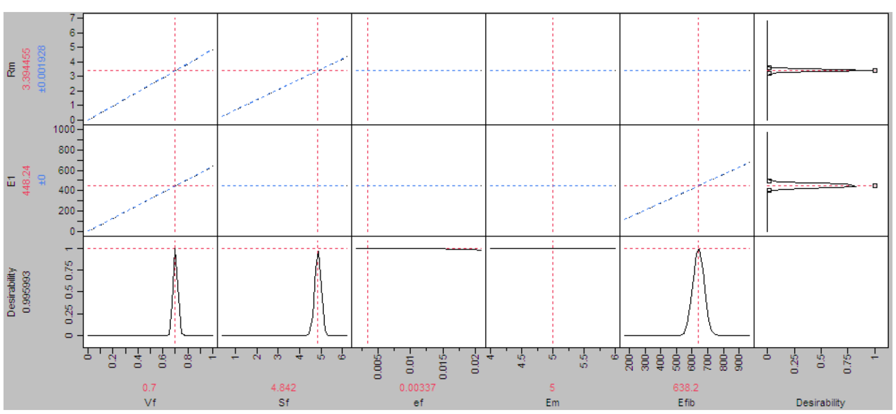


Fig. 6 Properties estimates according to the model

As seen in Fig. 6, the desirability is near the maximum for both responses.

The material properties of the constituents which could ensure the targeted longitudinal tensile strength and longitudinal elastic modulus are resumed in Table 5.

*Table 5*

**Model optimized values**

| Property | Value | Unit |
|---|---|---|
| Rm | 3.471934 | GPa |
| Vf | 0.7 | % |
| Sf | 4.842 | GPa |
| ef | 0.0033 | % |
| Em | 5 | GPa |
| Efib | 638.2 | GPa |
| Desirability | 0.995993 | - |

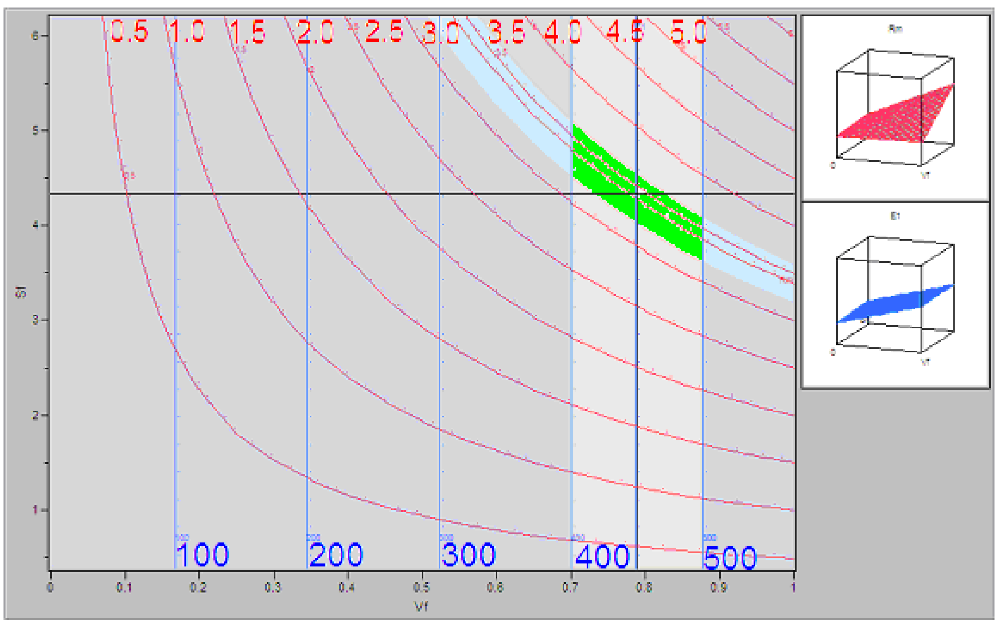



Fig. 7 Contour plots of Rm and E1 in (Vf, Sf) coordinates.
The highlighted area represents the range of the targeted values.

Keeping the Young's modulus of the matrix and the ultimate fiber strain of the matrix fixed, the projection of the response surfaces for the tensile strength and the longitudinal elastic modulus is shown in Fig. 7. The highlighted area represents the intersection of the range for tensile strength of the fiber and the elastic modulus of the fiber.

*Table 6*

**Comparison of results generated by the model**

| Property | Value | | Unit |
|---|---|---|---|
| | Target: Rm | Target: Rm and E1 | |
| Rm | 3.471934 | 3.471934 | GPa |
| Vf | 0.7 | 0.7 | % |
| Sf | 4.907 | 4.842 | GPa |
| ef | 0.020875 | 0.0033 | % |
| Em | 6 | 5 | GPa |
| Efib | - | 638.2 | GPa |
| Desirability | 0.965598 | 0.995993 | - |

For demonstration purposes, the same longitudinal tensile strength was targeted and the value for the fiber volume maintained.

To achieve the same tensile strength and a specified longitudinal modulus, the fiber strength falls as well as the ultimate failure strain of the fiber.

## 6. Conclusions

Data entry is crucial for the model, since faulty inputs will generate erroneous results.

Additional constraints to the model may improve its accuracy, but great care must be taken not to over-constrain it.

Some constraints are needed for the model to use the true material properties.

The maximum desirability is found at a target value represented by the mean of the range specified, but can be altered to user specifications.

The model provides multiple solutions, some with no practical or physical sense, thus a minimum knowledge of composite theory is required.

The model allows target response, maximization or minimization or any other type of scope.

If no combination is desirable, one should change the materials questioned.

The model supports multiple response factors and influence factors and the construction remains the same as presented.

This model was destined for polymer fiber reinforced composites, and should be used as.

This method is quick and easy to perform, especially due to the great number of software which includes DOE.

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