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CONTROL OF THE SATELLITES' ATTITUDE USING A PYRAMIDAL CONFIGURATION OF FOUR VARIABLE SPEED CONTROL MOMENT GYROS

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In this paper, the authors propose a new architecture for the control of the satellites' attitude by using a control law mainly based on a proportional-integrator component with respect to the quaternion error vector and to the satellite's angular velocity error vector. The designed control law modifies the cluster's equivalent gyroscopic moment, the equivalent kinetic moment and the angular velocities' vector, this leading to the modification of the quaternion vector and to the change of the satellite's attitude. The software implementation and validation of the new control architecture is achieved by using the Matlab/Simulink environment.

Keywords: satellite, variable speed control moment gyro, attitude control.

1. Introduction

Small satellites are becoming popular due to their low cost of development and shorter realization time. As a result, there has been a lot of effort to push satellite technology to smaller sizes and mass which would enable small satellites to accomplish missions to complement the larger satellites. The two major components of the attitude control system are *the actuator* and the *control algorithm* [1]. The satellites must have good rotational handling and agility to answer well to multiple tasks; to control the motion of the satellites, these must be equipped with an automatic system for the control of their attitude. First, the satellite must be oriented through the rapid action of its thrusters; these produce large moments. During its motion, the satellite must track the Sun and a terrestrial station; the maneuver of the two targets' tracking means the rotation of the satellite from its initial position to a target frame (the initialization maneuver). Then, by means of the VSCMGs (variable speed control moment gyros) or other type of actuators, the satellite tracks a reference frame. The aims of an attitude controller are: *attitude stabilization* (the process of maintaining an existing orientation) and *attitude maneuver control* (the process of controlling the satellite's orientation from one attitude to another) [2].

The parameterization of satellites' attitude is mainly described by the cosine rotation matrix which is associated to the orthonormal group *SO*(3) [3]; the Euler

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equations associated to the satellite are sometimes difficult to use and, therefore, the attitude's dynamics must be put into a double integrator form with respect to the parameters describing the satellite's attitude [4]. In this paper, to describe and control the attitude of the satellite, one uses an attitude matrix expressed in terms of quaternion in order to avoid singularities for the large values of the Euler angles and a large amount of calculation in the case of trigonometric functions' usage.

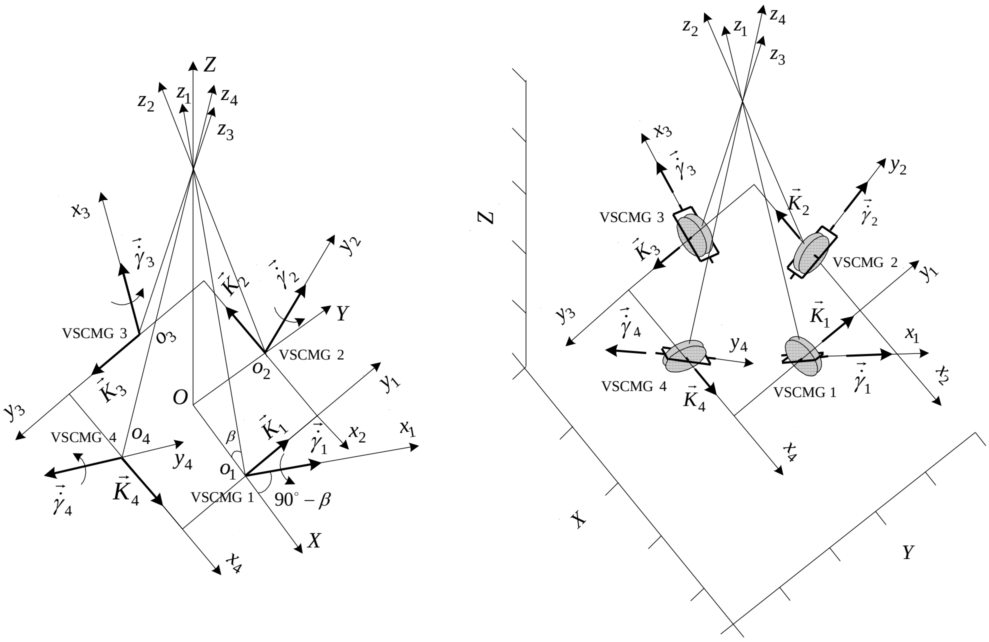
The type and also the number and arrangement of VSCMGs used by the attitude control systems are a compromise between performance, cost, reliability, performance and control algorithm's complexity. The VSCMGs are most efficient in terms of generation torque but these require at least four units to ensure control on the three axes avoiding singularities [2]; thus, a pyramidal configuration (cluster of VSCMGs) results. Each pyramid face is inclined at an angle $\beta \cong 55\,\mathrm{deg}$ from the horizontal, which gives an angular momentum envelope almost spherical.

The present study involves the design of a new architecture for the control of the satellites' attitude by using a proportional-derivative control law depending on the satellite's quaternion and angular velocity vectors, a reference model and, as the actuators' system, a pyramidal cluster consisting of four VSCMGs. The new architecture is implemented and validated through complex numerical simulations for the case of a mini-satellite involved in a typical maneuver around its own axis.

2. Pyramidal configuration with four VSCMGs

Control moment gyroscope (CMG) is a spacecraft attitude control actuator acting as a torque amplifier. It is suitable for three axis slew maneuvering by providing the necessary torques via gambling a spinning flywheel. The two main components of a gyroscope are the flywheel (a spinning rotor with inertia sufficient to provide the desired angular momentum) and the gimbal (a pivot about which the flywheel assembly can be rotated). The magnitude of torque produced is directly proportional to the inertia of the flywheel, the angular speed of the flywheel and the rate of rotation of the gimbal. Experimental results indicated the potential benefits of using CMGs. Specifically, a cluster of four single gimbal CMGs is used to practically demonstrate full 3-axis control for a microsatellite class spacecraft [5].

We consider the motion of a mini-satellite which performs a typical maneuver (a complete cycle) around its own axis (with constrained angular speed); the three phases of motion (the accelerated angular motion, the uniform angular motion and the braked motion) are described in detail in [6]. To control the satellite, a gyro system (a cluster having pyramidal configuration), consisting of four control moment gyros, is used. The pyramidal configuration is presented in Fig. 1. The gyros' rotation axes (the kinetic moments $\vec{K}_i, i=\overline{1,4}$) are initially oriented parallel to the sides of the pyramid base, the axes of the gyroscopic frames (the angular velocities associated to the frames' rotations $\vec{\dot{\gamma}}_i, i=\overline{1,4}$) are perpendicular to the side faces of the pyramid, while the transversal axes of the VSCMGs are perpendicular to the gyro axes and gyroscopic frames. In Fig. 2, one presents the rotations of the gyroscopic frames, the angular variables, the kinetic moments and the gyroscopic couples. The frame S (satellite linked) is denoted with *OXYZ*. Initially (in the absence of the gyroscopic frames' rotations), we have $\gamma_i = 0$; the frames are oriented towards the axes of the frames $o_i x_i y_i z_i$. The rotations of the gyroscopic frames with respect to the initial positions, the obtaining of the four gyroscopic moments, the projections of the gyroscopic couples on the axes of the frame S (*OXYZ* – satellite linked) have been deduced in [7].



a. b.

Fig. 1. Pyramidal configuration with four VSCMGs

3. Dynamics of the satellite using pyramidal clusters with *N* VSCMGs

The pyramidal system in fig. 1 is a particular case of the general cluster having *N* VSCMGs. In this section, one presents the main equations associated to the dynamics of the satellite. One denotes with $\boldsymbol{\gamma} = [\gamma_1 \quad \gamma_2 \quad \dots \quad \gamma_N]^T -$ the vector of rotation angles of the gyroscopic frames, $\boldsymbol{\Omega} = [\Omega_1 \quad \Omega_2 \quad \dots \quad \Omega_N]^T -$ the vector of angular rates of the gyros, $\dot{\boldsymbol{\gamma}} -$ the vector of angular rates of the frames and $\dot{\boldsymbol{\Omega}} -$ the vector of gyros' angular accelerations, $I_{g^*} = \text{diag}\left[I_{g_1^*} \quad I_{g_2^*} \quad \dots \quad I_{g_N^*}\right] -$ the matrix of inertia moments of the frames, $I_{r^*} = \text{diag}\left[I_{r_1^*} \quad I_{r_2^*} \quad \dots \quad I_{r_N^*}\right] -$ the matrix of inertia moments of the rotors and $I_{c^*} = I_{g^*} + I_{r^*} -$ the matrix of inertia moments of the ensembles frames-rotors; the sign "*" is *g*, *s*, *t* and it means the axis for the expressing the inertia moments (*g* – the axis of the gyro frame, *s* – the axis of own rotation of the gyro and *t* – the axis which is perpendicular to the axes *g* and *s*). Let $B_*(3\times N)$ be the transformation matrix having on each of the *N* columns corresponding of the *N* VSCMGs three elements (the cosines of the angles between the axes * of the "*N*" VSCMG and the axes of the satellite linked frame); only the matrices B_s and B_t depend on $\boldsymbol{\gamma}$. Having in mind all of these, the satellite inertia moment is [8]

$$J = J_b + B_s I_{cs} B_s^T + B_g I_{cg} B_g^T + B_t I_{ct} B_t^T \,, \tag{1}$$

where J_b is the matrix of the satellite's inertia moments without clusters; also, $I_{cs} = I_{gs} + I_{rs}, I_{cg} = I_{gg} + I_{rg}, I_{ct} = I_{gt} + I_{rt}$.

The total kinetic moment $(\boldsymbol{K}_c)$ of the *N* VSCMGs cluster with respect to the satellite's platform depends on kinetic moments of the VSCMGs with respect to the own rotation axes of the gyros' rotors $(I_{rs}\boldsymbol{\Omega})$ and on the kinetic moments of the VSCMGs with respect to gyroscopic frames' axes $(I_{cg}\dot{\boldsymbol{\gamma}})$ using the transformation matrices B_s and B_g, i.e.

$$\boldsymbol{K}_c = B_s I_{rs}\boldsymbol{\Omega} + B_g I_{cg}\dot{\boldsymbol{\gamma}} = \begin{bmatrix} B_g I_{cg} & B_s I_{rs} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{\gamma}} & \boldsymbol{\Omega} \end{bmatrix}^T . \tag{2}$$

Thus, the command kinetic moment is obtained by the modification of $\dot{\boldsymbol{\gamma}}$ and $\boldsymbol{\Omega}$. The satellite's absolute kinetic moment $\boldsymbol{K}$ is the resultant between the kinetic moment of the satellite's platform $J\boldsymbol{\omega}$ and the relative one $\boldsymbol{K}_c$, as below:

$$\boldsymbol{K} = J\boldsymbol{\omega} + \boldsymbol{K}_c = J\boldsymbol{\omega} + B_s I_{rs}\boldsymbol{\Omega} + B_g I_{cg}\dot{\boldsymbol{\gamma}} \,, \tag{3}$$

with $\boldsymbol{\omega} = \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 \end{bmatrix}^T -$ the absolute angular rates of the satellite with respect to its axes. With these, the satellite's dynamics is described by the equation: $\dot{\boldsymbol{K}} = \boldsymbol{\omega}^\times \boldsymbol{K} = \boldsymbol{u}, \boldsymbol{u} = \boldsymbol{u}_k + \boldsymbol{u}_g + \boldsymbol{u}_p$, where $\boldsymbol{u}$ is the vector of exterior moments applied to the satellite, $\boldsymbol{u}_k -$ the vector of command moments, $\boldsymbol{u}_g -$ the vector of moments due to the gravitational force, while $\boldsymbol{u}_p -$ the vector of disturbing moments. By time derivation of equation (3) and taking into account that *J* is less dependent on $\boldsymbol{\gamma}$, i.e. $\dot{J} = \dot{J}(\gamma)\dot{\gamma}$ can be neglected, it results: $\dot{\boldsymbol{K}} \cong J\dot{\boldsymbol{\omega}} + \dot{\boldsymbol{K}}_c$ and $J\dot{\boldsymbol{\omega}} + \dot{\boldsymbol{K}}_c + \boldsymbol{\omega}^\times \boldsymbol{K} = + \boldsymbol{\omega}^\times \boldsymbol{K} = \boldsymbol{u}_g + \boldsymbol{u}_p$; the component $\boldsymbol{u}_k$ is expressed through a sum of gyroscopic moments $(\dot{\boldsymbol{K}}_c + \boldsymbol{\omega}^\times \boldsymbol{K}_c)$ and, therefore, one gets: $J\dot{\boldsymbol{\omega}} = \boldsymbol{u}_g + \boldsymbol{u}_p$; in the following calculations, $\boldsymbol{u}_g$ and $\boldsymbol{u}_p$ will be omitted without ignore them.

The matrices B_* at the moment t=0 will be denoted with B_{*0}. Then, at moment t, one has:

$$B_g = B_{g0}, B_s = B_{s0}[\cos\boldsymbol{\gamma}]_d \pm B_{t0}[\sin\boldsymbol{\gamma}]_d,$$
$$B_t = \mp B_{s0}[\sin\boldsymbol{\gamma}]_d + B_{t0}[\cos\boldsymbol{\gamma}]_d, \tag{4}$$

where $\cos\boldsymbol{\gamma} = [\cos\gamma_1 \quad \cos\gamma_2 \quad \dots \quad \cos\gamma_N]^T, \sin\boldsymbol{\gamma} = [\sin\gamma_1 \quad \sin\gamma_2 \quad \dots \quad \sin\gamma_N]^T$ and $[x]_d$ – diagonal matrix $(N \times N)$, with the form: $[x]_d = \mathrm{diag}[x_1 \quad x_2 \quad \dots \quad x_N]$, where $x = \cos\boldsymbol{\gamma}$ or $x = \sin\boldsymbol{\gamma}$. By time derivation of equation (4) and taking into account the followings: $\dot{B}_g = 0, \dot{B}_s = -B_{s0}[\sin\boldsymbol{\gamma}]_d[\dot{\gamma}]_d \pm B_{t0}[\cos\boldsymbol{\gamma}]_d[\dot{\gamma}]_d = \pm B_t[\dot{\gamma}]_d, \dot{B}_t =$ $= \mp B_{s0}[\cos\boldsymbol{\gamma}]_d[\dot{\gamma}]_d - B_{t0}[\sin\boldsymbol{\gamma}]_d[\dot{\gamma}]_d = \mp B_s[\dot{\gamma}]_d$ and $[\dot{\gamma}]_d\boldsymbol{\Omega} = [\boldsymbol{\Omega}]_d\dot{\boldsymbol{\gamma}}$, one obtains:

$$\dot{\boldsymbol{K}}_c = B_s I_{rs}\dot{\boldsymbol{\Omega}} + \dot{B}_s I_{rs}\boldsymbol{\Omega} + B_g I_{cg}\ddot{\boldsymbol{\gamma}} \cong \pm B_t I_{rs}[\boldsymbol{\Omega}]_d\dot{\boldsymbol{\gamma}} + B_s I_{rs}\dot{\boldsymbol{\Omega}}; \tag{5}$$

one also had in mind that $I_{rs} \cong \text{const.}, I_{cg} \cong \text{ct.}, B_g I_{cg}\ddot{\boldsymbol{\gamma}}$ – negligible.

With (5), the dynamics of the satellite $(J\dot{\boldsymbol{\omega}} + \dot{\boldsymbol{K}}_c + \boldsymbol{\omega}^\times\boldsymbol{K} = \boldsymbol{u}_g + \boldsymbol{u}_p)$ becomes:

$$J\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}^\times\boldsymbol{K} + C\dot{\boldsymbol{\gamma}} + D\dot{\boldsymbol{\Omega}} = 0 \Leftrightarrow J\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}^\times\boldsymbol{K} = \boldsymbol{M}_c, \tag{6}$$

with $\boldsymbol{K}$ of form (3) and

$$C = \pm B_t I_{rs}[\boldsymbol{\Omega}]_d, D = B_s I_{rs}. \tag{7}$$

4. Design of a new control system with pyramidal cluster of VSCMGs, reference model and PD control law

To control the satellite, a cluster having pyramidal configuration, consisting of four control moment gyros, is used. In this section, the control of the satellite's attitude will be achieved by means of a complex system; it is mainly based on a proportional-integrator (PD) control law, a reference model and, as the actuators' system, the cluster presented in the previous section; the actuators' saturation will be considered both from the generated gyroscopic couples' point of view and from the gyroscopic frame angular velocities' point of view.

One chooses the following candidate of Lyapunov function: $V = \frac{1}{2}\boldsymbol{\omega}_e^T J^{-1}\boldsymbol{\omega}_e +$ $+ 2k_p \ln(1 + \boldsymbol{q}_e^T\boldsymbol{q}_e), k_p > 0$. The control law must assure the convergences $\boldsymbol{\omega}_e \to 0$ and $\boldsymbol{q}_e \to 0$, for $t \to \infty$, with $\boldsymbol{\omega}_e = \boldsymbol{\omega}_d - \boldsymbol{\omega}$ ($\boldsymbol{\omega}_d$ – the reference (desired) angular rate of the satellite expressed with respect to the inertial frame) and $\boldsymbol{q}_e = [q_{e1} \quad q_{e2} \quad q_{e3}]^T$ – the attitude error quaternion which is the solution of the equation $\dot{\boldsymbol{q}}_e = F(\boldsymbol{q}_e)\boldsymbol{\omega}_e$; $F(\boldsymbol{q}_e)$ has the form:

$$\boldsymbol{F}(\boldsymbol{q})=\frac{1}{2}\left\{I_{3\times 3}+\boldsymbol{q}_e^{\times}+\boldsymbol{q}_e\boldsymbol{q}_e^T-\frac{1}{2}\left[1+\boldsymbol{q}_e^T\boldsymbol{q}_e\right]I_{3\times 3}\right\},\boldsymbol{q}_e^{\times}=\begin{bmatrix}0 & -q_{3e} & q_{2e}\\ q_{3e} & 0 & -q_{1e}\\ -q_{2e} & q_{1e} & 0\end{bmatrix}. \qquad (8)$$

By time derivation of the Lyapunov function and imposing $\dot{V}=-k_d\boldsymbol{\omega}_e^T\boldsymbol{\omega}_e$, with k_d>0, one gets: $\dot{V}=\boldsymbol{\omega}_e^TJ\dot{\boldsymbol{\omega}}_e+k_p\boldsymbol{\omega}_e^T\boldsymbol{q}_e=-k_d\boldsymbol{\omega}_e^T\boldsymbol{\omega}_e\le 0$, for all $\boldsymbol{\omega}_e$ which implies the stability of the equilibrium point $\boldsymbol{\omega}_e=0$. From the condition: $J\dot{\boldsymbol{\omega}}_e+k_d\boldsymbol{\omega}_e+k_p\boldsymbol{q}_e=0$, it results $J\dot{\boldsymbol{\omega}}_e$, which is then replaced in (6) to obtain: $J\dot{\boldsymbol{\omega}}_d+k_d\boldsymbol{\omega}_e+k_p\boldsymbol{q}_e+\boldsymbol{\omega}^{\times}\boldsymbol{K}+C\dot{\boldsymbol{\gamma}}+D\dot{\boldsymbol{\Omega}}=0$. Now, from (6), the command moment $\boldsymbol{M}_c=-\left(C\dot{\boldsymbol{\gamma}}+D\dot{\boldsymbol{\Omega}}\right)=-[C \quad D]\left[\dot{\boldsymbol{\gamma}} \quad \dot{\boldsymbol{\Omega}}\right]^T$, is expressed; also, by using one of the above equations, one gets:

$$\boldsymbol{M}_c=J\dot{\boldsymbol{\omega}}_d+k_d\boldsymbol{\omega}_e+k_p\boldsymbol{q}_e+\boldsymbol{\omega}^{\times}\boldsymbol{K}. \qquad (9)$$

With the notation $\boldsymbol{u}_c=\left[\dot{\boldsymbol{\gamma}} \quad \dot{\boldsymbol{\Omega}}\right]^T$, it results:

$$\boldsymbol{u}_c=-[C \quad D]^{+}\boldsymbol{M}_c=-Q^{+}\boldsymbol{M}_c, \qquad (10)$$

with $Q_{(3\times 2N)}=\left[C_{(3\times N)} \quad D_{3\times N}\right], Q^{+}$ – the pseudo-inverse of the matrix Q and $\boldsymbol{M}_c$ of form (9). From the methods for the calculation of the pseudo-inverse of the matrix Q, we can choose the robust type inverse and the equation: $Q^{+}=Q^T\left(QQ^T+\lambda I\right)^{-1}$, with I – the identity matrix. This formula does not always guarantee the avoidance of VSCMGs' singularities. Such a VSCMG can get into the singularity zone in the presence of sensor noise. Furthermore, if the system is controlled such that it goes into the singularity zone, it cannot escape from this state. Therefore, to remove this drawback, we will use a simple but effective equation for the calculation of the pseudo-inverse of matrix Q [9], [10]; it is designed especially for reorientation maneuvers, stage in which it is not necessary to achieve certain accuracy. The pseudo-inverse is obtained by using the formula:

$$Q^{+}=Q^T\left(QQ^T+\lambda \mathrm{E}\right)^{-1}, \qquad (11)$$

where

$$\mathrm{E}=\begin{bmatrix}1 & \varepsilon_3 & \varepsilon_2\\ \varepsilon_3 & 1 & \varepsilon_1\\ \varepsilon_2 & \varepsilon_1 & 1\end{bmatrix};\varepsilon_i=0.01\sin\left(\frac{\pi}{2}t+\Phi_i\right), i=\overline{1,3}; \lambda=0.01\exp\left[-10\det\left(QQ^T\right)\right] \qquad (12)$$

and $\Phi_1=0,\Phi_2=\frac{\pi}{2},\Phi_3=\pi$; this guarantees that $Q^{+}\boldsymbol{u}_k=Q^{+}\dot{\boldsymbol{K}}\neq 0$. This solution does not ensure the avoidance of singularity, but rather its proximity and transit.

The new automatic system for the control of satellite's attitude, using a pyramidal cluster with N=4 VSCMGs, reference model and PD control law is presented in fig. 2. One must specify that the kinetic moment $\boldsymbol{K}_c$ is used to apply commands to the satellite. Thus, in its absence, the satellite's command moment is null and the equation of the satellite becomes: $\dot{\boldsymbol{K}} = -\boldsymbol{\omega}^{\times}\boldsymbol{K}$, with [11]:

$$\boldsymbol{\omega}^{\times} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}. \tag{13}$$

where $\omega_1, \omega_2, \omega_3$ are the components of the satellite's angular velocity upon the axes of the $OXYZ$ frame.

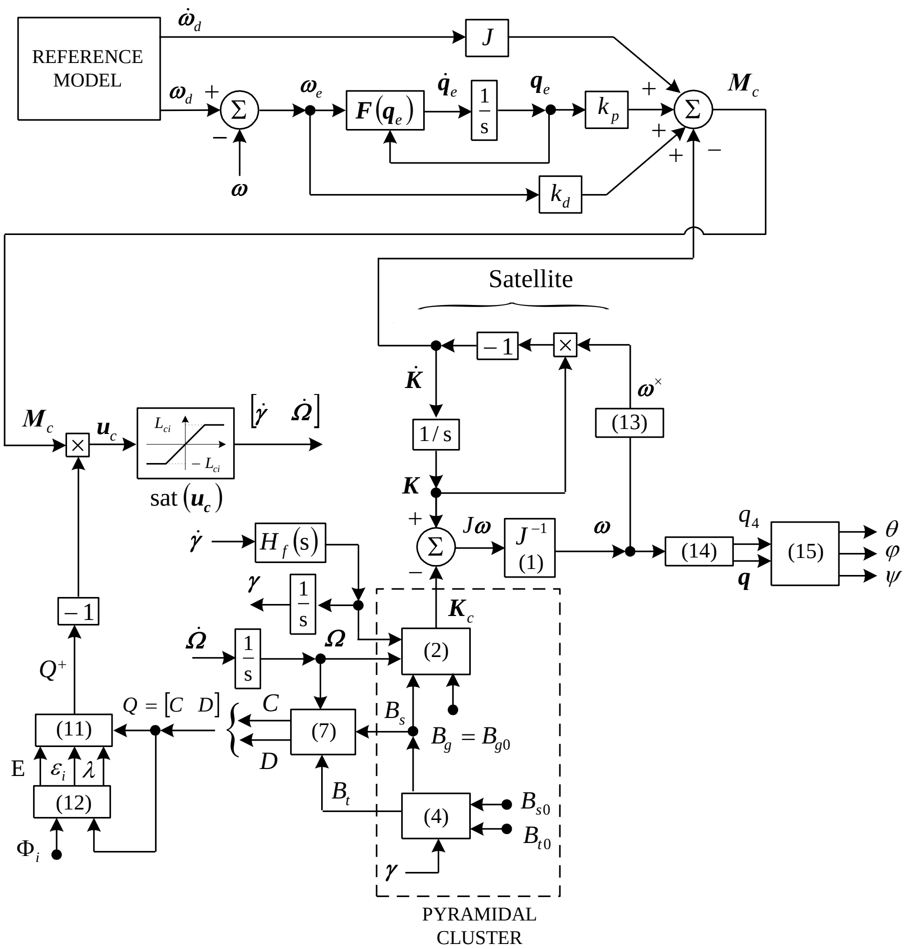


Fig. 2. The new automatic system for the control of satellite's attitude, using a pyramidal cluster with N=4 VSCMGs, reference model and PD control law

The satellite's attitude (Euler angles – θ, φ and ψ) may be defined by means of two quaternion vectors ($\boldsymbol{q} = [q_1 \quad q_2 \quad q_3]^T$ and $\hat{\boldsymbol{q}} = [q_1 \quad q_2 \quad q_3 \quad q_4]^T$); the significances of these angles are similar to the ones expressing the attitude of an aircraft with respect to the Earth tied frame: φ is associated to the roll angle, θ– associated to the pitch angle and ψ– associated to the direction angle. The differential equations of the quaternions are [12]:

$$\begin{aligned} \dot{\boldsymbol{q}} &= -\frac{1}{2}\boldsymbol{\omega}^{\times}\boldsymbol{q} + \frac{1}{2}q_4\boldsymbol{\omega}, \\ \dot{q}_4 &= -\frac{1}{2}\boldsymbol{\omega}^T\boldsymbol{q}; \end{aligned} \tag{14}$$

the correlation formulas between the components of the quarternion vector $\hat{\boldsymbol{q}}$ and the satellite's attitude angles are [13]:

$$\begin{aligned} \theta &= \operatorname{atan}\frac{2(q_1q_3 + q_2q_4)}{-q_1^2 - q_2^2 + q_3^2 + q_4^2}, \\ \varphi &= \operatorname{asin}\left[2(q_1q_4 - q_2q_3)\right], \\ \psi &= \operatorname{atan}\frac{2(q_1q_2 + q_3q_4)}{-q_1^2 + q_2^2 - q_3^2 + q_4^2}. \end{aligned} \tag{14}$$

5. Numerical simulation results

In this section, the satellite's attitude control system (fig. 2), using a cluster consisting of four VSCMGs (fig. 1), is software implemented and validated in Matlab/Simulink environment, for the case of a mini-satellite. The attitude of the satellite (the angles θ, φ and ψ) are controlled by means of the quaternion vectors $\boldsymbol{q}$ and $\hat{\boldsymbol{q}}$, of the total kinetic moment $(\boldsymbol{K}_c)$ and of the command moment $\boldsymbol{M}_c$; it is used for the obtaining of the PD control law (equation (10)). The gyroscopic moment $\boldsymbol{M}_c$ modifies the equivalent kinetic moment vector $\boldsymbol{K}$ of the satellite (see equation (6)) and, after that, according to (3), it also modifies the vector of angular velocities $(\boldsymbol{\omega})$. According to (14), the modification of the vector $\boldsymbol{\omega}$ leads to other expressions of the quaternion vectors ($\boldsymbol{q}$ and $\hat{\boldsymbol{q}}$); the modification of the quaternions is equivalent with the change of the satellite's attitude (see equation 15)).

For the system in fig. 2, the following matrices, vectors and initial values have been used:

$$J = \begin{bmatrix} 15.05 & 3 & -1 \\ 3.08 & 6.5 & 2 \\ -1 & 2 & 11.2 \end{bmatrix} \text{kg} \cdot \text{m}^2, \boldsymbol{\omega}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{rad/s}, \dot{\boldsymbol{\omega}}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{rad/s}^2, \boldsymbol{q}_e(0) = \begin{bmatrix} q_{e1}(0) \\ q_{e2}(0) \\ q_{e3}(0) \end{bmatrix} =$$

$$= \boldsymbol{q}(0) = \begin{bmatrix} 0.45 \\ 0.5 \\ -0.5 \end{bmatrix}, \gamma(0) = \begin{bmatrix} \pi/2 \\ -\pi/2 \\ -\pi/2 \\ \pi/2 \end{bmatrix} \text{rad}, \dot{\gamma}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{rad/s}, \boldsymbol{\Omega}(0) = \begin{bmatrix} 133.33 \\ 113.33 \\ 100 \\ 86.66 \end{bmatrix} \text{rad/s}, \; N = 4,$$

$$\beta \cong 55\,\text{grd}, \cos\beta \cong 0.6, \sin\beta \cong 0.8, B_g = B_{g0} = \begin{bmatrix} \sin\beta & 0 & -\sin\beta & 0 \\ 0 & \sin\beta & 0 & -\sin\beta \\ \cos\beta & \cos\beta & \cos\beta & \cos\beta \end{bmatrix},$$

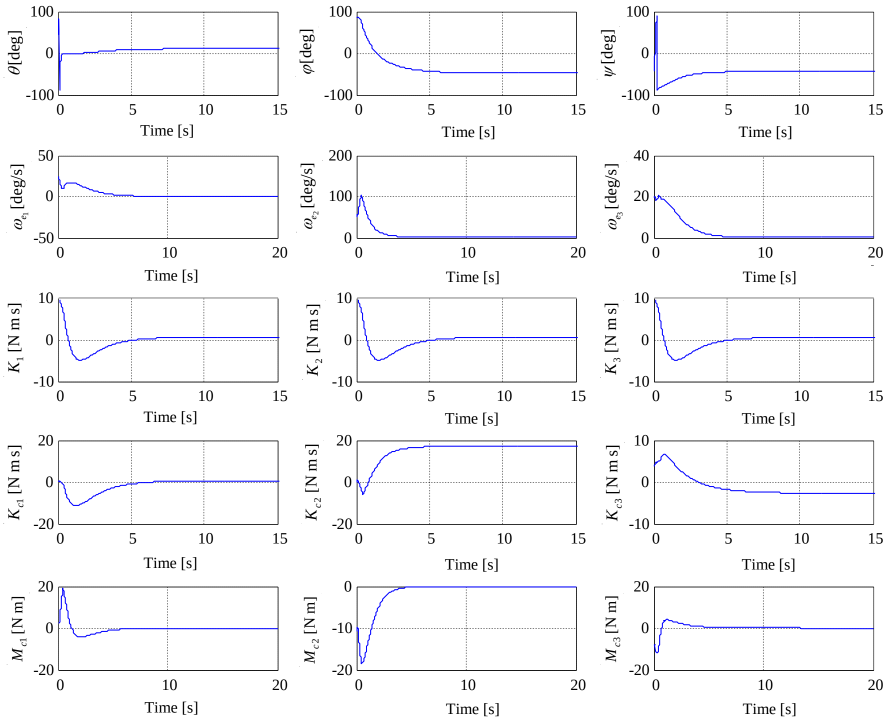
$$B_{s0} = B_s(0) = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B_{t0} = B_t(0) = \sin\beta \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix};$$

the matrices B_s and B_t have the forms (4), $I_{rs} = \text{diag}\,[0.7 \quad 0.7 \quad 0.7 \quad 0.7]\,\text{kg m}^2$, $I_{rg} = I_{rt} = \text{diag}\,[0.4 \quad 0.4 \quad 0.4 \quad 0.4]\,\text{kg m}^2, I_{gs} = I_{gg} = I_{gt} = \text{diag}\,[0.1 \quad 0.1 \quad 0.1 \quad 0.1]\text{kg m}^2$, $I_{cs} = I_{gs} + I_{rs} = \text{diag}\,[0.8 \quad 0.8 \quad 0.8 \quad 0.8]\text{kg m}^2, I_{cg} = I_{gg} + I_{rg} = I_{ct} = I_{gt} + I_{rt} =$ $= \text{diag}\,[0.5 \quad 0.5 \quad 0.5 \quad 0.5]\text{kg m}^2$; with these, using the equation (1), the matrix J is obtained. The vector $\boldsymbol{K}(0)$ is calculated with (3), i.e.: $\boldsymbol{K}(0) = J(0)\boldsymbol{\omega}(0) + B_s(0)I_{rs}\boldsymbol{\Omega}(0) +$ $+ B_g(0)I_{cg}\dot{\gamma}(0)$. To control the satellite's attitude, Q^+ is calculated with (11) and (12). Reference model's outputs are: $\boldsymbol{\omega}_d(t) = 0.02\begin{bmatrix} \sin\frac{2\pi}{800}t & \sin\frac{2\pi}{600}t & \sin\frac{2\pi}{400}t \end{bmatrix}^T$ rad/s, while $\dot{\boldsymbol{\omega}}_d(t)$ is the derivative of the $\boldsymbol{\omega}_d(t)$. The coefficients of the PD control law (k_p and k_d) are calculated by using the Ziegler-Nichols method; it resulted: $k_p = 4$ and $k_d = 10$. The limits of the saturation zone (nonlinearity $\text{sat}\,\boldsymbol{u}_c$ with 45 deg slope) are: $L_c = [L_{c1} \quad L_{c2} \quad L_{c3} \quad L_{c4} \quad L_{c5} \quad L_{c6}]^T = [[1 \quad 1 \quad 1]\text{rad/s} \quad [1 \quad 1 \quad 1]\text{rad/s}^2]$. The vector of the angular rates' signals $(\dot{\gamma})$ is applied to the drive motors of the gyroscopic frames through a dynamic correction filter having the transfer matrix

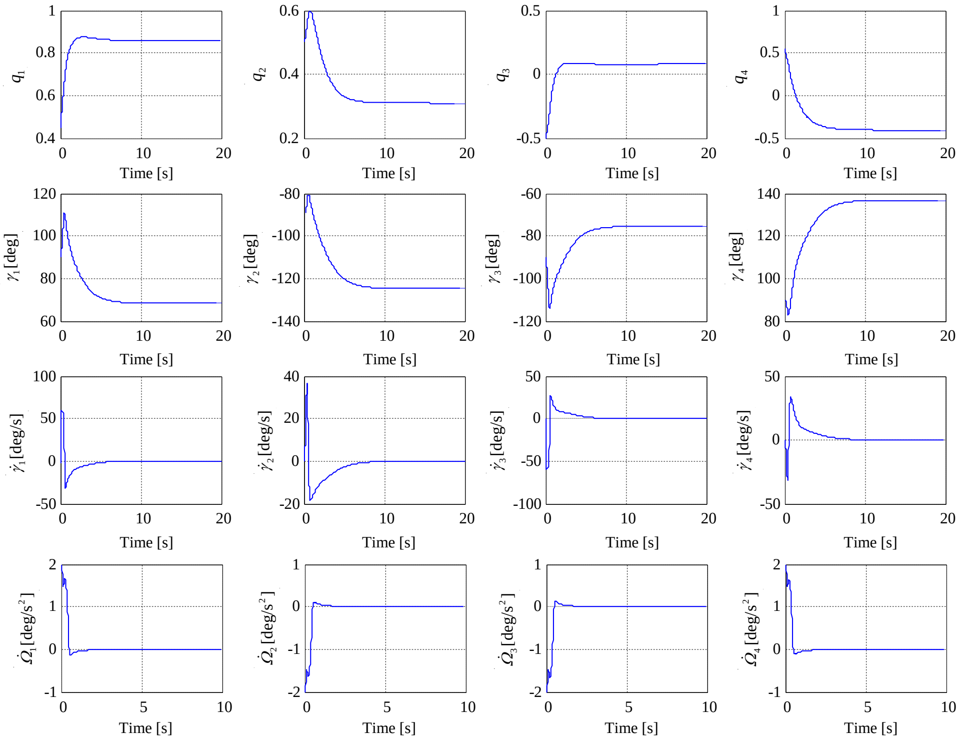
$$H_f(\text{s}) = \frac{\omega_{f0}^2}{\text{s}^2 + 2\xi_{f0}\omega_{f0}\text{s} + \omega_{f0}^2} I_{4\times4}; \tag{16}$$

one has chosen $\omega_{f0} = 50\,\text{rad/s}, \xi_{f0} = 0.7$.

In fig. 3.a one presents the time histories of the satellite's attitude angles (θ, φ, ψ), components of the satellite's angular velocity error vector $(\boldsymbol{\omega}_e)$, components of the kinetic moment vector $(\boldsymbol{K})$, components of the total kinetic moment $(\boldsymbol{K}_c)$ and the components of the command couple of the system $(\boldsymbol{M}_c)$; in


θ[deg]
φ[deg]
ψ[deg]
Time [s]
K1 [N m s]
K2 [N m s]
K3 [N m s]
Kc1 [N m s]
Kc2 [N m s]
Kc3 [N m s]
Mc1 [N m]
Mc2 [N m]
Mc3 [N m]

a.


q1
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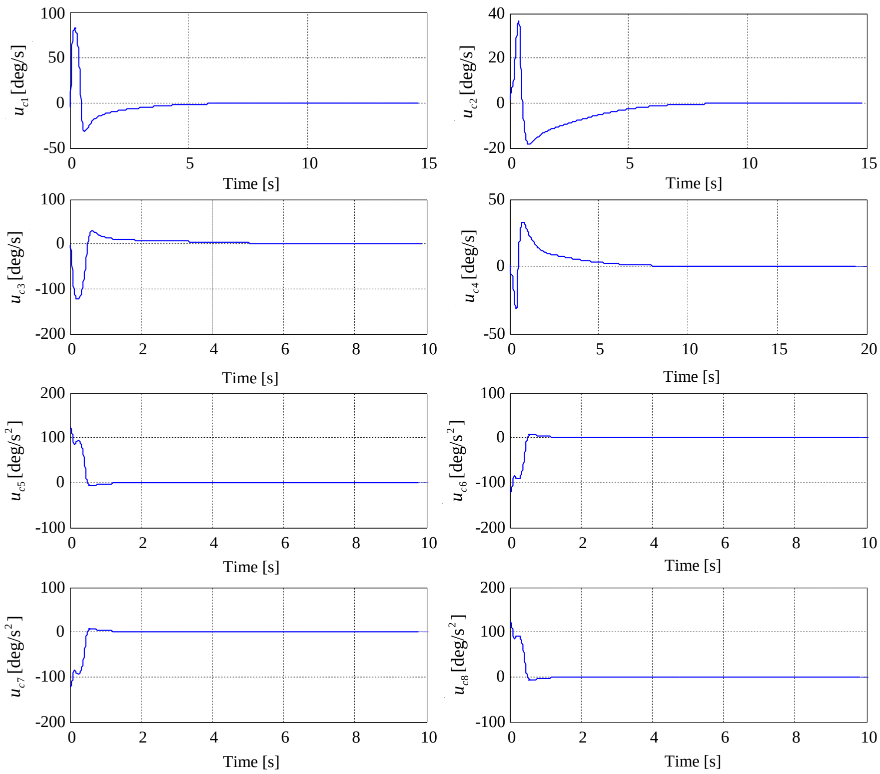
b.



c.

Fig. 3. Time characteristics of the new automatic system for the control of satellite's attitude, using a pyramidal cluster with N=4 VSCMGs

fig. 3.b one presents the time histories associated to the components of the quaternions $\hat{\boldsymbol{q}}$, to the components of the vector of gyroscopic frames' rotation angles $(\boldsymbol{\gamma})$, to the components of the vector of angular rates of the frames $(\dot{\boldsymbol{\gamma}})$ and to the components of the vector of gyros' angular accelerations $(\dot{\boldsymbol{\Omega}})$; the components of the vector of moments applied to the satellite ($\boldsymbol{u}_c$) are presented in fig. 3.c. The closed loop control system has good convergence, asymptotically stability and $u_c(t) \to 0$ (see fig. 3.c); on the other hand, as one can notice from fig. 3.b, the cancel of the system's control law is equivalent with the cancel of the frames' angular rates vector and of the gyros' angular accelerations vector, this leading to the cancel of the components of the command couple of the system ($\boldsymbol{M}_c$) (see fig. 3.a).

6. Conclusions

The purpose of this study was to design a new architecture for the control of the satellites' attitude using a pyramidal cluster consisting of four variable speed control moment gyros. The motion of the satellite is considered to be characterized by an elliptical trajectory in the plane containing the centre of Earth; during its motion, the satellite must track the Sun and a terrestrial station. The control law of the variable speed control moment gyros has been obtained from the stability condition of the closed loop system using the Lyapunov theory. The design of the new architecture's control law is also based on the usage of reference frame's dynamics and kinematics. All the steps of the new architecture's design procedure have been software implemented and validated in Matlab/Simulink environment; to validate the new automatic control system, a mini-satellite motion has been chosen. The obtained results are very good; the closed loop control system has been proved to be characterized by convergence and asymptotically stability.

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