ACHIEVING CONSENSUS IN SELF-ORGANIZED FRACTAL MULTI-AGENT SYSTEMS USING FRACTIONAL CALCULUS

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In this paper it is presented a personal opinion on two aspects of multi-agent systems (MAS) behaviour: 1. to what extent does a fractal structure of MAS favor the emergence of a solution of self-organization by adapting to the context; 2. to what extent the solution of the consensus problem through a fractional calculation procedure is favored by the fractal organization of the MAS architecture. The implications of these approaches are discussed for a consensus decision-making application in a fractal MAS structure. A fractional-order model of MAS that represents agents as nodes of a network whose topological structure admit a unique leader agent was proposed. Experimental results obtained by simulation show that consensus can be obtained with respect to time delays due to agents' different dynamics.

Keywords: Consensus, fractal MAS, fractional calculus, self-organization, context adaptation

1. Introduction

Consensus decision-making in multi-agent systems can be solved by rigorous mathematical procedures. The recent trend is to use fractional calculus, especially in the case of systems with an induced order structure, as in the case of self-organizing systems [1]. The essential problem of coordinating the agents' actions was strongly influenced by the practical aspects of achieving collaboration and obtaining consensus in social networks. For example in the field of health services, it is suggested to organize multi-agent systems (MAS) in social networks with FSO (Fractal Social Organization) nodes [2], on the principle of fractal topologies (eg Free Scale Networks [3]). The cooperative execution of tasks implies the existence of an evaluation mechanism for the selection of partner agents, so as not to compromise the success of an action due to less competent or less reliable partners. In other words, the decision on the selection of partners must be made after an assessment of the risk behavior of each agent and the level of trust that can be given to each agent. Increased confidence in selection can have the use of a strategy based on the concept of adaptive risk that at the level of the entire network leads to the idea of a global multi-agent system with self-

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organization, which allows decision making by consensus between agents. Starting from these considerations, it was proposed that this paper to answer of two specific problems: - To what extent a fractal structure of MAS favors the adaptation to the context and the emergence of a self-organization solution - To what extent the solution of the consensus problem through a fractional calculation procedure is favored by the fractal organization of the MAS architecture. Both problems are based on finding the correspondence between two basic concepts: fractional calculus and fractal behavior, respectively, or in other words about using fractional calculus on a fractal spatio-temporal model.

2. Related work

In this paper it is proposed a different approach to how to solve a "classical" problem in the theoretical approach to decision making in multi-agent systems, namely the problem of consensus. The novelty consists in trying to demonstrate that a certain structural order of the MAS architecture, of fractal type, obtained by self-organization, can lead to an optimal solution of consensual decision regarding the organization of information flows in MAS, obtained based on a fractional calculus procedure. In the literature there are numerous references to both aspects: 1) performance analysis of multi-agent systems with fractal structure, respectively 2) resolving consensus in distributed systems (as is the case of MAS) by fractional calculus, but it wasn't found any conclusive approach to both problems simultaneously. In the following, it will be listed only recent papers that fall within the two directions of research mentioned, in which it were found references to three keywords that are essential in this paper: multi-agent, consensus, fractional calculus. The intention was to signal that there are results that would suggest similar approaches in both directions and implicitly a tendency to reach the thematic area of our work. The work of Yang et al. [4] is one of the first to study the properties of multi-fractional agent systems structured in heterogeneous topologies involving delays on connection lines. The authors propose a solution for the particular case of consensus containment, but the proposed model is also applicable to integer order systems. The paper [5] describes a method of ensuring robust consensus in MAS whose agents have dynamic behavior (both linear and nonlinear) described by fractional differential equations. The paper focuses on the study of the stability of the global system in the presence of external disturbances. The paper [6] starts from a critical analysis of the consensus building solutions of fractional-order multiagent systems and proposes a generalized solution for systems with multiple integral dynamics. It is demonstrated that there is a method in the frequency-domain that ensures consensus in nonuniform time delays. The iterative learning procedural method used in [7] is very close to that used in this paper, being based on graph theory for

MAS structure modeling and fractional calculus in solving the leader-following consensus tracking problem. Moreover, the authors solve concrete control problems and provide consistent numerical results that demonstrate that after a sufficient number of iterations, all fractional agents with communication delays can achieve an exact consensus on the desired output trajectory. The paper [8] does not bring novelties in relation to those previously presented, but offers convincing results of achieving leader-following consensus of fractional-order MAS wih finite-time output. Like other papers cited above, [9] addresses the issue of consensus in multi-agent systems represented by models with multiple integral (in this case systems with double-integrator dynamics) subjected to constant disturbances, and proposes a fractional control protocol that ensures robustness of the system with an increased convergence rate. In paper [10] sampled-data control is used to establish asymptotic consensus in fractional-order MAS (FOMASs). The main novelty is a design scheme of distributed controllers using fractional order calculus. It should be mentioned that all the cited works confirm the efficiency of the fractional calculus in resolving the consensus in FOMASs, which is also a confirmation of the approach adopted in this paper. All the references commented above were a confirmation of the correctness of the way in which in the present paper was approached the problem of consensus in MAS systems. On the other hand, none of these works refer to the MAS with the fractal structure, nor to the problem of self-organization in such systems by adapting to the context, which represents the original ideas that were proposed in the current paper.

3. Basic of fractional calculus

The starting point in the argument for using fractional calculus in describing the dynamic behavior of a MAS with fractal structure (defined below as FMAS - Fractal Multi-Agent System) is the observation that in systems with free scale structure it is necessary to adopt a discontinuous model, especially when scaling tends to lower values of the unit of measurement [11]. In practical applications, any Scale Free Network (SFN) structure can be treated as a fractal boundary, and its fractal dimensions can be easily calculated. Therefore, fractional calculus can describe all the properties of a network topology in a fractal space-time location, and the value of the fractal dimensions corresponds to the fractional order. The problem now is to determine the cases in which it should be used the fractional model in practical applications. The classic solution for modeling the behavior of a dynamic system with fractal properties is to use differential equations involving fractional derivatives. Because fractional derivatives have increased flexibility compared to classical integer order models, it is recommended to use fractional calculus as tool for describing the properties of systems whose dynamics are expressed by a set of fractional differential equations

[12]. The fractional order controller is usually considered for system analysis. In this case, however, there is the problem of transferring the role of this controller to a decision block that ensures the collaboration between agents.

A. Fractional calculus

The principle of fractional calculus was stipulated by Leibnitz, as an answer to the question: Can the significance of full-order derivatives be generalized to non-integer-order derivatives? According to this principle, for a continuous function f(x), whose first-order derivative Df(x) is $\frac{df(x)}{dx} = f'(x)$, one can consider an operator H such as D = $H^{1/\alpha}$, α<1. Leibnitz's statement can be expressed as $Hf(x) = D^{\alpha} f(x)$, which represent the basic idea of the fractional calculus. In the same time $f'(x)$ is the standard part of the infinitesimal ratio:

$$f'(x) = st\left(\frac{\Delta f}{\Delta x}\right) = st\left(\frac{f(x_1)-f(x_2)}{x_1-x_2}\right) \quad (1)$$

It should be noted that Leibnitz's proposal is applicable not only to the definition of fractional derivatives, but also for that of a fractal derivative (see below).

B. Fractional Derivative

Numerous definitions of fractional derivatives can be found in the literature. A detailed exposition can be found in the review study of He [13], but for this article we summarize only the definition of the properties deriving from the application of the variational iteration method. It was considered the following linear equation of n-th order u(n)=f(t) for which the following iteration formula can be used:

$$u_{m+1}(t) = u_m(t) + \int_{t_0}^{t} \lambda(u_m^{(n)}(s) - f_m(s))ds \quad (2)$$

which after identifying the multiplier becomes:

(3)

For a linear equation (3) it takes the form:

$$u(t) = u_0(t) + (-1)^n \int_{t_0}^{t} \frac{1}{(n-1)!}(s-t)^{n-1}[u_0^{(n)}(s) - f(s)]ds \quad (4)$$

where u0 (t) satisfies the initial conditions. In equation (4) the integration operator In is defined by the relation:

$$I^n f = \frac{1}{\Gamma(n)} \frac{d^n}{dt^n} \int_{t_0}^{t} (x-t)^{n-1}[f_0(s) - f(s)]ds \quad (5)$$

where $f_0(t)= u_0^{(n)}(t)$ and $\Gamma(n)=(n-1)!$ for any positive integer n.

The formal definition for fractal derivatives deduced from equations (4) and(5)is:

$$D_t^\alpha f = D_t^\alpha \frac{d^n}{dt^n}(I^n f) = \frac{d^n}{dt^n}(I^{n-\alpha} f) = \frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_{t_0}^{t}(s-t)^{n-\alpha-1}[f_0(s) - f(s)]ds \tag{6}$$

In the proposed model will be used the simplified form of Caputo fractional derivative (expressed by equ. (7)), because it can be used only for differentiable functions:

$$D_x^\alpha\big(f(x)\big) = \frac{1}{\Gamma(n-\alpha)}\int_{t_0}^{t}(s-t)^{n-\alpha-1}\frac{d^n f(t)}{dt^n}dt \tag{7}$$

Fractal derivative

The fractal derivative is in fact a special fractional derivative which can be expressed as:

$$\frac{du(x)}{dx^D} = \lim_{s\to x}\frac{u(x)-u(s)}{x^D - s^D} \tag{8}$$

where D is the order of the fractal derivative. If a fractal is considered medium which has L0 as the smallest measure, using fractal geometry it can be calculated the distance ds between two points A and B as:

$$ds = \frac{Du}{Dx^\alpha} = \lim_{\Delta x\to L_0}\frac{u(A)-u(B)}{k L_0^\alpha} \tag{9}$$

where α is the fractal dimension, k is a weighting factor chosen such as $ds = kL_0^\alpha$ is the distance between two points in a discontinous space, i.e. k=k(α); obviously k(1)=1

4. Self-organization in FMAS

Fractal multi-agent systems (FMAS) offer the possibility to study the emerging behaviors of agents that co-evolve in a determined context. In FMAS, agents can adapt their behavior by changing action rules as they gain expertise. At the same time, each change of strategy in the behavior of an agent changes the context in which all partner agents evolve. Therefore, the process of mutual adaptation of a multiple population of agents is a coevolutionary process. The search for similarity models at different scales is favored by fractal mathematics, which offers a strong theoretical substantiation for coevolution, by development of action skills, conditioned by local constraints. More specifically, the variation in collaborative systems is characterized by the emergence of new tiers of self-organized order, subjected then to well-known selection and retention processes in organizational evolution. On the other hand, MAS are developed to solve problems that exceed the capabilities of an individual agent. That is why the cooperation between the agents is mandatory for the achievement of the objectives, but this in the conditions in which the autonomy of the entities is preserved. Respecting autonomy allows each agent to use his own experience to make choices (to make decisions) regarding the mode of action, but also can induce uncertainties regarding the relationship with partner agents. Due to the differences between the methods of reaching a certain objective, in MAS conflicting objectives can appear and implicitly a more pronounced vulnerability of the system as a whole. In order to avoid unpleasant situations, it is recommended to have a software security control [14] that protects the agents (and the system as a whole) by preventing unwanted behaviors and also by increasing the degree of confidence in the reliability of the execution of tasks. There is a clear similarity between the association based on trust in a social network and the choice of collaborating partners in a MAS. In both situations it is necessary to have an algorithm for making association decisions, and the result of these decisions is a reorganization of the system. It can be said that the system is self-adaptive, and the result of adapting to the context is self-organization. But self-organization can be defined both in terms of infrastructure (reorganization of the communication network between agents, more precisely reorganization of network topology) and in terms of software architecture (recalculation of information flow when performing a task).

For both situations, however, the same model can be used - a network structure with nodes to represent the agent entities - in the case of structural self-organization, or subtasks that perform software services - in the case of behavioral self-organization. In both cases the role of the decision-making mechanism is the same, namely the establishment by consensus of an action plan to optimize the overall utility and the number of completed tasks, while minimizing the time lost in performing unprofitable tasks and downtime. The proposed model is that of a graph that represents the network of nodes, without imposing a specific topological structure, and to which consensus should be obtained with respect to time delays due to different dynamics (either in the transfer of information between agents or in the execution of sub-tasks). Obviously, in the case of behavioral self-organization, the problem is simpler, because the execution network of a task can be considered as a tree structure whose nodes represent subtasks, presenting sequential dependence between subtasks. In this model, each task is considered as a set of multiple services. Because a service instance needs a specific (different) computation time, the task is considered completed only when all nodes have been traversed by the flow of information. In the following it will be presented an algorithm that solves the problem of consensus in a network of nodes that represent entities of a FOMAS, for which the structure resulting from self-organization is fractal, which justifies the choice of a fractional calculus method to achieve the goal. - minimizing the execution time in conditions of constraints determined by time delays.

5. Consensus in FOMAS with time delays

The problem of consensus for multi-agent systems, a particular facet of distributed consensus, is in principle to find a control protocol ensuring that the states of all agents converge on a common goal. The problem of consensus can be classified into two categories: the one in which a leader is followed (leader-following consensus) and the one in which there is no leader designated initially (leaderless consensus). In the situations of the first category there is a single leading agent, and all other agents must follow the leader agent and eventually converge to the common target through the local exchange of information mediated by an inter-agent communications network. The problems in the second category, in which there is not a single leading agent, are in fact problems in which in the structure of the multi-agent system several leaders can be designated (multiple leaders). A particular case of problems of this type is that of containment consensus in which potential leaders are connected in a fully interconnected network (each with each other) ensuring a geometric formation of convex hull type. All other followers must be checked to converge to this convex hull. In this paper it was studied only the situation in which there is a single dynamic leader and several followers, who may have a different reaction dynamic, which leads to communication delays. The solution involves finding a consensus protocol that can maintain the connectivity of the communications network. Topological model For a MAS with n agents, the exchange of information between agents can be represented by an interaction graph G = (V, E), where V = $\{s1 ,. . . , sn\}$ is the set of vertices) and E ∈ VxV is the set of edges [1]. The node indices belong to a finite set I = $\{1,2, . . . , n\}$. The weighted adjacency matrix is denoted as A = $[aij]\times n$. The connection state between agents si and sj is represented by the element placed at the intersection of the i-th row and the j-th column. If nodes si and sj are connected, then $aij > 0$, and sj is considered neighbor of node si. We note $Ni = \{j \in I, j \neq i\}$ the index set of all neighbors of agent i. If nodes si and sj are connected and $aij \neq aji$, G is a called a directed graph. It means that a directed path is a sequence of edges by $(s1, s2)$, $(s2, s3)$, and so on, where $(sj, si) \in E$. The Laplacian matrix of the graph G is defined by $L = \Delta - A \in R^{n \times n}$, where $\Delta \triangleq \mathrm{diag}\{deg_{out}(s_1),. . . , deg_{out}(s_n)\}$ is a diagonal matrix with $deg_{out}(s_i) = \sum_{j=1}^{n} a_{ij}$

Problem Statement

Let consider a FOMAS with n agents, represented by a graph G where each node corresponds to an agent. If $aij > 0$, we suppose that the i-th agent can get information from the j-th agent. The dynamics of the i-th agent of the FOMAS is represented by:

$$x_{i_1}^{(\alpha_1)}(t) = x_{i_2}(t), x_{i_2}^{(\alpha_2)}(t) = x_{i_3}(t), \dots x_{i_m}^{(\alpha_m)}(t) = u_i(t) \tag{10}$$

with $i \in I$, where $x_{i_1}(t)$, …, $x_{i_m}(t)$ represent m different states of the i-th agent, $x_{i_j}^{(\alpha_j)}(t)$ is the s_j α_j – order Caputo derivative of $x_{i_j}(t)$, with $\alpha_j \in (0,1], j = 1,2,\dots,m$ and $u_i(t) \in R$ is the control input.

The consensus can be reached if and only if all the following conditions are fulfilled:

$$\lim_{t\to\infty}(x_{i_1}(t) - x_{j_1}(t)) = 0, \lim_{t\to\infty}(x_{i_2}(t) - x_{j_2}(t)) = 0, \dots \lim_{t\to\infty}(x_{i_m}(t) - x_{j_m}(t)) = 0;\ i,j \in I \quad (11)$$

Definition. The control protocol for FOMAS described by (10) is given by:

$$u_i(t) = -\sum_{k=1}^{l-1} P_{k+1} x_{i_{k+1}}(t) + \sum_{j\in N_i} a_{ij} [x_{j_1}(t-\tau_{ij}) - x_{i_1}(t-\tau_{ij})] \quad (12)$$

where Ni denotes the index of the agent i neighbors, aij is the (i, j)-th element of A, $\tau ij > 0$ is the time delay of information transfer from the j-th agent to the i-th agent, and $Pk+1 > 0$ are scale coefficients. If all $\tau ij = \tau ji$, then the time delays are symmetrical; if other the time-delays are asymmetrical (non-uniform). We consider that τm denote M different time-delays of FOMAS described by Equ. (10), then the following control protocol is provided to resolve consensus problems of FOMASs.

$$u_i(t) = -\sum_{k=1}^{l-1} P_{k+1} x_{i_{k+1}}(t) + \sum_{j\in N_i} a_{ij} [x_{j_1}(t-\tau_m) - x_{i_1}(t-\tau_m)] \quad (13)$$

In the following section, a simulation is conducted to evaluate the performance of the proposed method on a simple MAS network with an unique leader.

6. Experimental results obtained by simulation

Consider the time-delay FOMASs with four follower agents, which are labeled as 1, 2, 3, and 4. The virtual leader agent is labeled as 0. The communication network topology of the leader-follower FOMAS is illustrated in Fig. 1.

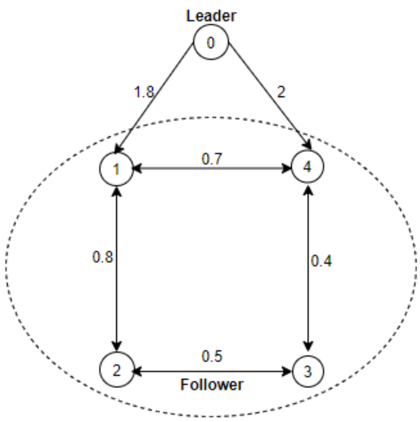

Fig.1 Communication network topology

According to Fig. 1, the weighted adjacency matrix M and the in-degree D_{in} can be obtained as follows:

$$M = \begin{bmatrix} 0 & 0.8 & 0 & 0.7 \\ 0.8 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0.4 \\ 0.7 & 0 & 0.4 & 0 \end{bmatrix};$$

D_{in}=diag $[1.6 \quad 1.7 \quad 1.4 \quad 1.3]$;

$$L=D_{in}-M=\begin{bmatrix} 1.6 & -0.8 & 0 & -0.7 \\ -0.8 & 1.7 & -0.5 & 0 \\ 0 & -0.5 & 1.4 & -0.4 \\ -0.7 & 0 & -0.4 & 1.3 \end{bmatrix};$$

D= diag $[1.8 \ 0 \ 0 \ 2.0]$.

In the simulation, the sampling time is selected as 1 s and the matrices and coefficients of the jth agent that are as follows:

$A=\begin{bmatrix} 0.1 & 0.3 \\ 0.2 & -0.1 \end{bmatrix}$; $B=\begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}$; $C=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $P=\begin{bmatrix} 0.2 & 0.6 \\ 0.8 & -0.7 \end{bmatrix}$,

where P is A's adiacent matrix, $\alpha = 0.9$.

The delay times for the followers are h1 = 0.4, h2 =0.45, h3 = 0.5, h4 = 0.6.

The desired reference trajectory is:

$$y_d = \begin{bmatrix} t + \sin(t * 2pi) \\ t - 1 + \cos(t * 2pi) \end{bmatrix}, \ t \in [0, 1]$$

In Fig. 2 were showed the trajectories of the 5 agents at the first iteration. In Fig. 3 was showed the positioning of the trajectories after 12 iterations. In Fig. 4 it was indicated that after 24 iterations the trajectories of the tracking agents overlap with the trajectory of the leader.

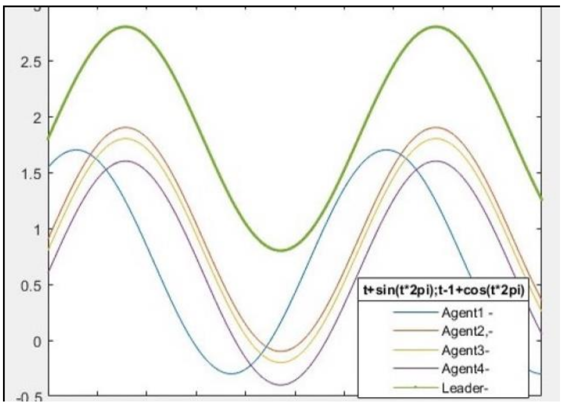

Fig.2 The initial trajectories of 4 followers and a leader

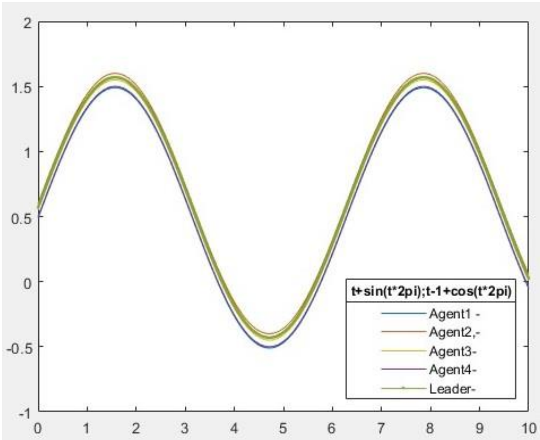

Fig.3 The output trajectory at the 12th iteration

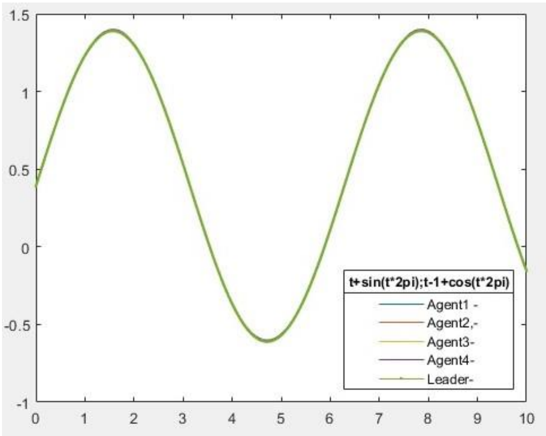

Fig.4 The output trajectory of all agents at the 24th iteration

This simulation example does not allow a correct evaluation of the performance, but only to show the correctness of the protocol used. In the future, the time to reach convergence for complex scale-free topologies will be computed, in an attempt to demonstrate that a communication network with a fractal structure leads to an increase in convergence speed in consensus decision-making.

7. Conclusions

Multi-agent systems are finding increasing use in several applications that involve collaborative actions between the so-called components of a complex distributed dynamic system. Ensuring the achievement of a common goal is solved through various procedures, but the one that is gaining the most ground at present is that of consensus in allocating the tasks performed by each agent. The problem of consensus is an issue that is addressed in decision theory and is treated extensively in the recent literature. Given the distributed nature of the MAS structure (most often represented by a graph that models the communication network between the agents, considered to be network nodes) most of these papers recommend that the dynamics of actions (behavior) of each agent be treated by fractional calculus methods, as indicated by the references cited in this paper.

Another trend in the study of collaborative MAS is that of structural self-organization in response, on the one hand, to environmental changes and unpredictable events (uncertainties) - the problem of adaptation to context, and on the other hand to capitalize on gain in experience achieved as the system evolves - the problem of iterative learning. Previous research of the authors (see reference [3]) has demonstrated the advantages of MAS with fractal architecture (FMAS) - robust behavior by adapting to external disturbances - and highlighted the capability to self-organize (including structural emergence).

Taking into account these considerations, in the present paper three study objectives were discussed as original contributions. First, that the problem of decisional consensus in collaborative actions of MAS is favored by the reorganization (more precisely self-organization) in fractal structures represented by scale-free networks. The second novelty is the proposal to use only the fractal derivative (as a particular form of Caputo fractional derivatives) in solving the problem of consensus in FMAS with particular topologies. The third issue discussed in the paper is that of a FOMAS model that can be applied equally in matters of structural self-organization (dynamic reconfiguration of the agent nodes network) and behavioral self-organization (reconfiguration of subtasks of a task having multiple integral dynamics). It should be noted that for both approaches the same optimal objective is expected - minimum time for establishing the consensual decision, respectively minimum time for execution of the task.

The validation of the three aspects mentioned above was done by simulation, with solving the problem of leader-following consensus on a simple MAS with a single leader, in which convergence is ensured even if the followers agents present dynamics with different time-delays. Although the results would indicate a good performance (reaching the target after a relatively small number of iterations) they cannot be considered conclusive for a real application, but only demonstrate the validity of the calculus method based on the Caputo-type fractal derivative.

For the future, the purpose is to study more complicated problems of establishing consensus in MAS with undirected dynamic communication topologies, with multiple leaders, in the presence of perturbations and uncertainties, with non-uniform delays, aiming in all situations to demonstrate that fractal organization is favorable (higher efficiency and speed of convergence).

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