U.P.B. Sci. Bull., Series D, Vol. 69, No. 2, 2007 ISSN 1454-2358

A DYNAMIC MODEL FOR THE IMPACT BETWEEN THE WHEEL FLAT AND RAIL

Traian MAZILU[1]

Locul plan este un defect important al suprafeței de rulare a roții unui vehicul feroviar. El apare datorită uzurii când roata este blocată şi alunecă pe şină în timpul frânării. Rularea roții cu loc plan este periculoasă din cauza forței de impact care solicită atât calea cât şi roata şi produce zgomot de impact. În articol este prezentat modelul de studiu al interacțiunii dintre roata cu loc plan şi şină. Şina este considerată ca o grindă infinită Euler rezemată pe un suport continuu cu două etaje elastice. Vehiculul este redus la un sistem cu două mase legate elastic. Este considerat de asemenea contactul neliniar Hertzian dintre roată şi şină. Soluția ecuațiilor de mişcare este obținută prin aplicare unei originale metode bazată pe funcțiile Green. Influența geometriei locului plan asupra forței de impact este prezentată pentru diferite situații de circulație a vagonului.

The flat is a serious fault of the rolling surface of the railway wheel. It appears when the wheel is blocked and slides on the rail during braking process due to the wear. The rolling of the wheel flat is dangerous due to the impact force, which applies stresses both to the track and the wheel and generates impact noise. This article presents the model for the analysis of wheel flat/rail interaction. The rail is considered as an infinite Euler beam supported on the continuous foundation with two elastically layers. The vehicle is reduced at a system of two elastic connected masses. The non-linear Hertzian wheel/rail contact is considered too. The solution of the motion equations are obtained by using an original Green's functions method. The influence of the flat geometry and rail pad stiffness for different cases of the vehicle traffic are presented.

Keywords: wheel flat, vehicle, rail, Green functions

1. Introduction

Sometimes, during braking process, the wheel is blocked and slides along the rail due to braking system malfunctioning. In the contact zone, the wheel suffers severe wear and a flat appears. Usually, the flat can reach up to 50 – 60 mm but it may reach even 100 -120 mm. At first, the flat appears to be a plane zone, but then, as the wear progresses, its edges are becoming rounder and rounder. This is caused by the wheel's modified structure due to its thermal regime – high heating followed by quick cooling – during wheel blocking.

[1] Lecturer, Dept. of Railway Vehicles, University "Politehnica"of Bucharest, ROMANIA

The flat may occur at any type of railway vehicles, including the high speed ones. For instance, on the 14th of December 1992, one bogie of the TGV train derailed as passing the station Mâchon-Loché at a speed of 270 km/h. The incident is assumed to be caused by a flat wheel [1].

The wheel flat/rail interaction resides in a huge periodical impact force. As the wheel speed increases, the impact force frequency increases. The rolling of wheel flat is dangerous. It can actually damage the track – fatigue cracks on the rails or sleepers, and the wheel - the cracks or even material losses occurring on the rolling surfaces. As seen before, in extreme situations, the vehicle might derail. Another consequence of a rolling wheel with flat fault is the periodic impact noise.

The dynamics of the wheel with flat fault has begun to be studied recently. It was studied among others by Ver [2], related to the impact noise. He introduced the concept of *critical speed*, defined as the one for which the wheel/rail contact is lost (momentary). In the quoted work, a series of basic formulas for critical speed and impulse variations' calculation are presented.

Newton and Clark [3] used a more complex model with the aim to study the wheel flat/rail impact force. They have considered the vehicle, taken as a three masses system connected through the elastic and damping elements, which rolls on an infinite rail mounted over an elastic foundation. The non-linear elastic Hertzian wheel/rail contact is modelled through an elastic element placed between the wheel and the rail. The wheel flat/rail geometry is modelled as an equivalent indentation on the rail head, the wheel is perfectly rounded. Thereby, they avoided the difficulties related to locating the impact spot on the rail during the experimental researches. This experimental measurements have shown that the maximal impact force increases along with the increase of the speed until reaching 30 km/h, then it has a small decreasing until 60 km/h and increases again for speeds beyond that limit. The theoretical results concurred with the experimental ones for speeds up to 80 km/h.

The two axles bogie track interaction was studied by Nielsen and Igeland [4]. The track was modelled using the finite element method and the bogie was taken as a system of three rigid bodies, the car body, the bogie and the wheel, connected through the elastic and damping elements of the primary and secondary suspension. The first wheel has a flat fault and the following wheel is in perfect condition. The speeds of losing contact for two types of flats, having the wavelengths of 60 and 90 mm, are determined. The maximal impact force is calculated too.

Wu and Thompson analyzed this issue in two of their works [5, 6]. They chose as a starting point two analytic track models. In the first one, the rail is a Timoshenko beam on a continuous two-level elastic support. The sleeper effect is neglected. The second model considers the rail as two Timoshenko beams on a discrete support. The sleeper is taken as a rigid with one degree of freedom. Each time, they calculated the parameters of an equivalent model which has the best approximation for the frequency response of the initial model. This way, the track system was transformed into a system of constant or variable parameters. The last variant is considered for parametric excitation due to track stiffness variation across a span length. The integration of the wheel/rail model equations was made using the Runge – Kutta method.

They studied the influence of the speed and flat dimensions over the impact force and impact noise. They also revealed that the sleeper has a marginal effect over the impact force.

Hou, Kalousek and Dong [7] applied a finite element model in order to study the vehicle-track system as an asymmetric system. The case of a wheel with flat fault was studied as well. The effect of a flat wheel on the other wheel of the same bogie was determined.

The current work comes with a different solution for solving the issue of this kind of interaction. Starting from the idea of Wu and Thompson, that the impact force is little influenced by the sleepers, the model of a beam on a continuous two-layer elastic support is considered. For a model like this one, Grassie and al. [8] revealed that there are no significant differences between the Euler and Timoshenko beam models. Differences appear if the periodic sleeper support is considered. As a result, the Euler beam model is adopted, because of its simplicity. The case of the vehicle with wheel flat is analysed.

The equations of motion are integrated using the Green functions method in original manner. This way, the errors occurring when using the Wu and Thompson equivalent model may be avoided. On the other hand, the Green function method is simpler and faster than the modal analysis. Using this model, some aspects of the flat wheel/rail interaction are studied.

2. Mathematical model for railway vehicle wheel flat/rail interaction

An analytic model of the railway vehicle wheel flat/rail interaction has been developed (fig. 1).

The particular case of a vehicle having a single suspension layer is considered. The vehicle model consists in two masses, the wheel and the car body, connected through the suspension. The car body mass is marked as M_b and the wheel mass as M_w. The suspension has one elastic element by the stiffness of k and one damping element having the damping constant of c. The car runs at constant V speed and its position compared to the referential $\Omega\xi\zeta$ is $x = Vt$ where t stands for time. The vertical displacements of the car body and the wheel are $z_b(t)$ and $z_w(t)$ respectively.

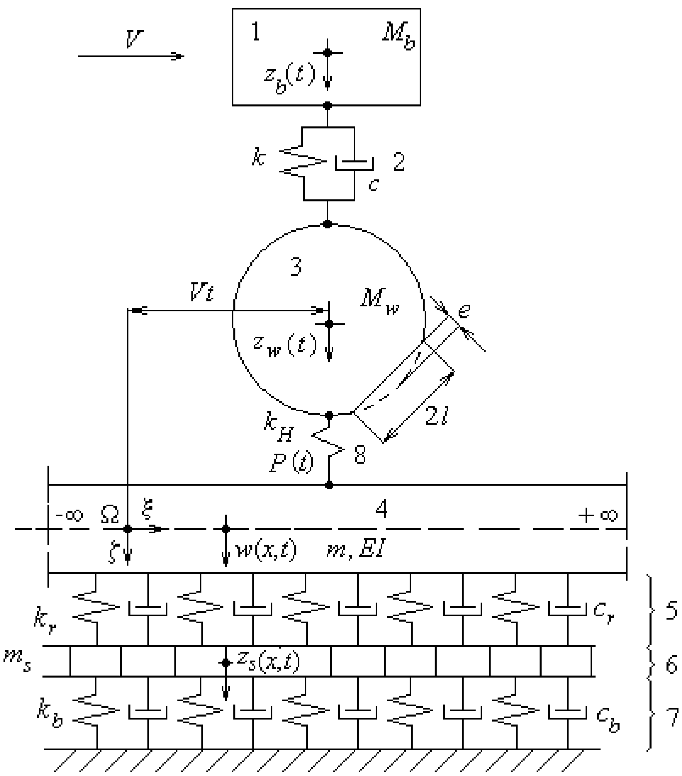

Fig. 1. Mechanical model of railway vehicle wheel flat/rail interaction: 1. car body; 2. suspension; 3. wheel; 4. rail; 5. rail pad; 6. semi-sleeper; 7. ballast; 8. contact stiffness.

The track model is a rail resting on a continuous two-layer support. The rail is taken as a uniform infinite Euler beam with specific m_r mass per length unit, the Young's modulus E and the area moment of inertia I. The loss factor of the rail is neglected. The vertical beam displacement is $w(x, t)$.

The rail pad is modelled as a uniform damped elastic layer. The elastic constant k_r and the damping constant c_r are constants per unit length. They are calculated from the corresponding parameters for discrete support by division by the sleeper bay. The ballast is represented by a uniform damped elastic layer with the elastic constant per unit length k_b and the damping constant c_b per unit length. The semi-sleeper is considered a layer devoid of shear or bending stiffness with specific m_s mass per length unit and vertical displacement $z_s(x, t)$. All elastic and damping elements have linear characteristics.

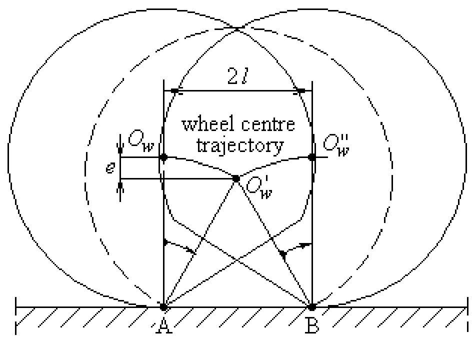

Fig. 2. Wheel flat / rigid rail geometry.

The wheel has a flat of length 2*l* and depth *e*. Fig. 2 presents the geometry of the wheel having a fresh flat fault, running on a rigid rail. When the wheel steps on the flat, the trajectory of the wheel centre is described by two juxtaposed circle arches by the radiuses equal to the wheel radius. The wheel motion has actually two phases. During the first one, the wheel spins around "A" and the centre O_w descends by *e* and reaches $O_w^{'}$. During the second phase, the wheel spins around "B" and its centre climbs back to its original distance, reaching $O_w^{''}$. The spin angle is the same for both phases of the motion and it's small. When the elasticity of the wheel/rail contact is considered, the relative wheel/rail displacement is equal to the deformation in the contact zone of the two bodies. In addition to that, the relative displacement due to the flat has to be considered.

It's obvious that during the running over a flat, the wheel centre and the contact point are not on the same vertical axis. The torque generated by the vertical load is negligible though and the wheel spin is practically constant. The wheel sliding along the rail and the friction force are negligible as well. As a result, the wheel dynamics is dictated only by the vertical forces.

The vehicle motion is described by the matrix equation

$$\mathbf{M}\{\ddot{\mathbf{z}}\}+\mathbf{C}\{\dot{\mathbf{z}}\}+\mathbf{K}\{\mathbf{z}\}=\{\mathbf{P}\} \tag{1}$$

where **M** is the mass matrix, **C** is the viscous damping matrix, **K** is the stiffness matrix and {**P**} is the column vector of forces on the vehicle

$$\mathbf{M}=\begin{bmatrix} M_b & 0 \\ 0 & M_w \end{bmatrix},\ \mathbf{C}=c\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix},\ \mathbf{K}=k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix},\ \{\mathbf{P}\}=\begin{bmatrix} 0 \\ P_0-P(t) \end{bmatrix}, \tag{2}$$

$P(t)$ is the rail reaction in the contact point and the static load is $P_0 = g(M_b+M_w)$ with g as the gravitational acceleration. The $\{\mathbf{P}\}$ vector has this particular form because the elastic element from the suspension is pre-loaded by the car body weight.

The track's differential equations of motion can be written in matrix form

$$\mathbf{L}_{x,t}\{\mathbf{q}\} = \{\mathbf{p}\} \tag{3}$$

where

$$\{\mathbf{q}\} = \{\mathbf{q}(x,t)\} = \begin{bmatrix} w(x,t) & z_s(x,t) \end{bmatrix}^t \tag{4}$$

is the column vector of displacements and

$$\{\mathbf{p}\} = P(t)\delta(x - Vt)\{\mathbf{e}\} \tag{5}$$

is the column vector of forces on the rail with $\{\mathbf{e}\} = [1 \quad 0]^t$ and $\delta(.)$ is Dirac's delta function. The differential operator $\mathbf{L}_{x,t}$ from equation (3) is defined by

$$\mathbf{L}_{x,t} = \begin{bmatrix} EI\dfrac{\partial^4}{\partial x^4} + m_r\dfrac{\partial^2}{\partial t^2} + c_r\dfrac{\partial}{\partial t} + k_r & -c_r\dfrac{\partial}{\partial t} - k_r \\ -c_r\dfrac{\partial}{\partial t} - k_r & m_s\dfrac{\partial^2}{\partial t^2} + (c_r + c_b)\dfrac{\partial}{\partial t} + + k_r + k_b \end{bmatrix}. \tag{6}$$

The wheel and the rail are solid elastic bodies and the deformation at the contact point can be expressed by Hertz's theory of elastic contact. According to this theory, the relationship between the contact force $P(t)$ and the Hertzian deflection,

$$z_\delta(t) = z_w(t) - w(Vt,t) - z_r(Vt), \tag{7}$$

is

$$[P(t)/C_H]^{2/3} = z_\delta(t)\sigma[z_\delta(t)] \tag{8}$$

where $z_r(Vt)$ is the relative displacement wheel/rail due to the flat, C_H represents the Hertzian constant and $\sigma[.]$ is the Heaviside function.

Wu and Thompson proposed a second degree polynomial for the relative wheel/rail displacement due to the wheel flat [6]. Another function

$$z_{rk}(x) = \frac{e}{2}\left(1 - \cos\pi\frac{x - x_k}{l}\right)[\sigma(x_k + l - x) - \sigma(x_k - l - x)], \tag{9}$$

where x_k is considered to be the position where the wheel steps for the "k"th time on the flat, was proposed by Wu and Thompson, too [7].

The boundary conditions are

$$\lim_{|x-Vt|\to\infty}\{\mathbf{q}(x,t)\}=[0 \quad 0]^T, \tag{10}$$

and the initial conditions are

$$\{\mathbf{z}\}=0,\ \{\dot{\mathbf{z}}\}=0,\ \{\mathbf{q}(x,0)\}=[0 \quad 0]^T,\ \{\dot{\mathbf{q}}(x,0)\}=[0 \quad 0]^T \tag{11}$$

where $\{\dot{\mathbf{z}}\}$ and $\{\dot{\mathbf{q}}(x,0)\}$ are speed vectors.

An original method for solving the motion equations is applied. It is based on the Green functions for both, the track and the vehicle.

The vector for vehicle displacement (for null initial conditions) is

$$\{\mathbf{z}(t)\}=\int_0^t \mathbf{g}_v(t-\tau)\{\mathbf{P}(\tau)\}\mathrm{d}\tau \tag{12}$$

where

$$\mathbf{g}_v(t)=\begin{bmatrix} \dfrac{1}{m_w+m_b}\left[\dfrac{m_b}{m_w}t+\varphi(t)\right] & \dfrac{1}{m_w+m_b}[t-\varphi(t)] \\ \dfrac{m_b/m_w}{m_w+m_b}[t-\varphi(t)] & \dfrac{1}{m_w+m_b}\left[t+\dfrac{m_b}{m_w}\varphi(t)\right] \end{bmatrix} \tag{13}$$

is the vehicle's Green functions matrix, with

$$\varphi(t)=\exp(-\alpha t)\sin\beta t/\beta,\ \alpha=c\frac{M_w+M_b}{2M_wM_b},\ \omega^2=k\frac{M_w+M_b}{M_wM_b},\ \beta^2=\omega^2-\alpha^2. \tag{14}$$

The Green functions matrix may be calculated using the Laplace transform applied to equation (1).

The time-domain analysis of the track has its fundament on the track's real Green functions

$$\{\mathbf{g}\}=\{\mathbf{g}(x,\xi,t-\tau)\}=\begin{bmatrix} g_w(x,\xi,t-\tau) \\ g_z(x,\xi,t-\tau) \end{bmatrix} \tag{15}$$

The column vector $\{\mathbf{g}\}$ contains the track's response in the x section at the t-τ moment, if at the τ moment in the ξ section an impulse force occurred. The column vector of the real Green functions is the solution for the following equation

$$\mathbf{L}_{x,t}\{\mathbf{g}\} = \delta(x-\xi)\delta(t-\tau)\{\mathbf{e}\} \tag{16}$$

and it may be calculated using the Fourier transform method. If the complex Green functions for the rail are defined through the Fourier transform $F[.]$

$$\{\mathbf{G}\} = \begin{bmatrix} \mathbf{G}_w(x,\xi,\omega) \\ \mathbf{G}_z(x,\xi,\omega) \end{bmatrix} = F[\{\mathbf{g}(x,\xi,t)\}] \tag{17}$$

then, they contain the amplitudes of the track's harmonic response in the x section at the ω angular frequency, if in the ξ section an impulse force occurred. The column vector of the complex Green functions is the solution of the following equation

$$\mathbf{L}_{x,\omega}\{\mathbf{G}\} = \delta(x-\xi)\{\mathbf{e}\} \tag{18}$$

where $\mathbf{L}_{x,\omega} = F[\mathbf{L}_{x,t}]$.

The vector for the real Green functions is calculated beginning with

$$\{\mathbf{g}\} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\{\mathbf{G}\}\exp(\mathrm{j}\omega(t-\tau))\mathrm{d}\omega \tag{19}$$

where $\mathrm{j}^2 = -1$.

The calculation method for the Green functions is similar to the one presented in reference [8].

The rail displacement at the contact point results by applying the Green function method

$$w(Vt,t) = \int_{-\infty}^{\infty}\int_0^t g_w(Vt,\xi,t-\tau)P(\tau)\delta(\xi - V\tau)\mathrm{d}\tau\mathrm{d}\xi = \int_0^t g_w(Vt,V\tau,t-\tau)P(\tau)\mathrm{d}\tau \tag{20}$$

where $g_w(Vt, V\tau, t - \tau)$ is the Green function for rail displacement. In other words, for any contact point $x = Vt$, there is a corresponding Green function $g_w(Vt, V\tau, t - \tau)$ which depends on $\tau \in [0, t]$ and is calculated from $g_w(x, \xi, t - \tau)$.

The rail's Green function has its property of being damped. There is a certain T for which the Green function's contribution may be neglected for $t - \tau > T$ (apriority $t \geq T$)

$$g_w(Vt, V\tau, t-\tau) \cong 0\,. \tag{21}$$

For numerical application purposes, a time partition - $t_0, t_1, \ldots t_n$ with $t_0 = 0$, $t_n = t$ and $\Delta t = t_i - t_{i-1}$ where $i = 1 \div n$ - has to be considered. Equations (12) and (20) are becoming

$$\{\mathbf{z}(t_n)\} = \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} \mathbf{g}(t_n - \tau)\{\mathbf{P}(\tau)\}\mathrm{d}\tau \tag{22}$$

$$w(Vt_n, t_n) = \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} g_w(Vt_n, V\tau, t_n - \tau)P(\tau)\mathrm{d}\tau\,. \tag{23}$$

The integrations will be calculated adopting the hypothesis that in the time interval $[t_{i-1}, t_i]$, the contact force $P(\tau)$ and the Green functions have a linear variation. It is obvious that the wheel and rail displacement in the contact point are depending on the amplitude of the contact force $P(t_n)$. By substituting those displacements in the equation of the contact force (4), a non-linear $P(t_n)$ based equation will result. By solving it in an iterative matter, the contact force results for each integration step. The wheel and rail displacements are resulting together with the contact force. Finally, the following integration step might be taken.

2. Numerical application

Next, the particular case of a railway vehicle having the wheel mass M_w = 750 kg and the car body mass (distributed on each wheel) M_b = 9250 kg and static load P_0 = 100 kN running with a wheel flat on a rail having the linear mass m_r = 56 kg/m on concrete sleepers, is studied. Other parameters for the considered vehicle are: k = 1.37 MN/m, c = 90 kNs/m and wheel diameter 840 mm. The values for the involved track parameters are: $I = 23.14\cdot10^{-6}$ m^4, E = 210 GPa, m_s = 184 kg, k_r = 400 MN/m, c_r = 90 kNs/m, k_b = 257 MN/m and c_b = 117 kNs/m.

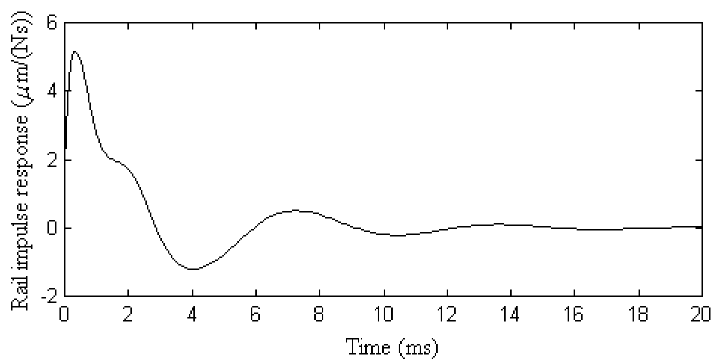

Fig. 3. The Green's function $g_w(0,0,t)$

The Green function for the rail was calculated integrating equation (19) from 0 to 10000 Hz at a step of 10 Hz. The time step Δt = 40 μs and the resulting step for rail mesh is $\Delta s = V\Delta t$.

Fig. 3 displays the rail's response in the point where the unitary impulse is applied. The maximal value is 5.15 μm/(Ns) and the function is damped. It's considered that after T = 60 ms, the function is negligible.

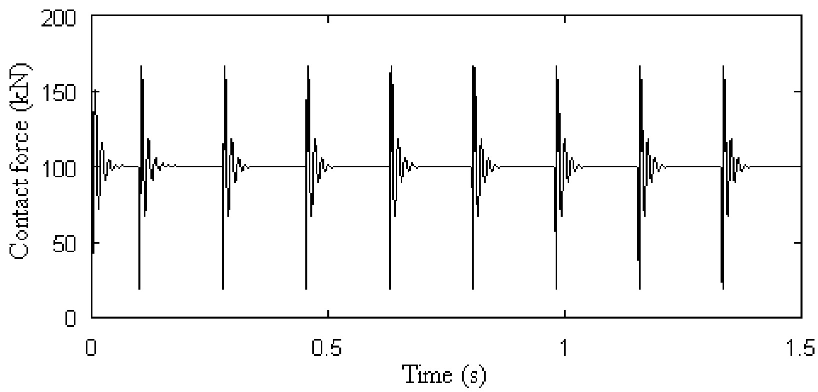

Fig. 4. Contact force.

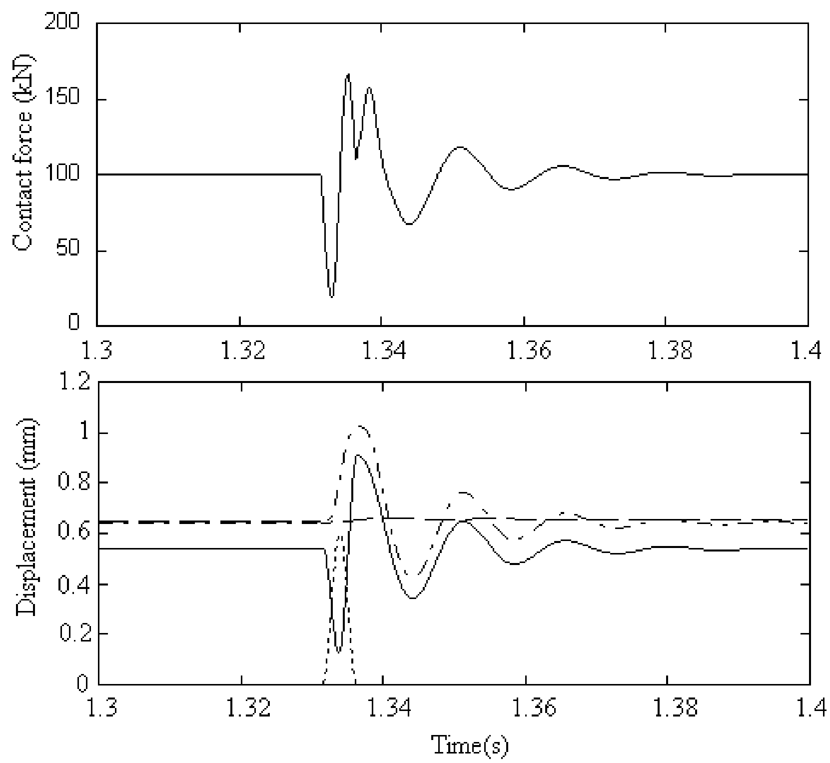

Fig. 5. Vehicle/rail response: a) contact force; b) —— rail, —·— wheel, — — body, - - - wheel flat/rigid rail displacement.

Figs. 4 and 5 are displaying the wheel/rail system's evolution at a speed of 15 m/s, when the wheel repeatedly steps over a 72 mm long and 0.6 mm deep flat. At the beginning of the simulation, the wheel has a transitory behaviour with the wheel's own frequency on the rail, which is about 70 Hz. First, the wheel steps on the flat at 0.1 s and then, at fixed time intervals of 175.9 ms. The maximal impact force stabilizes quickly and has very little variations after. The maximal impact force is 166.14 kN at 1.335 s. At first impact, its amplitude is 166.55 kN.

The wheel displacement is higher than the rail displacement because elasticity reasons. When the wheel reaches the flat zone, due to inertia, it has the tendency of splitting apart from the rail. Thus, the contact force decreases to its minimum of 18.46 kN. Then, the wheel is pushed back to the rail by the force from the suspension. The rail is also pushed up by the reactions occurred in the rail pad and the ballast. As a result of these contrary motions, the wheel/rail impact occurs. After the impact, the motion is quickly damped. The car body displacement is totally insignificant due to the suspension.

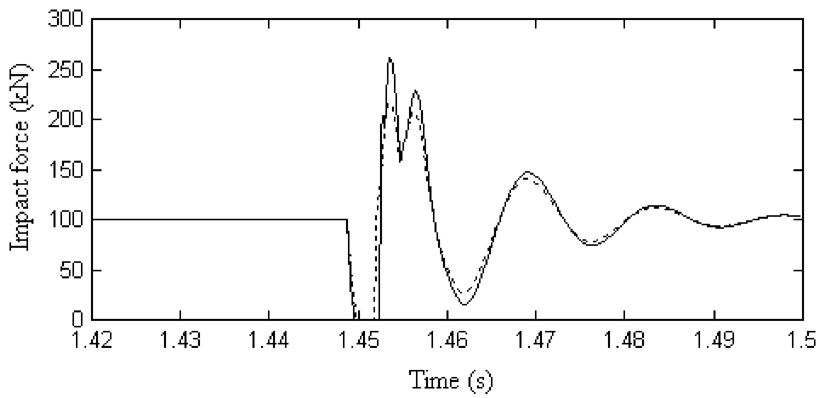

Fig. 6. Wheel/rail interaction force at speed 15 m/s due to 120 mm wheel flat: —— depth 1.6 mm, ---- depth 1.2 mm.

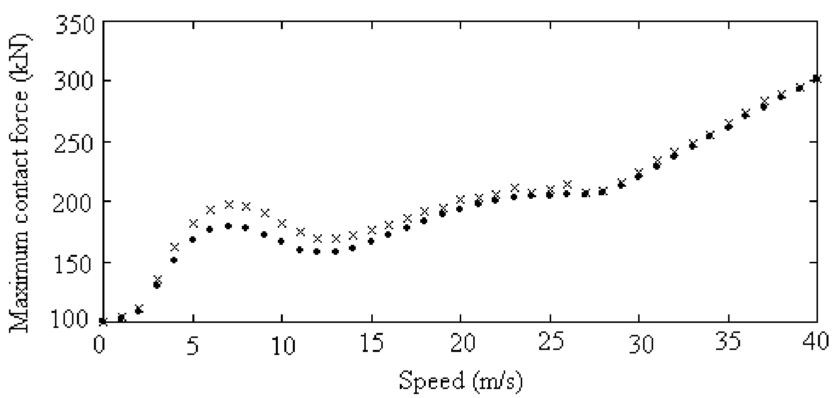

Fig. 7. Maximum contact force due to flat with length $2l = 72$ mm and depth $e = 0.6$ mm: • vehicle/rail interaction, × only wheel/rail interaction.

The impact force depends on the dimensions of the flat fault. Its value increases if the flat is longer or deeper. Fig. 6 displays the impact force for a speed of 15 m/s, a 120 mm long and 1.2 mm / 1.6 mm deep flat. The impact force has very high values, of 216.7 kN and 261.0 kN respectively. For both situations, in phase one, the wheel is completely un-loaded for 2.03 ms and 2.73 ms respectively.

The maximal impact force is influenced by the running speed, as shown in fig. 7. The result of the numerical simulation in case of a single wheel is presented as well. It is quite clear that a maximum at about 5 – 6 m/s has a strong connection with the approach of the wheel's own frequency to the frequency of passing over a flat. The numerical simulation shows that the vehicle suspension reduces the impact force at low speed especially.

The numeric results are in accordance with the experimental ones [3, 6].

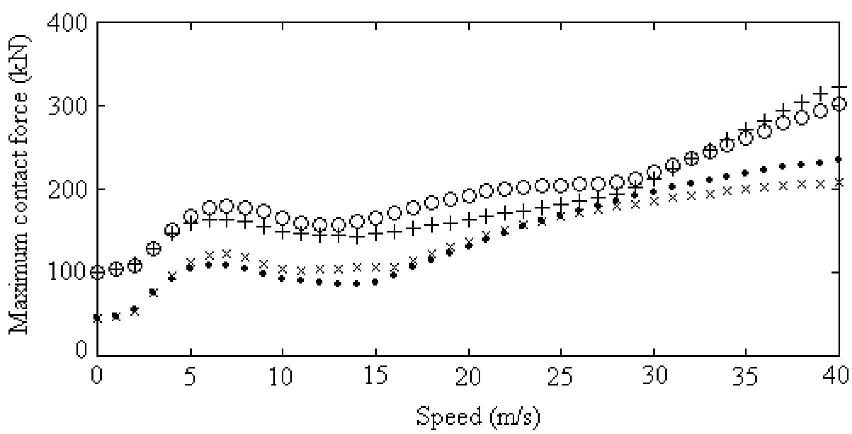

Fig. 8. Maximum contact force due to flat with length $2l = 72$ mm and depth $e = 0.6$ mm: •, $P_0 = 44$ kN and $k_r = 200$ MN/m^2, ×, $P_0 = 44$ kN and $k_r = 400$ MN/m^2, +, $P_0 = 100$ kN and $k_r = 200$ MN/m^2, ○, $P_0 = 100$ kN and $k_r = 400$ MN/m^2.

The impact force is also influenced by the stiffness of the rail pad. The rail on continuous two-layer pad has two corresponding own frequencies [8, 10], as known. If dealing with a rigid rail pad, the rail receptance is:

- smaller than the rail/sleepers first own frequency at low frequencies;
- greater than the rail/sleepers second own frequency at high frequencies.

The excitation frequency depends on the speed. As a result, the maximal impact force will be lower at low speeds if the rail pad is elastic, but the trend reverses at high speeds (see figure 8). This note is valid no matter how loaded the vehicle is.

3. Conclusions

The flat fault is a dangerous wheel defect because it may lead to very high overloads and unpleasant noise.

A numerical model has been developed to predict the vehicle/track interaction due to wheel flat excitation. The Green functions method was applied for both the rail and the vehicle, for purposes of solving the equations of motions. The predicted results are matching the experimental ones.

The vehicle/track evolution during the wheel flat/rail interaction has been detalied. The car body is isolated through the suspension, but the wheel and the rail are subject to high forces. When the speed increases or the flat has larger dimensions, the wheel/rail contact might be lost during impact. The impact force depends on the length and the depth of the flat. The result of the numerical simulation at the speed of 54 km/h reveals a 57 % increase of the impact force for a 120 mm long and 1.6 mm deep flat compared to the 72 mm long and 0.6 mm deep flat.

The study of the wheel flat/rail interaction overrates the impact force in slow speed range, up about 70 km/h for analysed case, compared to the study of vehicle with wheel flat/rail interaction.

As the vehicle speed increases, the impact force increases too. Although, a relative maximum may be observed if the frequency of ‘passing over the flat’ is a little higher than the wheel/rail first own frequency.

If the rail pad stiffness is smaller, the impact force is small at low speeds and increases at high speeds. Combining this aspect with the one presented above, the conclusion is that the rigid rail pad is preferred.

The time domain analysis of the flat wheel/rail interaction is interesting not only for determining the aggressiveness of the flat fault on the wheel/rail system, but for the noise level calculation as well, starting with the spectra of the impact force [6].

REFERENCES

[1]. *Brabie*, *D*. On the Influence of Rail Vehicle Parameters on the Derailment Process and it Consequences, Licentiate Thesis, ISSN 1651-7660, Royal Institute of Techology (KTH), Stockholm, 2005.

[2]. *Vér, I.L., Ventres, C.S. Myles, M. M.* Wheel/rail noise. Part III: impact noise generation by wheel and rail discontinuities. In: Journal of Sound and Vibration 46 (1976) 395-417.

[3]. *Newton, S. G., Clark, R. A.* An investigation into the dynamic effects on the track of wheel flats on railway vehicles. In: Journal of Mechanical Engineering Science 21, 1979, 287-297

[4]. *Nielsen, J.C.O., Igeland, A.* Vertical dynamic interaction between train and track-influence of wheel and track imperfections. In: Journal of Sound and Vibration 187 (1995) 825-739.

[5]. *Hou, K., Kalousek, J., Dong, R.* A dynamic model for an asymmetrical vehicle/track system. In: Journal of Sound and Vibration 267 (2003) 591-604.
[6]. *Wu, T. X., Thompson, D.J.* A hybrid model for the noise generation due to railway wheel flats. In: Journal of Sound and Vibration 251 (2002) 115-139.
[7]. *Wu, T. X., Thompson, D.J.* On parametric excitation of the wheel/track system. In: Journal of Sound and Vibration 278 (2004) 725-747.
[8]. *Grassie, S. L., Gregory, R. W, Harrison, D, Johnson, K.L.* The dynamic response of railway track to high frequency vertical excitation În: Journal Mechanical Engineering Science 24, 1982, 77-90.
[9]. *Mazilu, Tr.* The rail's response to the action of vertical sliding force. In: Scientific Bulletin Series D: Mechanical Engineering, vol. 68 no. 2 (2006) 41-58.
[10]. *Mazilu, Tr.* Confortul la materialul rulant. Ed. MatrixRom, Bucureşti, 2003.