AN EXTRACTION OF THE DOMINANT ROTOR-STATOR INTERACTION MODES BY THE USE OF POD TECHNIQUE IN A LOW PRESSURE CENTRIFUGAL COMPRESSOR

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Metoda descompunerii în moduri proprii ortogonale (POD) este folosită pentru analiza câmpurilor de viteză si de presiune dintr-un compresor centrifugal cu grad de comprimare mic, obținute prin simulări numerice. În comparație cu orice altă descompunere, metoda descompunerii în moduri proprii ortogonale ne furnizează cea mai bună aproximatie modală pentru orice număr luat de moduri proprii. Modurile proprii ortogonale arată caracteristicile interacțiunii nestaționare rotor-stator precum și erorile numerice.

The Proper Orthogonal Decomposition (POD) method is applied to the velocity and pressure fields from a low-pressure centrifugal compressor. The processed data come from numerical simulations. In comparison with any other decomposition, the POD gives the best modal approximation in terms of the energy expressed on any given number of modes to be taken into account. The POD modes show the behavior of unsteady rotor-stator interaction as well as the numerical errors.

Keywords: unsteady rotor-stator interaction, proper orthogonal decomposition, CFD.

1. Introduction

The flow in turbomachinery is complex: three-dimensional, viscous, and unsteady, with time scales that vary considerably. This behavior makes difficult the complete flow analysis. Usually, in the industrial practice, the flow is assumed steady in the reference frame linked to the studied row. Furthermore, one can consider that every flow is composed by a main flow and a secondary flow that contains physical phenomena with no null velocity rotor. The secondary flows are characterized by vorteces that lead to three-dimensional behavior of flow and generate the losses due to the entropy increase.

There are some sources of unsteady phenomena, in turbomachinery and they are described after their origin [1]. Because the unsteady rotor-stator interaction can affect dramatically the turbomachinery performance, we paid it a

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special attention in this paper. The majority of researchers that studied this interaction from the numerical point of view focused their research on transonic turbomachinery; therefore, there is very few information about the unsteady rotor-stator interaction for low velocity turbomachinery. Moreover, a recent study showed important discrepancies between experimental and numerical results for a low-pressure centrifugal stage [2]. Unfortunately, this study did not succeed to identify the effects that caused the major discrepancies between experimental and numerical results. For this reason, we have considered that it is useful to study the rotor-stator interactions in a low-pressure centrifugal stage.

Up to now, the Fourier transform is a common tool for data storage or for the analysis of any periodic signal. Some recent studies [3, 4] clearly showed that POD is a more efficient method to extract the dominant modes involved in unsteady flow field. Unfortunately, these studies applied POD only for one-dimensional decompositions. In order to take the full advantage of POD method, we have applied it for decomposition of full three-dimensional flow field.

2. Nomenclature

e internal energy (J/kg); f external acceleration (m/s^2)
h static enthalpy (J/kg) I rothalpy (m^2/s^2)
p static pressure (Pa); q_v heat source (W/m^3)
R gas constant (J/(kg·K)); T static temperature (K); t time (s)
u_i Cartesian components of velocity (m/s)
v_i Cartesian components of absolute velocity (m/s)
w_i Cartesian components of relative velocity (m/s)
x_i Cartesian components of position vector (m)
δ_{ij} Kronecker delta; κ thermal conductivity (W/(m·K))
μ dynamic viscosity (kg/(m·s)); μ_t eddy viscosity (kg/(m·s))
ρ static density (kg/m^3); τ shear stress tensor (Pa)
ε_{ijk} Levi-Civita symbol;

Ω_i Cartesian components of angular velocity (rad/s)

Subscript
t turbulent; Superscript
eff effective (laminar + turbulent)

3. Governing equations

For a three-dimensional rotating Cartesian coordinate system, the unsteady Reynolds-averaged Navier-Stokes equations using the Favre averaging (a mass-weighted averaging) could be written in the conservative form as [5, 6]

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j) = \rho f_i + \frac{\partial}{\partial x_j}(\tau_{ij}^{eff} - p\delta_{ij}) \tag{2}$$

$$\frac{\partial}{\partial t}(\rho E) + \frac{\partial}{\partial x_j}(\rho u_j I) = \rho f_j u_j + q_v + \frac{\partial}{\partial x_j}\left(\kappa^{eff}\frac{\partial T}{\partial x_j} + \tau_{ij}^{eff} u_i\right) \tag{3}$$

where energy E and rothalpy I are defined by:

$$E = e + \frac{w_i^2}{2} - \frac{v_{ri}^2}{2}. \tag{4}$$

$$I = h + \frac{w_i^2}{2} - \frac{v_{ri}^2}{2} \tag{5}$$

$$w_i = u_i - v_{ri}, v_{ri} = \varepsilon_{ijk}\Omega_j x_k \tag{6}$$

According to the Boussinesq hypothesis and Stokes postulates and hypothesis for a Newtonian fluid, the shear stresses τ^{eff} may be written as

$$\tau_{ij}^{eff} = \left(\mu + \mu_t\right)\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\left(\mu + \mu_t\right)\frac{\partial u_k}{\partial x_k}\delta_{ij} \tag{7}$$

The Sutherland's formula could be used to determine the dynamic viscosity μ as a function of temperature, while the eddy viscosity μ_t is computed with a turbulence model

For gases, the gravitational acceleration is very small, therefore it can be neglected. Moreover, we can assume that the thermal conductivity is the single heat source; therefore the heat source q_v is null:

$$q_v = 0 \tag{8}$$

The pressure is obtained from the equation of state of ideal gas

$$p = \rho RT \tag{9}$$

4. Numerical simulation

The numerical simulations of the three-dimensional viscous flow were carried out on a centrifugal compressor designed, manufactured and tested by COMOTI, with commercial CFD code FLUENT that is based on finite volume method where each unknown takes an average value on each discretization cell. The computational domain generated in Gambit was split into eight blocks to

facilitate the building of a fully structured mesh as shown in Fig. 1. The mesh for which, the results are given, has about 253 000 hexahedral cells for the impeller passage and 127 000 hexahedral cells for the vaned diffuser passage.

In order to decrease the computational time, impressively, the time discretization is made with a backward implicit first order scheme and multigrid technique is used. To take into account the physical properties of flow, the convective fluxes are discretized with Roe scheme, which is a Godunov-type scheme [5, 6]. Because the turbulence is not a critical issue of this study, we used the Spalart-Allmaras model, which is a one-equation model [7].

At the inlet, a uniform stagnation pressure (96 310 Pa) and temperature (300 K) are imposed, turbulent viscosity ratio μ_t/μ is 10 and the flow is normal to inlet. At the outlet, a uniform static pressure (156 000 Pa) is imposed. At the left and right sides of computational domain, the rotational periodic boundary conditions are imposed. All the walls have been assumed adiabatic. The shaft speed of impeller is 14 915 rpm.
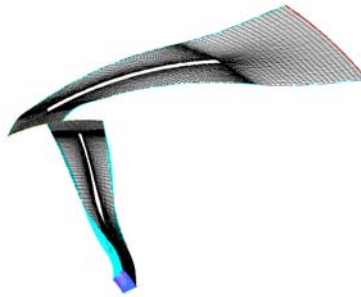

Fig. 1. Computational domain of centrifugal compressor

5. Proper orthogonal decomposition - POD

In the field of fluid mechanics two approaches have been used for the POD. Historically the method of Continuous POD (or the classical method) of Lumley [8] is proceeded by the Snapshot POD of Sirovich [9]. More information regarding the application of the proper orthogonal decomposition in the analysis of turbulent flows together with a detailed bibliography is given in [10]. In this paper, we used the Snapshot POD because it is much more efficient from the numerical point of view.

The POD is a method that reconstructs a data set from its projection onto an optimal base. Besides using an optimal base for data reconstruction the POD does not use any prior knowledge of the data set. That is why the basis is only data dependent and this is reason for which the POD is used also in analyzing the natural patterns of the flow field.

For the reconstruction of the dynamic behavior of a system, the POD decomposes the data set in two parts: a time dependent part, $a_k(t)$, that forms the orthonormal amplitude coefficients and a space dependent part, $\psi_k(x)$, that forms the orthonormal basis. The reconstructed data set is:

$$u(x,\, t) = \sum_{k=1}^{M} a_k(t) \cdot \psi_k(x) \tag{10}$$

where M is the number of time instant observations in the data set.

We denote the error of the reconstructed data set as:

$$\varepsilon(x,t) = u(x,t) - \sum_{k=1}^{m} a_k(t) \cdot \psi_k(x) \tag{11}$$

The base from which the data set is reconstructed is said to be optimal in the sense that the average least squares truncation error is minimized for any given number ($m \leq M$) of basis functions over all possible sets of orthogonal functions:

$$\varepsilon_m = \left\langle (\varepsilon,\, \varepsilon) \right\rangle \tag{12}$$

where the $\langle . \rangle$ is the ensemble average and $(.,.)$ is the standard Euclidian inner product.

It was shown that the minimization condition for the error $\varepsilon(x,t)$ translates into maximum condition for:

$$\lambda = \frac{\left\langle |u,\psi|^2 \right\rangle}{(\psi,\psi)} \tag{13}$$

This maximization can be proven to take place if the time independent base functions $\psi(x)$ are obtained from the Fredholm integral equation:

$$\sum_{j=1}^{M} \int R_{ij}(x,x') \cdot \psi_j(x') dx' = \lambda \psi_i(x) \tag{14}$$

where R_{ij} is the correlation kernel. In this way, we transform this into an eigenvalue problem and λ_k is the eigenvalue corresponding of the eigenvector ψ_k. Because we can consider the inner product as being the equivalent of an "energy", the value of λ_k is linked to the energy contained in mode ψ_k and the involved optimization process can be summarized as follows: the data set is projected onto a basis that maximizes the energy content.

In the classical approach of Lumley [8], the correlation matrix is constructed as a space correlation matrix. Solving the eigenvalue problem, we obtain directly the eigenvectors as the spatial modes and then use them in order to obtain the time-dependent coefficients

$$a_k(t) = (u(x,t), \psi_k(x)) \tag{15}$$

In the Snapshot POD of Sirovich [9], the correlation matrix is a time correlation matrix:

$$C = \frac{1}{V}\int_V u(x,t)\cdot u(x,t')\,dV \tag{16}$$

which is of the size of the square of the number of snapshots. From the time correlation matrix, we get the eigenvalues λ_k and time dependent eigenvectors $\phi_k(t)$. The spatial eigenmodes that are time independent, are computed according to the formula:

$$\psi_k(x) = \frac{1}{\mu_k}\int_t \phi_k(t) u(x,t)\,dt \tag{17}$$

where

$$\mu_k = \sqrt{|\lambda_k|} \tag{18}$$

For the reconstruction of $u(x,t)$, we take into account only a small number of modes that contain the most energy:

$$u(x,t) = \sum_{k=1}^{m}\mu_k \phi_k(t)\psi_k(x) \tag{19}$$

6. Results

The processed data are the absolute velocity magnitude and static pressure fields obtained from numerical simulations using the commercial CFD code Fluent. For each period, we took 20 snapshots and the time between adjacent snapshots is of $\Delta t = 9.5781\mu s$; therefore, the Snapshot POD of Sirovich yields 20 eigenmodes for each considered field.

Table 1

Fraction of total energy for the most energetic modes

Mode	Fraction of total energy for variation of static pressure field	Fraction of total energy for variation of absolute velocity magnitude field
1	6.40E-01	5.49E-01
2	2.65E-01	4.06E-01
3	6.63E-02	2.13E-02
4	1.72E-02	1.58E-02
5	8.97E-03	5.73E-03
6	1.41E-03	7.57E-04
7	6.29E-04	4.50E-04
8	1.60E-04	8.52E-05
9	4.58E-05	7.30E-05
10	4.02E-05	5.25E-05

The very high efficiency of the proper orthogonal decomposition is clearly underlined by table 1. The sum of the first two modes represents 90.5% and 95.5% of the total energy, respectively for the variations of static pressure and absolute velocity magnitude fields while the sum of modes 6 to the last mode represents only 0.163% and 0.236% of the total energy, respectively for the variations of static pressure and absolute velocity magnitude fields. Therefore, both variations of static pressure and absolute velocity magnitude fields can be very accurately reconstructed using only the first five modes. Furthermore, these results confirm that the base from which the data set is reconstructed is indeed optimal.

It the following, we give results for some control points placed in a section at mid height of blade of vaned diffuser, at the middle distance between the blade and the right periodic as shown in Fig. 2. The numbering of these control points is from upstream to downstream.
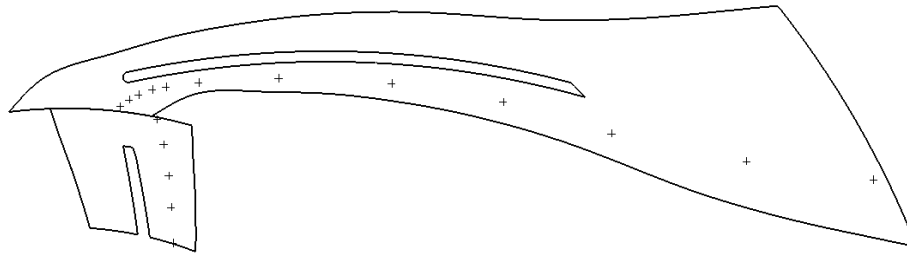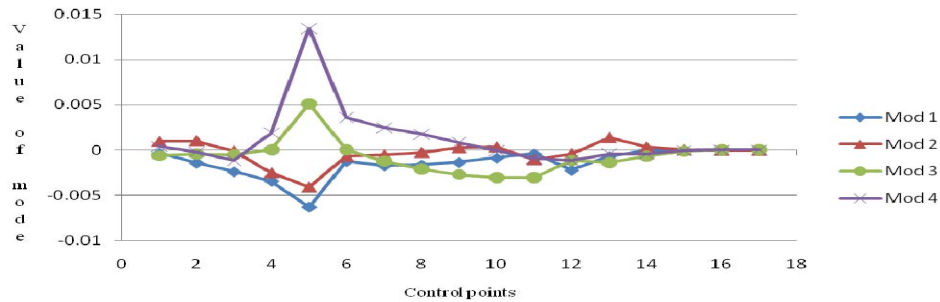

Fig. 2. Placement of control points

Fig. 3. The first four most energetic modes of variation of static pressure field

The sum of the first three most energetic modes of variation of static pressure field is 97.1% of the total energy. These modes are physical because they show how the potential and wake effects affect the flow, especially in the impeller region. More exactly, the first mode that contains 64% of the total energy shows especially the potential effects that affect the flow in the impeller region. For this reason, the peak of this mode is located on the interface between impeller region and vaned diffuser region. The second mode contains 26.5% of the total energy

and it shows especially, the interaction between the wakes due to the blunt trailing edge of impeller blade and potential effects. The peak of this mode is also placed near the interface between impeller region and vaned diffuser region because, at the middle distance between rows, this interaction is usually maximal. The third mode has 6.6% of the total energy and it represents mainly, the potential effects and the interaction between the wakes due to the blunt trailing edge of impeller blade and potential effects that cannot be captured by the first two modes. The last modes contain only 2.9 of the total energy and represent mainly, the numerical errors that occur at the interface between rotating region and stationary region and due the rotational periodicity condition that is not too correct. Fortunately, they contain little energy.
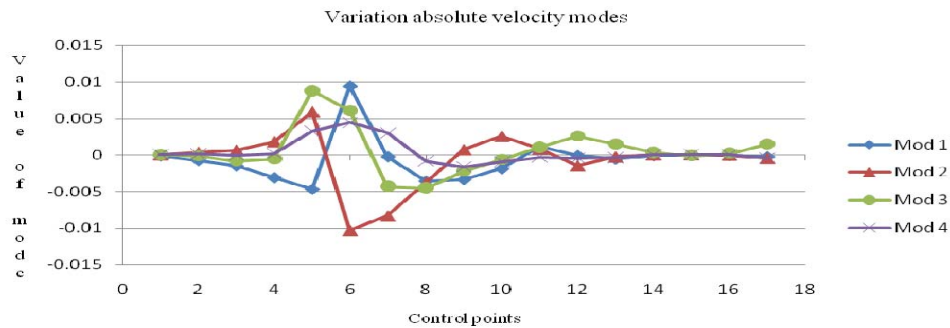

Fig. 4. The first four most energetic modes of variation of absolute velocity magnitude field

The first two most energetic modes of variation of absolute velocity magnitude field contain as much as 95.5% of the total energy. The first mode has 54.9% of the total energy and it represents the interaction between wakes due to the blunt trailing edge of impeller blade and potential effects. According to theory of characteristics, this interaction affects especially the vaned diffuser region and its peak is located near the middle distance between impeller and vaned diffuser. The second mode contains 40.6% of total energy and it represents the interaction between wakes and potential effects in the vaned diffuser region as well as the propagation of potential effects in the impeller region. The third and fourth modes have 3.7% of the total energy and they contain both physical and numerical information. From the physical point of view, they contain the information regarding the interaction between potential effects as well as the influence of potential effects in the impeller region. From the numerical point of view, they represent the numerical errors that occur at the interface between rotating region and stationary region and due the rotational periodicity condition. Furthermore, one sees that the value of the third mode is not close to zero at the outlet boundary of computational domain because we imposed a uniform static pressure on this frontier and this is not too correct according to the theory of characteristics [5, 6].

Because the POD is a method that reconstructs a data set from its projection onto an optimal base, we need only four modes to rebuild the variations

of static pressure and absolute velocity fields accurately. We will rebuild them for two snapshots placed at half the period (the number of impeller blades is equal to the number of vaned diffuser blades) as shown in Figs. 5 and 6.
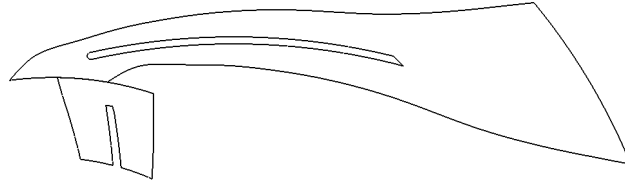

Fig. 5. The first snapshot for which the reconstruction is built

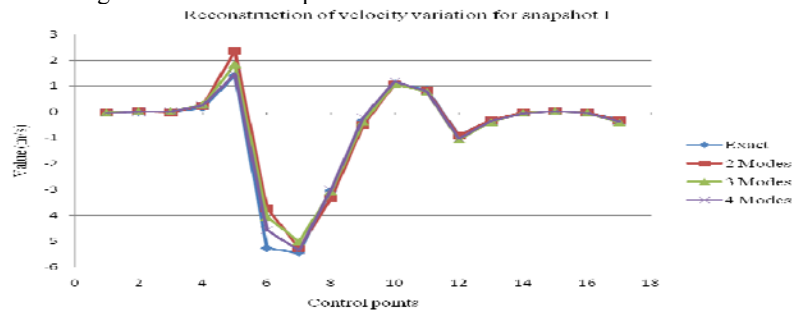Fig. 6. The second snapshot for which the reconstruction is built

Fig. 7. Reconstruction of absolute velocity magnitude field for snapshot 1
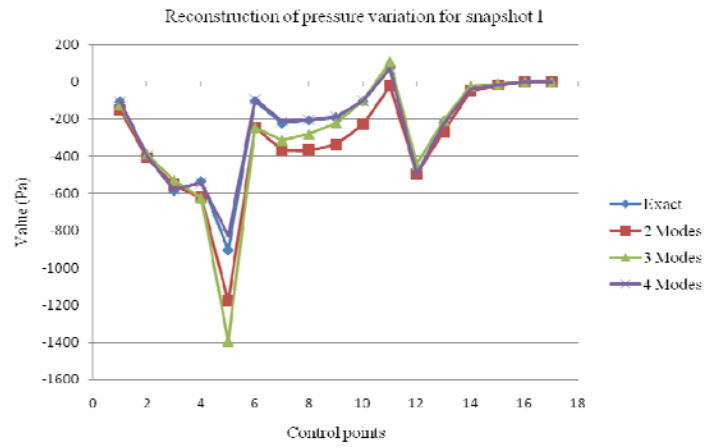
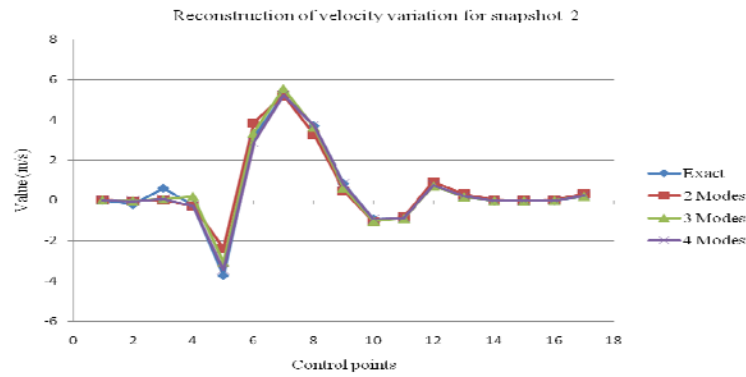Fig. 8. Reconstruction of static pressure field for snapshot 1

Fig. 9. Reconstruction of absolute velocity magnitude field for snapshot 2
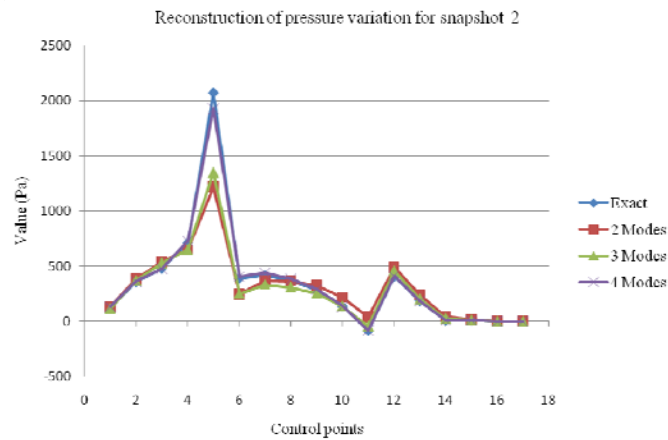

Fig. 10. Reconstruction of static pressure field for snapshot 2

Analyzing the Figs. 7-10, one observes that only four modes are enough to reconstruct accurately the variations of static pressure and absolute velocity magnitude fields. Furthermore, the points where the field u(x,t) has high absolute values are better reconstructed than the points with small absolute values because the data set is projected onto a basis that maximizes the energy content. In other words, points with high energy (information) are reconstructed more accurately than points with low energy.

7. Conclusions

The POD method has been successfully applied to the decomposition and reconstruction of variations of fully three-dimensional static pressure and absolute velocity magnitude fields obtained from numerical simulations using the commercial CFD code Fluent. Both variations of static pressure and absolute velocity magnitude fields can be accurately reconstructed using only the first four modes; therefore, the proper orthogonal decomposition method is a very efficient method for the data storage of unsteady flows. Moreover, the POD technique is able to capture the relevant features of the unsteady rotor-stator interaction, especially, the potential effects and the interaction between wakes and potential effects. Furthermore, the POD method clearly shows the numerical errors such as those errors that occur at the interface between rotating region and stationary region because the information exchange does not use the characteristic variables, the reflection of numerical waves at rotational periodic and outlet boundaries as well as their magnitude. In order to obtain more accurate results [3, 11], we should impose the phase-lagged condition, which is not yet available in Fluent, on the left and right sides of computational sides, instead of the rotational periodicity condition.

Acknowledgments

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