ON THE *-FOLD FUZZY CONTEXTUAL GRAMMARS WITH CHOICE

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*Se prezintă o modalitate nouă de introducere a conceptului de "fuzzy" în structura gramaticilor contextuale selective (Marcus), prin definirea gramaticilor contextuale selective *-valente. Este investigată capacitatea generativă a acestor gramatici.*

*One presents a new way of introducing the concept of fuzziness in the structure of the contextual grammars with choice (Marcus), by defining the * -fold contextual grammars with choice. The generative capacity of these grammars is investigated.*

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1. Introduction

In the theory of the formal languages, one may consider two different classes of general devices, namely, grammars which use nonterminal symbols in the string derivation process, and grammars without nonterminal symbols, respectively. The contextual grammars introduced by Marcus [1-3] belong to the last category. In a previous paper [4] based on the concept of fuzzy set introduced by Zadeh [5], Q-fuzzy contextual grammars (simple, generalized, and with choice) were defined over different algebraic structures Q (the [0,1] lattice, Boole Algebra, semiring, semipartial semiring) and their generative capacity was investigated. In this work it is studied another way of introducing the concept of fuzziness in the structure of the contextual grammars with choice.

2. The *-fold fuzzy contextual grammars with choice

Definition 2.1 A ***-fold Q-fuzzy contextual grammar with choice** is the ordered system

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$$QG = (V, B, C, \varphi, Q)$$

where V is a finite nonempty vocabulary

$B \subset V^*$ is a finite language, in addition B is a Q fuzzy set in V* (called the base of grammar QG)

$C \subset V^* \times V^*$ is a finite set of contexts

$\varphi : (\text{Init } C)^* B (\text{Fin } C)^* \to \mathcal{P}(C \times Q)$

where $\text{Init } C = \{ u_i \mid \text{there is } < u_i, v_i > \in C \}$

$\text{Fin } C = \{ v_i \mid \text{there is } < u_i, v_i > \in C \}$

The fuzzyfication is made over the algebraic structure Q.

The relation

$$(< u_i, v_i >, \mu_i) \in \varphi(u_{i-1} ... u_1 x v_1 ... v_{i-1})$$

has the following meaning: the context $< u_i, v_i >$ is applied with the grade μ_i after the sequential application of the contexts $< u_1, v_1 >, < u_2, v_2 >, \ldots, < u_{i-1}, v_{i-1} >$ to the word $x \in B$. In this case $\mu_i \neq 0$. For the words y which have not the form $u_{i-1} ... u_1 x v_1 ... v_{i-1}$, $\varphi(y) = \emptyset$.

Definition 2.2 A Q-fuzzy contextual generation in the grammar QG of a word $y \in V^*$ of the form $y = u_n ... u_1 x v_1 ... v_n$, where $x \in B$ with the grade $\mu_B(x)$ and the context $< u_i, v_i > \in C$ is applied with the grade μ_i after the contexts $< u_1, v_1 >$, $< u_2, v_2 >, \ldots,$ $< u_{i-1}, v_{i-1} >$ were sequentially applied to the word x , for $i = 2, 3, ..., n$ is the string :

$$CG : (x, \mu_B(x)), (< u_1, v_1 >, \mu_1), ..., (< u_n, v_n >, \mu_n)$$

The grade of the Q-fuzzy contextual generation of the word $y \in V^*$ by the grammar QG is defined as:

$$\mu_{QG}(y) = \oplus \left[\mu_B(x) \odot \bigodot_{i=1}^{n} \mu_i \right], \ n \geq 0 \tag{1}$$

where $\oplus$ is taken over all the contextual generations CG of the word y. (the symbols "$\oplus$" and "$\odot$" denote the additive law and the multiplication law, respectively, from the algebraic structure Q).

Definition 2.3 The Q-fuzzy language L(QG) generated by *-fold Q-fuzzy contextual grammar with choice QG is a Q-fuzzy set in V* characterized by the membership function $\mu_{QG}(y)$ given by (1).

Let λ be an element of Q. Then the language $L(QG, \lambda)$ generated by QG with a threshold λ is a subset of V*:

$$L(QG, \lambda) = \{ y \mid y \in V^* \text{ and } \mu_{QG}(y) \geq \lambda \}$$

One can also define the language $L(QG, =, \lambda)$ as

$$L(QG,=,\lambda)=\{\,y \mid y\in V^* \text{ and } \mu_{QG}(y)=\lambda\,\}$$

The family of the languages generated by the *-fold Q-fuzzy contextual grammar with choice with a threshold λ, $\lambda \in Q$, will be denoted by $\mathcal{C}^{\lambda}_{Q*}$.

The next theorem establishes the relation between $\mathcal{C}^{\lambda}_{Q*}$ and $\mathcal{C}^{\lambda}_{Q}$ (the family of the languages generated by Q-fuzzy contextual grammars with choice and a threshold λ defined in [6]).

Theorem.

$$\mathcal{C}^{\lambda}_{Q} \subseteq \mathcal{C}^{\lambda}_{Q*} \tag{2}$$

Proof. Let $L = L(QG,\lambda)$ be the language generated by the Q-fuzzy contextual grammar with choice $QG=(V,B,C,\varphi,Q)$ with a threshold $\lambda\in Q$ (introduced in [4]). We construct the *-fold Q-fuzzy contextual grammar with choice $QG'=(V,B,C',\varphi',Q)$ where:

$$C'=\{<u,v>\in C \mid \mu_C(<u,v>)\neq 0\} \text{ and}$$

$$\varphi' : (\text{Init } C')^* B (\text{Fin } C')^* \to \mathcal{P}(C'\times Q)$$

is defined in the following way:

$$(<u_i,v_i>,\mu_i)\in\varphi'(u_{i-1}...u_1xv_1...v_{i-1})$$

if $\mu_C(<u_i,v_i>)=\mu_i$ and $<u_i,v_i>\in\varphi(u_{i-1}...u_1xv_1...v_{i-1})$ for $i=2,3,...,n$. We denote by $L'=L(QG',\lambda)$ the language generated by the above grammar QG', with the threshold $\lambda,\lambda\in Q$. One can verify that the languages L and L' are equal and thus the inclusion (2) is proved.

Since [4] one has

$$\mathcal{C} \subset \mathcal{C}^{\lambda}_{Q} \tag{3}$$

if Q is a Boole algebra, it results from (2) and (3) that $\mathcal{C}\subset\mathcal{C}^{\lambda}_{Q*}$.

We give an example of a language $L\in\mathcal{C}^{\lambda}_{Q*}$ which does not belong to $\mathcal{C}$. The language $L=\{ba^{2^n}b \mid n\geq 0\}$ can be generated by a *-fold Q-fuzzy contextual grammar with choice, with the threshold $\lambda,\lambda\in Q$.

Let us consider the *-fold Q-fuzzy contextual grammar with choice

$$QG=(\{a,b\},B,C,\varphi,Q)$$

where

(i) Q is a semipartial semiring

(ii) B is a Q-fuzzy set in V* caracterised by the membership function

$$\mu_B(x)=\begin{cases} x_1 & \text{if } x=\varepsilon \\ 0 & \text{if } x\neq\varepsilon \end{cases}$$

where $x_1 \in Q$.

(iii) $C = \{<a,a>, <b,b>\}$

(iv) $\varphi : (\text{Init } C)^* B(\text{Fin } C)^* \to \mathcal{P}\ (C \times Q)$

$\varphi(a^m) = \{(<a,a>,1)\}$, $m \neq 2^n$

$\varphi(a^{2^n}) = \{(<a,a>,1),(<b,b>,x_1)\}$

$\varphi(x) = \emptyset$ for the other cases.

It can easily be seen that the language generated by the grammar QG with a threshold equal to x_1^2 is $L(QG,=,x_1^2) = \{ba^{2^n}b \mid n \geq 0\}$. Thus $L \in \mathcal{C}^{\lambda}_{Q^*}$.

Suppose now that $L \in \mathcal{C}$. Let us define the homomorphism

$$h : \{a,b\} \to \{a,b\}^* \text{ such that } h(a) = a\ ,\ h(b) = \varepsilon$$

Since [7] the family of the contextual languages with choice is closed to the arbitrary homomorphism it follows that

$$L_1 = h(L) = \{a^{2^n} \mid n \geq 0\} \in \mathcal{C}$$

which is a contradiction because $L_1 \notin \mathcal{C}$[2]. This proves that $L \notin \mathcal{C}$.

3. Conclusions

A new class of grammars is obtained by defining the * -fold contextual grammars with choice in which the fuzzy application of a context is made after the sequential application of a number of contexts. The generative capacity of these grammars is increased if the fuzzyfication is made over a Boole Algebra.

R E F E R E N C E S

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