LOSS FUNCTIONS USED IN THE QUALITY THEORY

Constantin TÂRCOLEA[1], Adrian Stere PARIS[2]

Filosofia Taguchi reprezintă un nou punct de vedere în decizia statistică, în locul tipului clasic dichotomic, bun/defect, cu scopul principal de a îmbunătăţi calitatea produselor în practica industrială. În lucrare sunt definite câteva noi modele ale funcţiei de pierdere Taguchi determinate cu software statistic. Se propun legi statistice de pierdere a calităţii, distribuţii simetrice şi asimetrice, şi astfel se obţin modele adecvate, care dau o aproximare mai bună în lumea reală.

The Taguchi philosophy is a new point of view in the statistical decision instead of classical dichotomic type of decision good/bad, with the principal goal of improving quality of products in the industrial practice. In the present paper are introduced some new adaptive models of Taguchi's loss functions using statistical software. We propose quality loss laws, which are symmetrical and asymmetrical distributions, and therefore more adequate models, which give a better approximation in the real world.

Keywords: Quality loss functions, statistical distributions, software application

1. Introduction

From the practical point of view, the goal of Taguchi methods is to find a trade-off between quality loss and product price. The equilibrium between levels of different factors, robust tolerance design, and costs is based on two main concepts proposed by Taguchi: quality loss function and signal/noise ratio [1;2;3;4].

In the competitive market, the company that holds customer confidence with on-time delivery of defectfree, reliable products and services is the company that will succeed. Customer satisfaction cannot be a frill or a fluke; it is imperative for companies wishing to earn or maintain world-class stature. Quality is then a gateway to success in the global free-for-all for customer satisfaction and loyalty. Quality in products and product related processes is now, more than ever, a critical requirement for success in manufacturing [5].

In 1986, Taguchi presented the quadratic quality loss function for reducing deviation from the target value.

[1] Prof., Applied Sciences Faculty, University POLITEHNICA of Bucharest, Romania e-mail: constantin_tarcolea@yahoo.com,

[2] Assist., Dept. of Material Technology and Welding, University POLITEHNICA of Bucharest, Romania

The objective of this quality improvement method is to minimize total losses to society. Taguchi's concept is different from the traditional concept of conformance to specifications.

Subsequently, the quadratic quality loss function has been applied in on-line and off-line quality control, for obtaining the economic design of control charts, of sampling plans, and of specification limits. The advantage of the quadratic loss function is that it is simple and many mathematical methods are readily available.

In order to reduce expected losses with respect to the quadratic loss function, the process mean should be close to the target and process standard deviation should be small.

Thus, if the quality characteristic concentrates on the target value with minimum standard deviation, then it is said that the product has minimum quality loss. However, evidently the quadratic loss function is inappropriate in many situations [6].

According to Taguchi's quality engineering philosophy and methodology, there are three important steps in designing a product or process: system design, parameter design and tolerance design.

The aim of system design is to create a product that indeed possesses the properties intended for it at the planning stage. This involves the development of a prototype, choice of materials, parts, components, assembly system and manufacturing processes, so that the product fulfils the specified conditions and tolerances at the lowest costs.

Parameter design tries to determine the connections between controllable and noise factors, in order to ascertain the best combination of factor levels in the manufacturing process, having the purpose of achieving robustness, and improving quality, without increasing costs.

In the last stage, tolerance design tries to narrow the ranges of the operating conditions, so that the most economical tolerances are obtained.

According to Taguchi's viewpoint, the quality loss function is a measure for the evaluation of deviations from the target values of the product, even when these lie within specifications. The literature indicates three type of tolerances:"the nominal - the best", "the smaller - the better", "the larger - the better", and, therefore, there are three resulting classes of loss functions. "The nominal - the best" type is required in many cases when a nominal characteristic can vary in two directions. Various studies proposed different loss functions for evaluating the quality level in one-dimensional case [7].

Many researchers proposed quadratic loss function, based on approximating the continuous loss function by its Taylor theory expansion up to its quadratic term. In 2007 Fathi and Poonthanomsook [8] used series expansion up to quartilic form and developed corresponding quartilic loss functions.

Consider a quality characteristic whose value is represented by Y, and we assume that the target value of Y is τ. Let L(y) denote a function representing the monetary value of the losses incurred by a unit whose quality characteristic is Y = y. We refer to L(y) as the loss function for this quality characteristic, and we assume that this function is continuous and differentiable everywhere. To approximately determine this function we expand it in the Taylor series about the target value τ up to its quartic term:

$$L(y) \approx L(\tau) + \frac{L^{'}(\tau)}{1!}(y-\tau) + \frac{L^{''}(\tau)}{2!}(y-\tau)^2 + \frac{L^{'''}(\tau)}{3!}(y-\tau)^3 + \frac{L^{''''}(\tau)}{4!}(y-\tau)^4 \quad (1)$$

Clearly we have L(τ)=0; also, since the minimum value of the function is attained at the target τ, the first derivative of the function at this point is also zero, i.e., L'(τ)=0. Therefore equation (1) reduces to:

$$L(y) = \frac{L^{''}(\tau)}{2!}(y-\tau)^2 + \frac{L^{'''}(\tau)}{3!}(y-\tau)^3 + \frac{L^{''''}(\tau)}{4!}(y-\tau)^4, \quad (2)$$

which we can rewrite as:

$$l(y) = k_2(y-\tau)^2 + k_3(y-\tau)^3 + k_4(y-\tau)^4 \quad (3)$$

This expression represents the general form of the quartic loss function that they propose and its coefficients k_2 , k_3 and k_4 are constants, that are we refer to as the *second, third and fourth order.*

The new approach, that we introduced for loss functions, is based on approximating continuous loss functions ,using regression models.

2. New laws used as quality loss functions

In the following it is proposed some types of models using polynomial, exponential, gamma and beta pattern [9;10;11]. The canonical distributions have usually unbounded range, while the quality loss function can be non-zero only on bounded interval. The distribution analysis of experimental data points indicated that curve profiles can be described more adequately different of the degrees of skewness and peakedness of the curves.

For a sample of product the average loss can be decomposed as a sum of the variance and bias. According to this approach, a manufacturing process must to have two complementary goals: zero bias and the smallest possible variance. Taguchi philosophy of quality control focuses on the design stage. The deviations from control target should be evaluated in terms of the loss of quality they cause.

A loss occurs even if the outcome is still within pre-specified tolerance bounds. The analysis of experimental data were performed on CurveExpert [12] software developed for curve fitting (nonlinear regression - least squares method, Levenberg-Marquardt algorithm). In this paper were used the following functions as example:

a. Polynomial model

$$y_1 = a + bx + cx^2 + dx^3 \quad (4)$$

For the experimental data it results:

$$y_1 = -460 + 189x - 24x^2 + 0.966x^3 \quad (5)$$

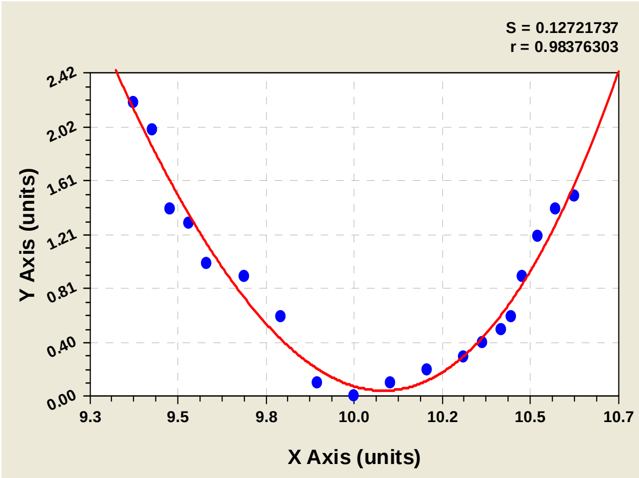

Fig..1 Plot of the polynomial model

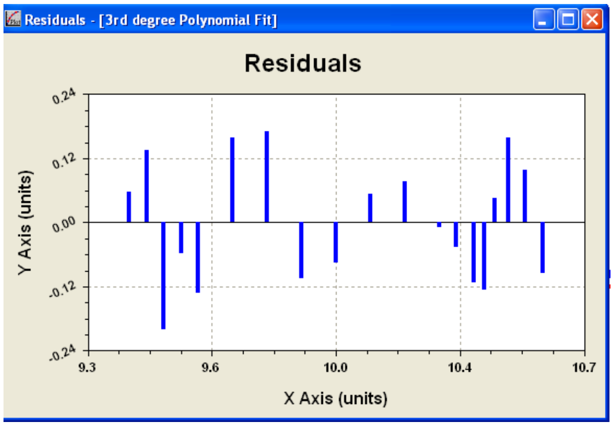

Fig.2 Residuals of the polynomial model

b. Sinusoidal model

$$y_2 = a + b\cos(cx + d) \quad (6)$$

For the experimental data it results:

$$y_2 = 5.53 + 5.5\cos(1.39x + 39.5) \quad (7)$$

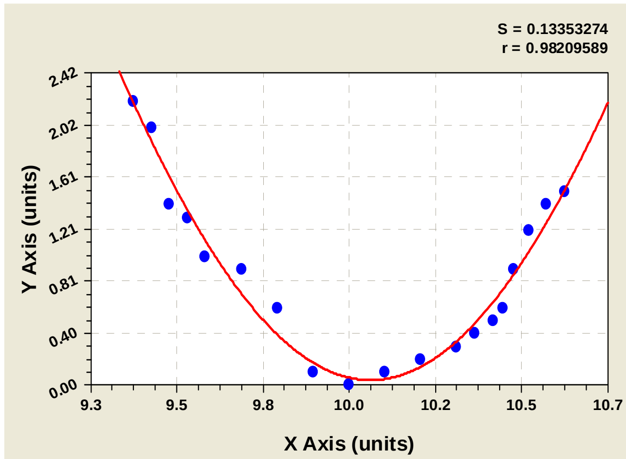

Fig.3 Plot of the sinusoidal model

c. Heat capacity model

$$y_3 = a + bx + \frac{c}{x^2} \quad (8)$$

For the experimental data it results:

$$y_3 = -450 + 32.5x + \frac{16461}{x^2} \quad (9)$$

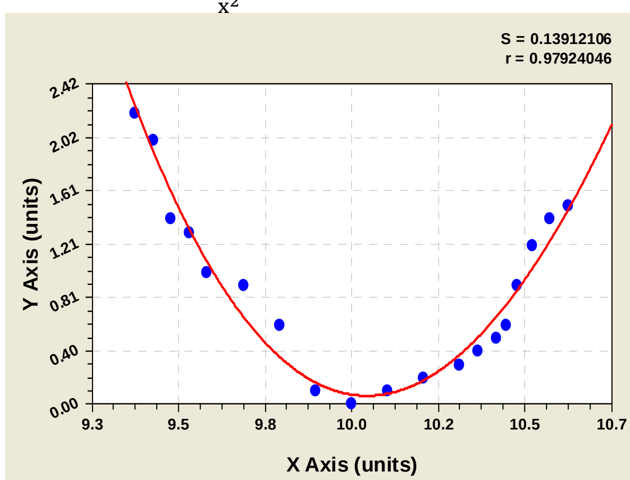

Fig.4 Plot of the heat capacity model

d. A hyperbolic cosine model

$$\boldsymbol{y_4 = ae^{b(x-10)} + ae^{b(10-x)} - 2a} \quad (10)$$

and for the experimental data it results:

$$\boldsymbol{y_4 = 1.036e^{2.067(x-10)} + 1.036e^{2.067(10-x)} - 2.072} \quad (11)$$

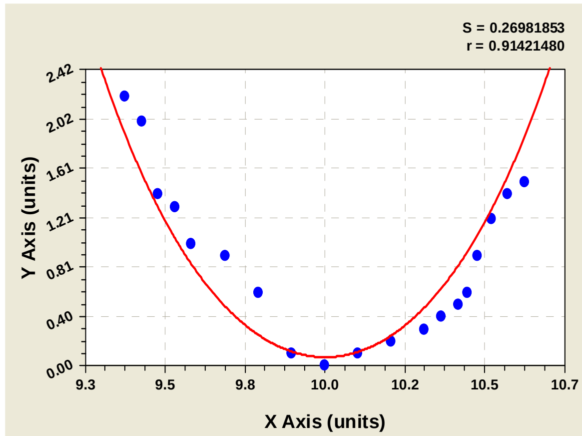

Fig.5 Plot of the hyperbolic cosine model

e. An adapted hyperbolic cosine model

$$y_5 = ae^{b(x-10)} + ae^{c(10-x)} - 2a \tag{12}$$

and for the experimental data it results:

$$y_5 = 1.05e^{1.875(x-10)} + 1.05e^{2.2(10-x)} - 2.1 \tag{13}$$

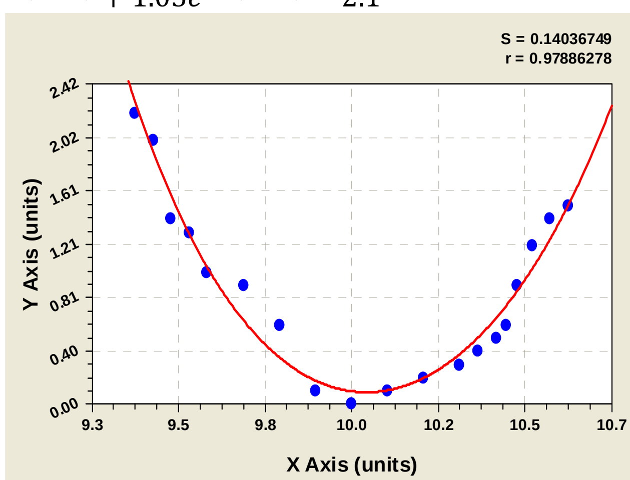

Fig.6 Plot of the adapted hyperbolic cosine model

f. A generalized hyperbolic cosine model

$$y_6 = ae^{b(x-10)} + ce^{d(10-x)} - a - c \quad (14)$$

and for the experimental data it results:

$y_6 = 3.74e^{1.274(x-10)} + 7.212e^{0.764(10-x)} - 3.74 - 7.212$ (15)

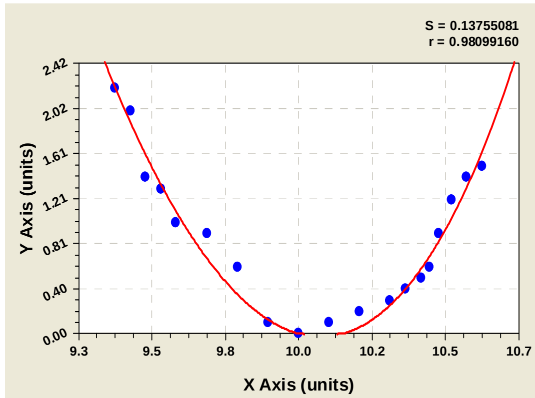

Fig.7 Plot of generalized hyperbolic cosine model

g. Sum of generalized shifted gamma

$$y_7 = ax^b e^{c(x-10)} + dx^e e^{f(10-x)} - g \quad (16)$$

and for the experimental data it results:

$$y_7 = 0.313x^{0.585} e^{1.974(x-10)} + 0.494x^{0.7} e^{1.373(10-x)} - 3.576 \quad (17)$$

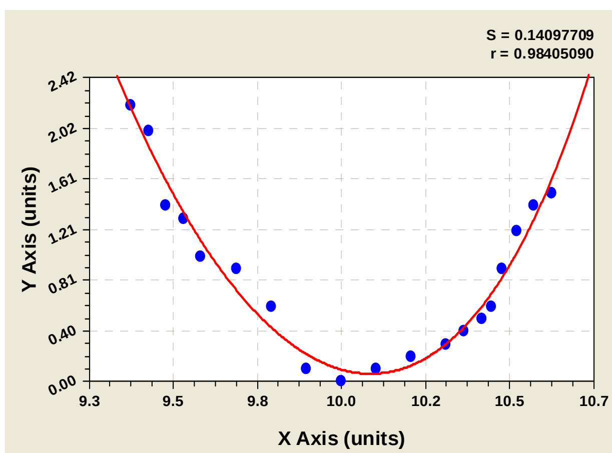

Fig. 8 Plot of sum of generalized shifted gamma model

h. Adapted beta model

$$y_8 = a(x-9.2)^b (10.8-x)^c - a0.8^{bc} \quad (18)$$

and for the experimental data it results:

$$y_8 = -5.27(x-9.2)^{2.2}(10.8-x)^{1.77} + 5.27 * 0.8^{2.2*1.77} \quad (19)$$

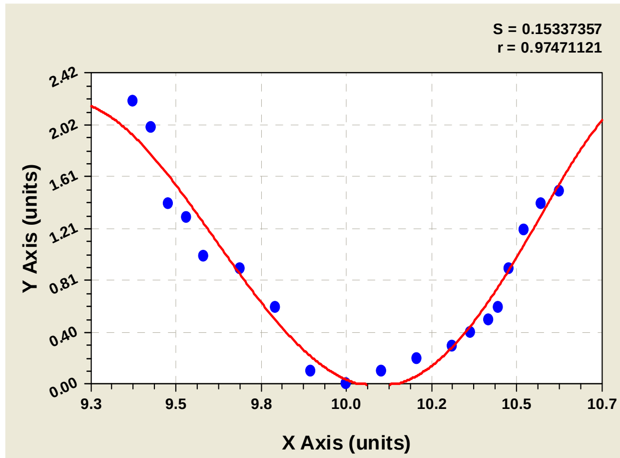

Fig.9 Plot of the adapted beta model

i. Generalized adapted beta model

$$y_9 = a(x-9.2)^b(10.8-x)^c - d \tag{20}$$

and for the experimental data it results:

$$y_9 = 5.46(x-9.2)^{-0.33}(10.8-x)^{-0.283} - 6.068 \tag{21}$$

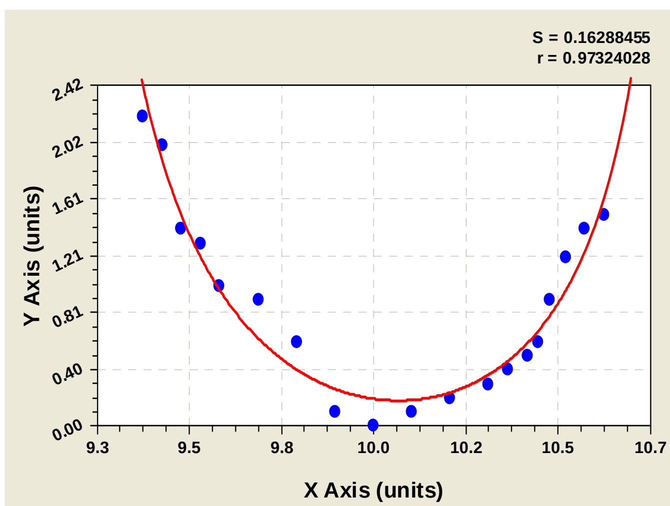

Fig.10 Plot of the generalized adapted beta model

In the last part of the paper it is developed a general mixed bivariate model that will consider the correlation between the two responses of general quality loss function [7]. Mixed bivariate vectors occur when a sampling unit has two different types of response. This is a common occurrence in many manufacturing processes. The traditional optimization approach for such a problem is to analyse each response separately and to determine vital factors for that response, then choose optimal factor settings by making trade-off adjustments among all factors. For the multidimensional case with independent components, the following loss function is used:

$$L(x_1, x_2, \dots\dots, x_l) = \sum_{i=1}^{l} a_i (x_i - m_i)^2 \quad (13)$$

For products with several measurable functional quality characteristics, potential interactions may apear, which can be used for improvements of models. In case of a system with dependent components, the quality can be measured by

$$L(x_1, \dots\dots, x_p) = \sum_{i=1}^{p} c_i (x_i - m_i)^2 + 2\sum_{1\le i<j\le p} k_{ij} |(x_i - m_i)(x_j - m_j)| \quad (14)$$

For equipment batches, or manufacturing systems and lines, the following formula of the average loss function results:

$$I = \sum_{1\le i\le p} d_i [s_i^2 + (\bar{x}_i - m_j)^2] + 2\sum_{1\le i<j\le p} h_{ij} [|s_{ij}| + |(\bar{x}_i - m_i)(\bar{x}_j - m_j)|] \quad (15)$$

The signal/noise ratio for the dependent case has the expression

$$\frac{S}{N} = 10 log_{10}\left(\prod_{1\le i\le p} \frac{x_i^2}{s_i^2} * 2\prod_{1\le i<j\le p} \frac{|\bar{x}_i \bar{x}_j|}{|s_{ij}|}\right) \quad (16)$$

The quality of products must be continuously improved, and these two concepts, loss function and signal/noise ratio, are of utmost importance in process-oriented manufacturing.

3. Conclusions

In the paper following issues will be pursued:

- the comparative analysis of the proposed models for experimental data
- identification of adequate stochastic laws.

The best results obtained with the adapted models are with the values of the correlation coefficient closed to one and the coefficient of variance very small. In the presented case results values for correlation coefficient greater than 0.98 are obtained for polynomial, sinusoidal, generalized hyperbolic cosines, generalized shifted gamma and values for the coefficient of variance less than 0.15 are obtained for polynomial, sinusoidal, adapted hyperbolic cosines, generalized hyperbolic cosines, and generalized shifted gamma.

Hence the modeling by truncated models on a given interval is an adequate quality loss function for symmetrical / asymmetrical cases. The analyzed laws prove their efficiency if are applied in the design stage. The results can be extended in the n-dimensional space, taking into consideration multidimensional variables with/without the potential interactions, which can cause a fraction of quality loss too [3;4]. It exists the possibility to adapted the obtain results in the reliability area [13;14].

In the paper it was proposed adaptive stochastic distributions for describing the loss functions of the usual manufacturing models [13;14]. The study utility consists in the fact that the obtained results can be useful in the design of the actual strategies and in the practice allowing the prediction of costs for the manufacturing systems.

R E F E R E N C E S

[1] *M. Demetrescu, ,A.S. Paris, C. Tarcolea,* Uni-and multivariate loss functions and the Taguchi theory, ,BSG PROCEEDINGS 17, (DGDS-2009) October 8-11, 2009, Bucharest, ROMANIA, Geometry Balkan Press, ISSN 1843-2654 (printed version), ISSN 1843-2859 (online version), 2010, pp. 79-83

[2] *A. S. Paris,* Contribuții la studiul fiabilității şi disponibilității sistemelor tehnologice de fabricație (Contributions to the study of the reliability and the availability of technological manufacturing systems), PhD Thesis, Univ. Politehnica Bucharest, 2009.

[3] *G. Taguchi*, Introduction to quality engineering, Asian Productivity Organization Press, Osaka, 1990.

[4] *G.Taguchi, E.A. Elsayed, T.C. Hsiang,* Quality Engineering in Production Systems, McGraw-Hill, New York,1989.

[5] *J.Teeravaraprug,* Incorporating Kano's Model în Quality Loss Function , International Journal of Production Research,vol. 47, no.1, 2002, pp 37-54

[6] *C-H. Chen*, A Note on the Appropriateness of Taguchi's Loss Function, Heldermann Verlag Economic Quality Control ISSN 0940-5151 Vol 20, No. 2, 2005, pp 271 – 275

[7] *S. Maghsoodloo, L., H. Huang,* Quality loss functions and performance measures for a mixed bivariate response (1997), Benchmarking: An International Journal, Vol. 4, Iss.:2, pp.121 – 147

[8] *Y.Fathi, C.,Poonthanomsook,* A Quartic Quality Loss Function and Its Properties, Journal of Industrial and Systems Engineering,Vol. 1, No. 1, Spring 2007, pp 8-22

[9] *A.S.Paris, C.Târcolea,* Taguchi applications on manufacturing systems, International Conference on Manufacturing Systems - ICMaS 2004 08-09.10.2004 UPB, Romanian Journal of Technical Sciences APPLIED MECHANICS, Editura Academiei Române ISSN 0035-4074, ISBN 973-27-1102-7, *Proceedings of the International Conference on Manufacturing Systems, ICMAS'2004*, vol. 49, n° Special 2004, pp.459-462

[10] *C.Târcolea, A.S. Paris,A. Târcolea,* Statistical Models Applied to Manufacturing Systems, BSG PROCEEDINGS 12 THE 3-rd INTERNATIONAL COLLOQUIUM OF MATHEMATICS IN ENGINEERING AND NUMERICAL PHYSICS (MENP-3) October 7-9, 2004, Bucharest, Geometry Balkan Press ISBN 973-8381-11-8, 2005, pp. 259-264.

[11] C. *Târcolea, A.S.Paris,* Modele bidimensionale de tip Taguchi(Taguchi type bidimensional models), Management Industrial,vol.2, no.1, 1995, pp.29-32.

[12] *** http://www.curveexpert.net

[13] *M. Demetrescu*, Some Statistical Methods with Relevance for Robust Design Under General Loss Functions, PhD Thesis, Univ. Politehnica Bucharest, 2009

[14] *C. B. Rae, S. Kaewkuekool, S. R. Bowling, M.T. Khasawneh,* The Optimum Target Value for a Process Based on a Quadratic Loss Function, Quality Engineering, Volume 17, Issue 2, April 2005 , pp. 227 – 241