OPTIMIZATION OF PARAMETERS OF ELECTRIC VEHICLE POWER TRAIN ACCORDING TO THE CRITERIA OF ACCELERATION CAPABILITY

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In the paper a rigorous method for the determination of the minimum value of the rated power of the motor as well as the selection of the gear ratios of an electric vehicle transmission so that to ensure a given acceleration time from rest to a certain vehicle velocity is presented. After the acceleration characteristic of an electric vehicle is examined, the rated power optimization problem is treated when there is a single-gear and a multi-gear transmission. Taking into account different constraints, including that relating to the adhesion limit of the driven wheels, one reaches a nonlinear programming problem with constraints. One investigates also the case when motor inertia is neglected. The numerical exemplifications allow drawing more general conclusions.

Keywords: acceleration time, base motor speed, constraints, electric motor, electric vehicle, gear ratio, optimization, rated power, transmission.

1. Introduction

One of the important requirements for electric vehicles (EV) is that regarding the acceleration capability. It can be expressed by several quantities: a) t_{aV} [s]-the acceleration time from rest to a certain velocity V_a [km/h]; b) t_{a500} [s]-the time of covering the first 500 m in acceleration from rest; c) t_{ar} [s]-the acceleration time from velocity V_{ai}[km/h] to velocity V_{as} [km/h]. Generally, the above mentioned quantities have comparable values with those of a conventional automobile. As a rule, for t_{aV} one considers V_a =50 km/h and V_a=100 km/h. As illustration of t_{aV} values, one can mention: t_{a50}= (7 -8)s, $t_{a100} \geq 10$s [1,2,3,4] (in the case of the hybrid vehicles, for the electrical pure mode t_{a50} =8s [5]). Also, as a rule, the following values are chosen: V_{ai}= 80 km/h, V_{as}=110 km/h, t_{ar}= 6s.

The theoretical determination of the mentioned quantities is made by integration of the differential equation for the vehicle rectilinear motion. The integration may be done analytically or numerically. The first method is presented in literature assuming certain simplifications [6, 7]. The numerical methods can be used as in the case of the conventional automobile [8] or as in [3]. One can use either rather simple computer programs or powerful subroutines of certain software packages. Also, in a great extent, simulation tools with different degrees of complexity are used [9, 10].

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To determine the rated power of the electric motor and the transmission gear ratios so that one of the above mentioned parameters of the acceleration performances should be accomplished, a first possibility consists in using a verification calculation. With the values of the rated power and the transmission gear ratios, which have been calculated for other conditions, such as those relating to the maximum velocity and maximum grade, the mentioned parameters are determined using one of the above mentioned methods. The obtained value is compared with required values. The calculations are repeated until the required value is accomplished [9]. A second possibility is more direct. So in [2] t_{aV} is expressed by two integrals which comprises the rated power. For an imposed t_{aV} a transcendental equation is obtained which is solved by a numerical method, inclusive the two integrals. Other details are not given in [2]; also, it does not take into account the motor inertia. In [3], the tractive power is expressed as a function of t_{aV} and V_a using some approximations and averaging operations as well as a simple relation between the vehicle velocity and the acceleration time. This relation has been established by a generalization of some results obtained by numerical integration of the motion equation. In [11], in the first step, the air resistance is neglected and the rated power is predetermined using a plotting, after that, in the second step, the motion equation is numerically integrated. Further, the determination of the rated power is made by an iterative procedure.

Obviously, the use of the first possibility is involved; it is difficult to make the selection of gear ratios. The second possibility is more systematic, but the variant out of [3] may produce large errors, which requires making additional corrections. As well in this case the selection of gear ratios is difficult. It can be observed that the motor inertia is ignored in some cases. Finally, it is necessary to notice that there are difficulties in solving certain optimization problems.

In the paper a rigorous method for determination of the minimum value of the rated power of motor as well as the selection of the gear ratios of the electric vehicle transmission so that to ensure the required acceleration capability expressed by t_{aV} is presented. After the acceleration characteristic of an electric vehicle is examined, the optimization problem of the rated power is treated when there is a single-gear transmission and a multi-gear transmission. Taking into account different constraints, including that relating to the adhesion limit of the driven wheels, one attains a nonlinear programming problem with constraints. One investigates also the case when motor inertia is neglected. The numerical exemplifications enable emphasizing the influence of different factors.

2. Acceleration characteristic

In the following we consider that the EV is equipped with the one of the next motor types: induction motor, permanent magnet motor (synchronous and

Brushless-DC), switched reluctance motor (at present, direct current motors are not longer used in actual fact [12]). The mechanical characteristics of these motors are similar. They are defined thus:

$$P = f_P(n) = \begin{cases} \dfrac{P_n}{n_b} \cdot n, \; n \le n_b, \\ P_n, n_b \le n \le n_{\max}, \end{cases} \tag{1}$$

where: n [1/min] is the motor speed; n_b [1/min]-the base motor speed; n_{max} [1/min]-the maximum motor speed; P_n [kW]-the rated power of the motor. For a certain time intervals these motors can provide powers greater than the rated power which corresponds to the continuous duty. During short-time duty or intermittent duty the mechanical characteristic is defined by the relation (1) in which P_n is replaced by $k_s.P_n$, where k_s represents the overload coefficient (generally, k_s=1.4÷3). The results obtained in this paper using the relation (1) and which contain P_n can be considered as valid when operation mode is intermittent as well. In this case the proper rated power is $P_n^*=P_n/k_s$ (k_s is chosen according to motor features and the directives of the motor designer).

The vehicle acceleration is given by the general relation [8]:

$$\frac{dv}{dt} = \frac{1}{\delta . m_a}(F_t - R_r - R_g - R_a)[\frac{m}{s^2}], \tag{2}$$

where: F_t [N] represents the tractive force; m_a [kg]-gross vehicle mass; R_a [N]-the air resistance; R_g[N]-the grading resistance; R_r [N]-the rolling resistance; t [s]-the time; v [m/s]-the vehicle velocity; δ [-]-the rotational inertia coefficient (coefficient accounting for rotating masses). Taking into account the expressions of F_t, R_a, R_g and R_r [13], the relation (2) becomes

$$\frac{dv}{dt} = \frac{g}{\delta}\left[\frac{Mi_0 i_g \eta_t}{m_a r_r g} - (f(v)\cos\alpha_s + \sin\alpha_s) - 0.6125\frac{c_x A}{m_a g}v^2\right], \tag{3}$$

where: A [m/s^2] is the frontal area of the vehicle; c_x [-]-air resistance coefficient (aerodynamic resistance coefficient); f [-]-rolling resistance coefficient; g [m/s^2]-acceleration due to gravity; i_0 [-]-gear ratio of final drive; i_g [-]- gearbox gear ratio; M [N.m]-torque of the motor; r_r [m]-rolling radius of the driven wheels; α_s [-]-slope angle; η_t [-]-overall transmission efficiency. The rolling resistance coefficient is depending on the vehicle velocity:

$$f(v) = f_0 + f_{01}v + f_{02}v^2. \tag{4}$$

The values of the coefficients f_0, f_{01} and f_{02} are established according to the considerations from [14].

There are the following relations:

$$M = 9549.3P/n, \tag{5}$$

$$v = (\pi / 30) r_r n / (i_0 i_g), \tag{6}$$

$$V = 0.377 r_r n / (i_0 i_g), \tag{7}$$

where V [km/h] is the vehicle velocity.

The maximum acceleration capability is obtained when the motor is running according to the mechanical characteristic, namely when the power P is given by the relation (1). Taking into account the relations (1), (3), (4), (5) and (6) the expression of the vehicle acceleration becomes:

$$a_v = \frac{dv}{dt} = \begin{cases} g\delta^{-1}(-c_1 v^2 - c_2 v + c_3), & for\, v \le v_b = (\pi / 30) r_r n_b / (i_0 i_g), \\ g\delta^{-1}(-c_1 v^2 - c_2 v - c_4 + c_5 / v) & for\ v \ge v_b, \end{cases} \tag{8}$$

where:

$$c_1 = 0.6125 c_x A / (m_a g), c_2 = f_{01} \cos\alpha_s, c_3 = 9549.3 i_0 i_g \eta_t P_n / (m_a r_r g n_b) - \\ - f_0 \cos\alpha_s - \sin\alpha_s,\ c_4 = f_0 \cos\alpha_s + \sin\alpha_s,\ c_5 = 10^3 \eta_t P_n / (m_a g). \tag{9}$$

The rotational inertia coefficient is given by the relation [8]

$$\delta = 1 + \frac{\sum I_w}{m_a r_r^2} + \frac{I_{me} i_0^2 i_g^2 \eta_t}{m_a r_r^2}, \tag{10}$$

where I_{me} [kg.m^2] is the mass moment of inertia of the rotor motor (briefly, the motor inertia moment) and $\sum I_w$ represents the total mass moment of inertia of all vehicle wheels. The mass moment of inertia of the electric motor rotor can be expressed as a function of the rated power [15]:

$$I_{me} = \alpha + \beta P_n + \gamma P_n^2, \tag{11}$$

where α, β and γ are some coefficients which are chosen taking into account the peculiarities of the electric motors [15].

The vehicle acceleration depends on the velocity and the engaged gear. For the gear j with the gear ratio i_{gj} the following velocities are defined

$$V_{bj} = 0.377 r_r n_b / (i_0 i_{gj}),\ V_{Mj} = 0.377 n_{\max} / (i_0 i_{gj}),\ j = 1,2,...N_g, \tag{12}$$

where N_g is the total number of the gears. For the gear j, the used velocity range is included in the interval [0, V_{Mj}]. If the curves of the acceleration as a function of velocity are extended, they will pass through the same point (V_{max}, 0) (V_{max} [km/h] is the maximum velocity of vehicle). Indeed, for $V \ge V_{bj}$ the expressions of the accelerations are given by the second relation (8), in which the expression out of parenthesis does not depend on gear. This expressions become zero for maximum velocity, namely a_v =0. If we consider two neighboring gears j and j+1 and assuming that $\eta_{tj}=\eta_{tj+1}$, the accelerations in the two gears are given by the following expressions

$$a_{vj} = \frac{g}{\delta_j}(-c_1\frac{V^2}{13} - c_2\frac{V}{3.6} - c_4 + 3.6\frac{c_5}{V}),$$
$$a_{Vj+1} = \frac{g}{\delta_{j+1}}(-c_1\frac{V^2}{13} - c_2\frac{V}{3.6} - c_4 + 3.6\frac{c_5}{V}) \quad (13)$$

for $V \geq V_{bj+1}$, where δ_j and δ_{j+1} are the rotational inertia coefficients for the gears j and j+1. Taking into account the relation (10) and that $i_{gj+1} < i_{gj}$ it follows that $a_{vj+1} > a_{vj}$ for $V \geq V_{bj+1}$. For a given gear ratios the acceleration has the maximum value for the velocity equal to zero (see (8)). It is given by the relation

$$a_{v\max} = \delta^{-1}[9549.3 i_t \eta_t P_n /(m_a r_r n_b) - (f_0 \cos\alpha_s + \sin\alpha_s) g], \quad (14)$$

where $i_t = i_0 i_g$ represents the overall gear ratio of the transmission. Generally, the maximum acceleration depends on i_t and the rated power. Taking into account the relation (10), after investigation of the function of the maximum acceleration given by (14), we attain the conclusions that this has a maximum for the overall transmission ratio (it is assumed that η_t=const.)

$$i_{tm} = \frac{f_0\cos\alpha_s + \sin\alpha_s}{9549.3\eta_t /(m_a g r_r \eta_t)} + \sqrt{\frac{1+\delta'}{\delta''} + \left[\frac{f_0\cos\alpha_s + \sin\alpha_s}{9549.3\eta_t /(m_a g r_r \eta_t)}\right]^2}, \quad (15)$$

where

$$\delta' = \sum I_w /(m_a r_r^2),\ \delta'' = I_{me}\eta_t /(m_a r_r^2). \quad (16)$$

The possible maximum acceleration of the vehicle, for a given rated power, is obtained by substituting i_t for i_{tm} in (14). We will exemplify considering the following specifications of the electric vehicle: m_a=1200 kg, A=1.90 m^2, c_x=0.34, f_0=0.012, f_{01}=0, f_{02}=0, n_b=2000 1/min, r_r=0.30 m, α_s=0, η_t=0.95, $\sum I_w$=2.5 kg.m^2. To calculate I_{me} by the relation (11) it is assumed that α=-0.01225, β=4.877.10^{-3}, γ=3.8377.10^{-5} [15]. The obtained results are showed by the curves in Fig.1 and 2 (the curve 1).

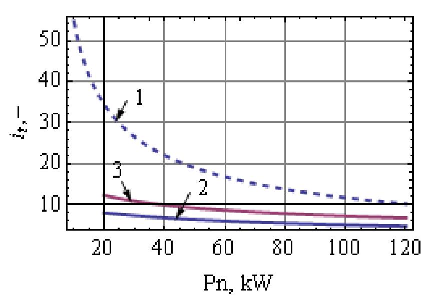


Fig.1. Influence of rated power on the overall transmission ratio: 1-for the possible maximum acceleration; 2, 3-for the minimum t_{a50} and t_{a100}, respectively

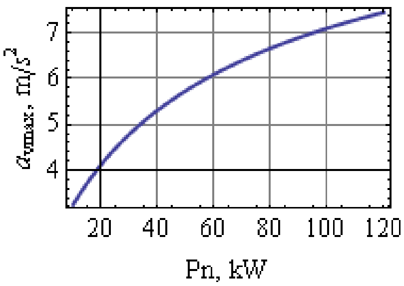


Fig.2. The possible maximum acceleration as a function of the rated power

It is found that the increase of the rated power leads to the decrease of the i_{tm} and the increase of the possible maximum acceleration. For a given P_n, the curves of the acceleration for two different gears may be situated in two different positions in a way which is showed in Fig. 3. The representations are in concordance with the before discussion. If the first gear has $i_{g1}.i_0< i_{tm}$, always there is other intersection of the curves of the accelerations for the neighboring gears, which is different from the point corresponding to the maximum velocity. If $V_{j,j+1}$ is the velocity corresponding to this intersection point (the gears are j and j+1), then $V_{j,j+1}<V_{bj+1}$. If $i_{g1}.i_0> i_{tm}$ then the one from the cases is that in Fig. 12b when, also, $i_{t2}>i_{tm}$, $i_{t1}>i_{t2}$.

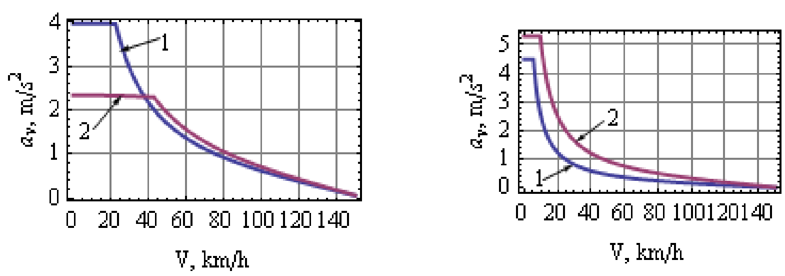


Fig.3. The relative position of the acceleration curves: a) $i_{t1}<i_{tm}$, $i_{t1}>i_{t2}$ (i_{t1}=10, $i_{t2}=i_{t1}/1.9$); b) $i_{t1}>i_{tm}$, $i_{t1}>i_{t2}$ (i_{t1}=40, $i_{t2}=i_{t1}/1.7$)

Obviously, in this case the starting from rest of the vehicle when the first gear is engaged is not recommendable because the vehicle acceleration is smaller than in the following gear. Therefore, this circumstance should be avoided. When $i_{t1}>i_{tm}$, depending on the value of i_{t2}, the curves are positioned as shown by Fig.3a. But, generally, i_{tm} has large enough values so that it is advantageous to choose i_{gt} smaller than i_{t1}. Therefore, generally, we choose $i_{t1}< i_{tm}$.

In the case when δ does not depend on i_t ($I_{me}\cong 0$) then, according to relation (14), the maximum acceleration increases linearly with i_t. Also, one can easily verify that for $V\geq V_{bj+1}$ the acceleration curves of the all gears with the order number greater or equal to j+1 coincide with each other (the velocity range is extended beyond V_{Mj+1}). Also there is the relation

$$V_{j,j+1} = V_{bj+1}. \tag{17}$$

It is necessary to take into account that the maximum acceleration of the vehicle is limited by the tire road grip (adhesion) of the driving wheels. For a front-drive vehicle and a rear-drive vehicle the expressions of the limited accelerations are the followings, respectively [14]:

$$\begin{aligned} a_\varphi(v)/g = \{[(\varphi_x + f)(b/L - fr_r/L) - f]\cos\alpha_s - [1 + (\varphi_x + f)h_g/L]\sin\alpha_s + \\ + (\varphi_x F_{az1} - fF_{az2} - R_a)/(m_a g)\}/[1 + (\varphi_x + f)h_g/L], \end{aligned} \tag{18}$$

$$\begin{aligned} a_\varphi(v)/g = \{[(\varphi_x + f)(a/L - fr_r/L) - f]\cos\alpha_s - [1 - (\varphi_x + f)h_g/L]\sin\alpha_s + \\ + (\varphi_x F_{az2} - fF_{az1} - R_a)/(m_a g)\}/[1 - (\varphi_x + f)h_g/L]. \end{aligned} \tag{19}$$

In the above relations the following notations are used: *a, b* [m]-distance between vehicle center of gravity and the normal plane on road which passes through the axes of the front axle and of the rear axle, respectively; h_g [m]-height of the center of gravity of vehicle; L [m]-wheel base of vehicle; F_{az1}, F_{az2} [N]-aerodynamic lift forces on the front axle and the rear axle, respectively (here, the positive direction is towards road); φ_x [-]-longitudinal adhesion coefficient.

The vehicle acceleration limited by adhesion is depended on the longitudinal adhesion coefficient and the vehicle velocity. The effect of velocity is depended on the aerodynamic lift coefficients c_{z1}, c_{z2} [13], c_x, A, $f(v)$ and h_g/L. Generally, even φ_x is influenced by the velocity. The aerodynamic effect of the velocity is small enough in the case of a common automobile, but it may be significant in the case of race cars. As an exemplification, the acceleration limited by adhesion as a function of the velocity is showed in Fig. 4 for two cases: front drive axle-a/Li=0.46, h_g/L=0.20, c_{z1}=0.20, c_{z2}=0.40; rear drive axle- a/Li=0.56, h_g/L=0.20, c_{z1}=0.20, c_{z2}=0.40.

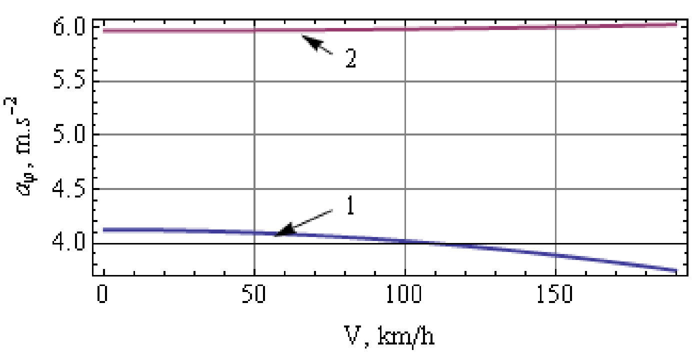


Fig.4. Vehicle acceleration limited by adhesion versus vehicle velocity: 1-front wheel drive; 2-rear wheel drive

3. Optimization according to criterion t_{aV}
3.1. Single-gear transmission
3.1.1. Mass inertia moment of electric motor is considered

The acceleration time from rest to a final velocity v_f is obtained by the integration of the inverse of the acceleration given by the relation (8). Assuming that $v_f > v_b i$, we can write

$$t_a = (\delta / g) I, \tag{20}$$

where

$$I = \int_0^{v_b} \frac{dv}{-c_1 v^2 - c_2 v + c_3} + \int_{v_b}^{v_f} \frac{v dv}{-c_1 v^3 - c_2 v^2 - c_4 v + c_5}. \tag{21}$$

It should be noticed that, generally, $c_1>0$, $c_2 \geq 0$, $c_3>0$, $c_4>0$ and $c_5>0$ (it is assumed that $\alpha_s \geq 0$). Therefore, $c_2^2+4c_1c_3>0$, so that, after integrations, it follows that:

$$I = -\frac{1}{\sqrt{c_2^2 + 4c_1c_3}} \ln \left| \frac{2c_1 v + c_2 - \sqrt{c_2^2 + 4c_1c_3}}{2c_1 v + c_2 + \sqrt{c_2^2 + 4c_1c_3}} \right| \Bigg\|_0^{v_b} - \sum_{j=1}^{3} \frac{v_j Ln(v - v_j)}{c_4 + 2c_2 v_j + 3c_1 v_j^2} \Bigg|_{v_b}^{v_f}, \tag{22}$$

where v_j (j =1, 2, 3) represent the roots of the equation $c_1 v^3 + c_2 v^2 + c_4 v - c_5 = 0$. This equation has a positive real root and two conjugate-complex roots. It should be mentioned that, in relation (22), the following equality $Ln(w) = \ln|w| + \sqrt{-1}.\arg(w)$ is valid, where, generally, w is a complex number.

Obviously, the acceleration time is depended on the rated power and the gear ratios. The integrals of (21) depend on two parameters: $i_t = i_0 . i_g$ and P_n. Using the known rules for calculation of the derivative of an integral with respect a parameter, the following expression is obtained:

$$\frac{\partial I}{\partial i_t} = -9549.3 \frac{\eta_t P_n}{m_a g r_r n_b} \int_0^{v_b} \frac{dv}{(-c_1 v^2 - c_2 v + c_3)^2} + \frac{dv_b}{di_t} . \frac{1}{-c_1 v_b^2 - c_2 v_b + c_3} -$$
$$- \frac{dv_b}{di_t} . \frac{v_b}{-c_1 v_b^3 - c_2 v_b^2 - c_4 v_b + c_5}.$$

Because the last two terms are equal each to the other (the acceleration is a continuous function of v: see (8)), the preceding relation becomes

$$\frac{\partial I}{\partial i_t} = -9549.3 \frac{\eta_t P_n}{m_a g r_r n_b} \int_0^{v_b} \frac{dv}{(-c_1 v^2 - c_2 v + c_3)^2}. \tag{23}$$

Because the integral of (23) is positive, it follows that $dI/di_t < 0$ always. Therefore, the increase of i_t leads to the decrease of I. At same time, the δ coefficient increases when i_t increases. Obviously, there is an optimum value of i_t for which the acceleration time has a minimum value. Consequently, if the effect of inertia of the rotational masses is not considered, the maximum acceleration capability is obtained when the overall transmission ratio has the maximum value. If i_g=1.0, then we choose i_{0max} given by the relation [15]:

$$i_{0\max} = 0.377 r_r n_{\max} / V_{\max}. \qquad (24)$$

There are electric motors which have small inertia moments, so that their effect is reduced on the vehicle acceleration capability. But, generally, to evaluate this effect and also to determine the optimum value of the transmission ratio it is necessary to resort to numerical calculations (an analytical expression for the optimum i_t is difficult to establish).

Using the preceding method it is easy to prove that $\partial I/\partial n_b > 0$ and $\partial I/\partial P_n < 0$. Consequently, when the base speed decreases, namely when the speed ratio $x_t = n_{max}/n_b$ increases, the acceleration time decreases. Also, when P_n increases the acceleration decreases.

Using the relations (10), (20) and (22) we can obtain an expression for the acceleration time. If one takes the relation (11) into account, then one can write under general form of

$$t_a = t_a(V_f, i_t, P_n, n_b). \qquad (25)$$

For the imposed values of $V_a = V_f$ and t_{aV}, the relation (25) becomes

$$t_{aV} = t_a(V_a, i_t, P_n, n_b). \qquad (26)$$

In the case when the effect of the rotational mass inertia is reduced, it follows that, according to those showed, the overall transmission ratio should have the maximum value given by the relation (24) (assuming that i_g=1.0). In this case the relation (26) becomes

$$t_{aV} = t_a(V_a, i_{0\max}, P_n, n_b). \qquad (27)$$

For a given value of n_b the relation (27) becomes an equation with one unknown quantity P_n, which can be numerically solved. It should be noticed that in this way the minimum value of P_n is obtained. With the same data of the preceding exemplification, using a proper program with *Mathematica*, the following results have been obtained. For electric vehicle with V_{max}=90 km/h, n_b=2000 1/min, n_{max}=6000 1/min, i_{0max}=7.54, V_a=50 km/h and t_{aV}=7.0 s, the rated power is P_n=26.0 kW, in comparison with 10.22 kW, which corresponds to the value obtained from the condition of the maximum velocity [15]. When V_{max}=180 km/h, V_a=100 km/h and t_{aV}=10.0 s (n_b and n_{max} are the same as in preceding case, respectively) the results are: i_{0max}=3.77, P_n=73.63 kW in comparison with requested power of 59.5 kW corresponding to the condition of the maximum velocity. A diminution of the rated power is possible by using a gearbox.

In the general case, when the effect of the rotational mass inertia can not be neglected one can write:

$$\frac{\partial t_a}{\partial i_t} = g\frac{\partial(\delta I)}{\partial i_t} = g\left[2\delta'' i_t I - 9549.3\frac{\eta_t \delta P_n}{m_a g r_r n_b}\int_0^{v_b}\frac{dv}{(-c_1 v^2 - c_2 v + c_3)^2}\right]. \quad (28)$$

The integral out of (28), which will be denoted by I_1, is written as

$$I_1 = \left\{\frac{2c_1 v + c_2}{c_3 - c_2 v - c_1 v^2} - 4c_1\frac{\arctan[(2c_1 v + c_2)/\sqrt{-c_2^2 - 4c_1 c_3}\,]}{\sqrt{-c_2^2 - 4c_1 c_3}}\right\}\frac{1}{c_2^2 + 4c_1 c_3}\Bigg|_0^{v_b} . \quad (29)$$

In the relation (29) one takes into account that if x, y are real numbers, one can write $\arctan(x + y\sqrt{-1}) = 0.5\sqrt{-1}.Ln[(1 + y - x\sqrt{-1})/(1 - y + x\sqrt{-1})]$. Substituting integral I_1 given by (29) in relation (28) gives the expression of the derivative of I with respect i_t as a function of i_t and P_n. The expression is intricate, but it may be obtained by a suitable program with *Mathematica*. The equalization of the derivative to zero leads to condition of extremum point, which is a minimum. Considering P_n as a parameter and solving the equation for i_t numerically gives the optimum value of i_t. With preceding data and V_f=50 km/h, V_f=100 km/h the results are showed in the graphs 2 and 3 in Fig. 1. It can be seen that these gear ratios are much smaller than the optimal gear ratio corresponding to the maximum acceleration.

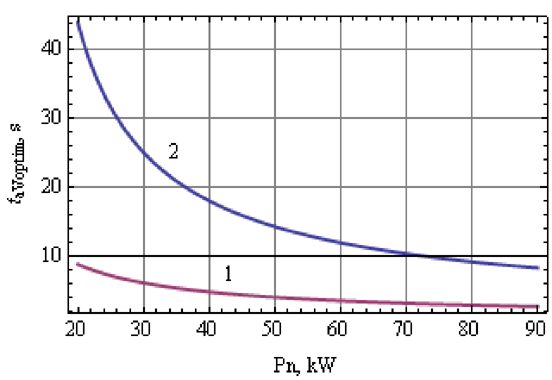


Fig.5. The optimal acceleration time versus the rated power: 1-V_f=50 km/h; 2-V_f=100 km/h

The minimum acceleration time corresponding to these optimal gear ratios as a function of P_n has the graph shown in Fig. 5. The representations in Figs. 1 and 5 are in accordance to those in Fig.6. It can be seen that from certain values of t_{aV}, the required power P_n changes slowly when i_t increases. In the same time, i_t is limited by the imposed maximum velocity. In Fig. 6 the dashed vertical lines correspond to the maximum values of i_t for V_{max}=90 km/h and V_{max}= 180 km/h. Obviously, P_n should be equal to or greater than the required power P_{Vmax} for the maximum velocity. One can clearly see that in order to obtain the minimum values for P_n, these values must be chosen: 27.84 kW and 78.66 kW, respectively. They are obtained by numerically solving the equation $t_{aV}=t_a(V_a,i_{tmax},P_n,n_b)$.

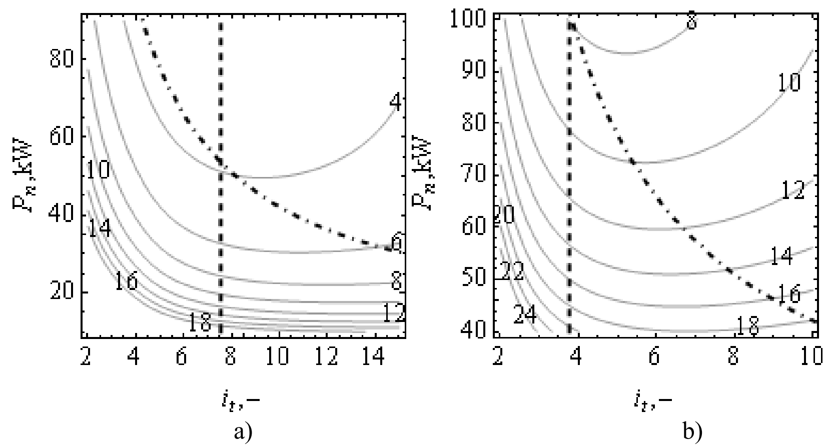


Fig.6. Dependence upon i_t and P_n of t_a represented by contour lines:a) V_f=50km/h; b) V_f=100 km/h

It is found that the values of the rated power are larger than those corresponding to the case when the effect of the rotational mass inertia is neglected.

3.2. Multi-gear transmission

To improve the acceleration performances one can use a gearbox. When there is a gearbox, the total acceleration time is given by the relation

$$t_{at} = t_a(V_{12},i_{t1},P_n,n_b) + \sum_{j=2}^{N_f-1}[t_a(V_{j,j+1},i_{tj},P_n,n_b) - t_a(V_{j-1,j},i_{tj},P_n,n_b)] + \\ + t_a(V_f,i_{tN_f},P_n,n_b) - t_a(V_{N_f-1,N_f},i_{tN_f},P_n,n_b), \quad (30)$$

where: i_{tj} is the overall gear ratio when the gear j is engaged; $V_{j,j+1}$ [km/h]-the vehicle velocity when the gear j is changed with the gear j+1; N_f –the order number of the gear at which the prescribed final velocity is attained. It is assumed that the time of the gear change is zero (there is not velocity drop during change). Determination of the gear ratios can be made so that P_n gets the minimum value. This is equivalent to the condition that the acceleration gets as large as possible value. In connection with this it has been found that there is an optimal value of the overall gear ratio given by (15), Consequently, to avoid the case of Fig.3b when, in fact, it would be necessary to renounce starting from rest when the first gear is engaged, it is imposed that

$$i_{t1} \le i_{tm}. \quad (31)$$

Also, it is obvious that the acceleration in any gear should not exceed the acceleration limited by adhesion. If the curve of $a_{\varphi}(v)$ has the shape as that of the curve 2 in Fig.4, the adhesion condition can be written as

$$9549.3 i_{t1}\eta_t P_n /(m_a r_r n_b g\delta(i_{t1}, P_n) \le a_{\varphi}(0). \quad (32)$$

Because, generally, the effect of velocity on $a_{\varphi}(v)$ is reduced, the condition (32) may be used as well in the case of the curve 1 in Fig.4.

The necessary condition for minimal t_{at} with respect to velocities $V_{j,j+1}$ is

$$\partial t_{at} / \partial V_{j,j+1} = 0,\ j = 1,2,...,N_f - 1. \quad (33)$$

The total acceleration time can be written as (the change time of gears is neglected)

$$t_{at} = \int_0^{V_{12}} [3.6 a_{vj}(V)]^{-1} dV + \sum_{j=2}^{N_f-1} \int_{V_{j-1,j}}^{V_{j,j+1}} [3.6 a_{vj}(V)]^{-1} dV + \int_{V_{N_f-1,N_f}}^{V_f} [3.6 a_{vj}(V)]^{-1} dV. \quad (34$$

In view of the relation (34), the condition (33) leads to (a more complete analysis is given in [14])

$$a_{vj}(V_{j,j+1}) = a_{vj+1}(V_{j,j+1}),\ j = 1,2,..,N_f - 1. \quad (35)$$

Taking into account that there are the circumstances of Fig.3a, it follows that

$$V_{bj} < V_{j,j+1} < V_{bj+1},\ j = 1,2,...,N_f - 1. \quad (36)$$

Obviously, the following conditions should be also fulfilled:

$$V_{j,j+1} \le V_{Mj},\ j = 1,2,3,..,N_f - 1, \quad (37)$$

$$i_{tN_f} \le 0.377 r_r n_{\max} / V_{\max}. \quad (38)$$

Following reasoning of [14] it is proved that

$$i_{gj} / i_{g,j+1} = i_{tj} / i_{t,j+1} \le n_{\max} / n_b,\ j = 1,2,3,..,N_g - 1, \quad (39)$$

where N_g is the total number of gears.

Taking above into account we reach a nonlinear programming problem with constraints. The variables are P_n, i_{tj}(j=1,2,..,N_f) and $V_{j,j+1}$(j=1,2,…,N_f-1). Consequently, the problem is

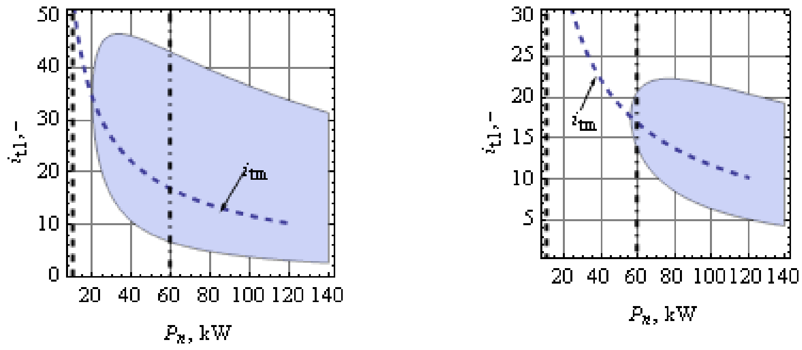
$$\min_{i_{gj}, V_{j.j+1}, P_n} P_n$$

with the constraints: -t_{at}=the imposed value of t_{at} for a given V_f (t_{at} is expressed by (30));

-$P_n \ge P_{Vmax}(V_{max})$; (40)

- (31), (32), (36), (37), (38) and (39);

$$- i_{t1} > i_{t2} > i_{t3} > \ldots > i_{tNf} \geq 0.377 r_r n_b / V_f. \quad (41)$$



a) b)

Fig.7. Graphical representation of the constraints (31), (32) and (40): a) front-drive wheels; b) rear-drive wheels

It is noticed that the constraint (35) has not been included because during the implementation of the computer program it has been found that the use of the constraints (36) is more favorable (they are as a result from (35); computer program itself leads to (35)). Since the constraints in optimization problems are not strict inequalities, the conditions (36) and (40) are used under the form

$$m_j V_{bj} \leq V_{j,j+1} \leq l_j V_{b,j+1}, \quad i_{tj} \geq q_j i_{t,j+1}, \quad (42)$$

where m_j, l_j and q_j are some constants which satisfy the inequalities $m_j>1$, $l_j<1$ and $q_j>1$. It is interesting to emphasize the constraints (31) and (32) which have been highlighted in Fig.7**.** In this figures the lines corresponding to the condition (40) are plotted for V_{max}=90 km/h and V_{max}=180 km/h. The domain in which the condition (32) is not fulfilled is marked by the gray area. From these representations it is found that the admissible domain is small enough especially for the large maximum velocity and in the case of the front-drive wheels.

To solve the mentioned nonlinear programming problem, a computer program with *Mathematica* has been designed. The obtained results are showed in tables 1 and 2 in the case of the front-drive wheels.

Table 1

The result of the optimization: t_{aV}=10 s, V_a=100 km/h, V_{max}=180 km/h

i_{t1}, -	i_{t2}, -	i_{t3}, -	i_{t4}, -	v_{12}, m/s	v_{23}, m/s	v_{34}, m/s	P_n, kW
5.773	4.475	3.515	2.790	13.370	17.370	22.144	68.11
5.746	4.105	2.996	-	14.414	20.365	-	68.40
5.682	3.461	-	-	16.83	-	-	69.10
3.77	-	-	-	-	-	-	78.66

Table 2

The result of the optimization: t_{aV}=7.0 s, V_a=50 km/h, V_{max}=90 km/h

i_{t1}, -	i_{t2}, -	i_{t3}, -	i_{t4}, -	v_{12}, m/s	v_{23}, m/s	v_{34}, m/s	P_n, kW
23.694	14.279	9.396	6.394	3.408	6.066	9.426	22.63
23.167	12.187	7.171	-	3.864	8.049	-	22.90
16.837	7.540	-	-	6.863	-	-	23.54
7.540	-	-	-	-	-	-	27.82

It has been found that the use of two gears instead a single gear leads to the decrease of the rated power with 12.5% and 15.34% for the two cases which are considered in tables 1 and 2. In proportion as the gear number increases the P_n decreases, but not significantly. Consequently, in general, the use of a gearbox with number gears greater than two does not give important advantages with respect to the reduction of P_n (it is assumed that the acceleration capability is the same). In the case of table 2, when there are two gears, the first gear does not lead to the adhesion limit as in the case of the gear number greater than 2.

In the case of rear-drive wheels, the optimization results differ from those of the preceding case to a certain degree because adhesion conditions are modified. The conclusions are similar to those which have been already explained. When N_g=4, V_a=100 km/h, t_{aV} =10.0 s, the optimization results are: i_{t1}=11.842, i_{t2}=7.120, i_{t3}=4.674, i_{t4}=3.180, v_{12}=6.845 m/s, v_{23}=12.210 m/s, v_{34}=18.980 m/s and P_n=64.44 kW. Because of the adhesion domain is more extended, the gear ratio i_{t1} can be larger and so P_n is reduced with 3.67 kW.

3.2.2. Mass inertia moment of electric motor is small

When δ is not depended on i_{gj} (I_{me}≅0) and η_t =const., the acceleration characteristic has a certain feature which is important for optimization. Because $\delta_j=\delta_{j+1}$, from the relations (8) and (9) one can deduce that $a_{vj}=a_{vj+1}$ for $V\geq V_{bj+1}$. Consequently, the acceleration characteristic shows as in Fig.8. It is obvious that $V_{j,j+1}\geq V_{bj+1}$. This fact is emphasized in Fig.8, where it is found that if $V_{j,j+1}<V_{bj+1}$ the acceleration in gear j+1 is smaller that one in gear j.

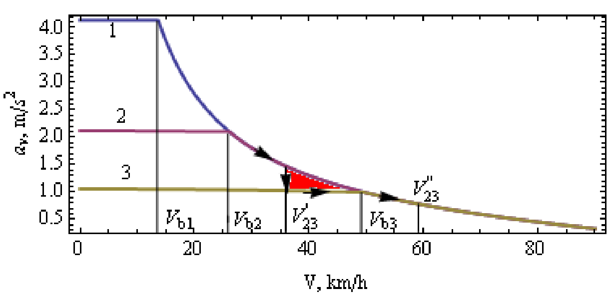


Fig. 8. Acceleration characteristic when I_{me}≅0.

In this conditions, the total acceleration time from rest to V_f depends explicitly only on the overall transmission ratio corresponding to the first gear.

Consequently $t_{at} = t_a(V_f, i_{t1}, P_n, n_b), t_{aV} = t_a(V_a, i_{t1}, P_n, n_b).$ (43)

For given gear ratios, the total acceleration time given by (43) represents a minimum acceleration time. In the second relation of (43), for given values of V_a and t_{aV} there are two unknowns: P_n and i_{t1}. The nonlinear programming problem becomes

$$\min_{i_{t1}, P_n} P_n$$

with the constraints: the second relation of (43); the relation (32); $i_{t1} \le 0.377 r_r n_{\max} / V_{\max}$. It is noticed that according to the above assumption it follows that $\delta^{"}$=0 and, in turn, in conformity with (15), i_{tm} tends to infinity.

By means of a proper computer program with *Mathematica,* the mentioned programming problem has been solved. The results are: a) for the vehicle with V_{max}=90 km/h and t_{a50}=7.0s-P_n=20.93 kW, i_{t1}=16.496; b) for vehicle with V_{max}=180 km/h and t_{a100}=10.0 s –P_n=64.81 kW, i_{t1}=5.326 (in both cases: front-drive wheels). It can be seen that the rated power is decreased with 19.5% and 11.9%, respectively, in comparison with the values obtained previously when i_t=i_{0max} and δ is depended on i_t. Assuming that the highest gear is in direct drive (i_{gN}=1) it follows that

$$i_0 \le 0.377 r_r n_{\max} / V_{\max}.$$

The value of i_0 is chosen and after that the values of i_{gj} are determined in conformity with (39). Obviously, there is not a unique solution for i_{gj}. Consequently, there is the possibility to consider different criteria as, for instance, that relating to operating motor with high values of efficiency. In fig. 8 it has been considered: i_0=4.555, i_{g1}=3.621, 3 gears and the selection of the gear ratios in accordance with a geometrical progression (V_{max}=90 km/h, i_{g2}=1.902, i_{g3}=1.0).

4. Conclusions

a) There exists an optimal overall transmission ratio for which the electric vehicle acceleration gets the maximum value. Generally, this value of gear ratio is greater than the optimal value corresponding to the minimum value of t_{aV}.

b) In the case of a single-gear transmission, when the effect of the rotational mass inertia is neglected, the overall transmission ratio, which leads to a minimum value of t_{aV}, has the maximum value determined by the maximum motor speed and the maximum vehicle velocity.

c) The study of the nonlinear programming problem, explained minutely in the paper, shows that the constraint with respect to the adhesion of the driven wheels is very important. The determination of the required rated power of the electric motor and the selecting of the gear ratios of transmission take place by solving of the mentioned nonlinear programming problems.

d) The use of two gears instead a single gear leads to a decrease of the required rated power with (12÷15) %. Generally, the use of a gearbox with a gear number greater than two does not give important advantages with respect to the reduction of the rated power. When rotational mass inertia is reduced, the required rated power decreases (in the exemplification- (12÷19) %).

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