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ON SOME STATISTICAL INDICATORS OF THE TRAFFIC IN WIRELESS LOCAL AREA NETWORKS

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În ultimul timp, capacitatea de protocol ale rețelelor fără fir, de exemplu pragul permis într-un canal de comunicație, joacă un rol important în aplicațiile Internet. Există mai multe modele teoretice şi variante pentru a estima întârzierile în transmisie datorate coliziunilor sau congestiei. În acest articol, se studiază un model bazat pe schema programării cadrelor, poziționat între stratul legăturii logice şi cel al controlului mediului de acces. Se dau câteva estimări privind mediile şi dispersiile unor variabile aleatoare, care descriu utilizarea curentă a rețelelor fără fir.

In the last time, the protocol capacity of the wireless networks, for instance the throughput as allowed in a given wireless channel, plays an important role in the Internet applications. There are many theoretical models and variants to take into account the transmission delay due to collisions or to subsequent transmission. In this paper, one studies a model – based frame scheduling scheme, positioned between the layers of the logical link and medium access control (MAC) and give some statistical estimations regarding the expected values and the variations of some random variables, which describe the collisions

Keywords: transmission control protocol (TCP), medium access control, statistical indicators, performance evaluation

1. The description of the Frame Service Time

By a <u>fluid unity</u> we mean any sequence of collision periods followed by a successfully transmission frame, where a collision period is composed of idle slots and a collided frame. In the figure 1, one indicates such a unit. In practice, one meets sequences of

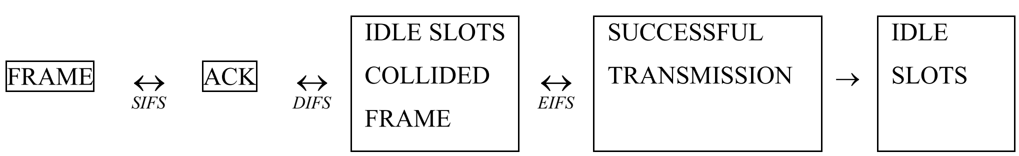

Fig.1

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such fluid unities (SIFS = “short interframes space” ; ACK = “acknoledgment”; DIFS = “distributed interframe space”; EIFS = “extended interface space”). Each fluid unity consists of zero or more collision periods, followed by a successful frame transmission. One can add some mechanisms like “request to send” or “clear to send”, but here we avoid them.

Let us fix a fluid unity. A fundamental indicator is the frame service time denoted , in what follows, by T, as being the time necessary for a successful transmission of a frame. In a sense, T can be identified with the length of the considered fluid unity. T is a random variable and can be expressed by some simpler components. Define the following random variables (relatively to the same probability field (Ω, K, P), which is understood) :

N = the number of collision periods;

C_k = the k-th collision period, $1 \le k \le N$;

C = the total length of collision periods;

S = the time taken by a successful transmission of a frame;

F = the size of a frame;

CF = the size of a collided frame;

S_k = the number of idles slots before the k-th collision or the successful transmission, $1 \le k \le N$.

Directly, with these, one can formulate:

PROPOSITION 1.

The following relations take place:

$$T = C + S ; \tag{1}$$

$$C = C_1 + C_2 + \ldots + C_N ; \tag{2}$$

$$C_k = S_k + CF + EIFS \text{ , for any } 1 \le k \le N ; \tag{3}$$

$$S = S_{N+1} + F + SIFS + DIFS + ACK . \tag{4}$$

One can mention that the random variables F and are identically distributed; EIFS, SIFS, DIFS and ACK are system parameters, with their values known (e.g. in IEEE 802.11 – operated WLAN, SIFS = 10 μsec, ACK = 112 bits/ 1 (Mb/s); EIFS = SIFS + DIFS + ACK);[1], [3]. On the other hand, the following hypothesis are legitimate : C and S are independent random variables; $C_1, C_2, \ldots, C_N$ are independent of each other, with the same distribution; S_k, CF and F are independent of each other. One also remark that the number of idle periods in a frame service time is N + 1 and put $CS = S_{N+1}$.

We will below give some statistical estimations regarding the means (expected values) and variations of the above random variables.

2. Some statistical indicators for a fluid unity

Recall that for any random variable X (relatively to the considered probability field), one can define its cumulative distribution function $F_X : R \to [0, 1]$, $F_X(t) = P(X \le t)$. Suppose that this function is derivable and its derivative $p_X : R \to R$, called the probability density function, belongs to the class L^1 of the absolutely integrable functions. Denote by $X(\omega) = F\{p_X(t)\}$, the Fourier transform of p_X , that is $X(\omega) = \int_{-\infty}^{\infty} p_X(t).e^{-j\omega t}\, dt = E(e^{-j\omega t})$, called also the <u>characteristic function</u> of the random variable X. Obviously:

$$X(0) = 1 \text{ , } |X(\omega)| \le 1 \text{ , } X^{(k)}(0) = \int_{-\infty}^{\infty} (-jt)^k . p_X(t)\, dt = (-j)^k.\nu_k \text{ ,}$$

where $\nu_k = E(X^k)$ is the moment of order k of X. Particularly, for the mean and the variance of X, it will follow that :

$$EX = \nu_1 = j.\frac{d}{d\omega} X(\omega)\Big|_{\omega=0} \text{ ; } \mathrm{Var}\, X = \nu_2 - (\nu_1)^2 = -\frac{d^2}{d\omega^2} X(\omega)\Big|_{\omega=0} - (EX)^2. \quad (5)$$

We will also apply the following fact: if X and Y are independent random variables, then $p_{X+Y} = p_X * p_Y$ (convolution) and thus:

$$(X+Y)(\omega) = X(\omega).\, Y(\omega) \text{ and } \mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y). \quad (6)$$

PROPOSITION 2. <u>With transparent notations, the following relations hold</u>

a) $T(\omega) = C(\omega).\, S(\omega)$, for any real ω ; (7)

b) $S(\omega) = CS(\omega).\, F(\omega).e^{-j\alpha\omega}$, (8)

where α = SIFS + DIFS + ACK;

c) $C(\omega) = \sum_{n=0}^{\infty} \Gamma(\omega)^n.P(N = n)$, (9)

where $\Gamma(\omega)$ is the characteristic function of the random variables C_k 's ;

d) $\Gamma(\omega) = \Gamma_1(\omega).\, CF(\omega).\, e^{-j\beta\omega}$, (10) where β = EIFS and $\Gamma_1(\omega)$ is the characteristic function of the random variables S_k's ;

<u>Proof.</u> a) By (1) and (6), $T(\omega) = (C + S)(\omega) = C(s).S(\omega)$, since C and S are independent.

b) By (4), one has $S = Y + \alpha$, where $Y = CS + F$. But $Y(\omega) = CS(\omega)$ $.F(\omega)$, according to (6). On the other hand, for the cumulative distribution functions:

$$F_S(t) = F_Y(t - \alpha),$$

hence

$$p_s(t) = p_Y(t - \alpha),$$

whence

$S(\omega) = F\{p_S(t)\} = F\{p_Y(t - \alpha)\} =$ $e^{-j\alpha\omega}.Y(\omega)$ and one gets (8).

c) By (2), $C(\omega) = E(e^{-j\omega C}) = E(e^{-j\omega C_1} \ldots e^{-j\omega C_N}) = E(e^{-j\omega C_1}) \ldots E(e^{-j\omega C_N})$. Since $C_1, \ldots, C_N$ are identically distributed, $C_1(\omega) = \ldots = C_N(\omega) \overset{\text{not}}{=} \Gamma(\omega)$, hence $C(\omega) = \sum_{n=0}^{\infty} \Gamma(\omega)^n . P(N = n)$.

d) From (3), it follows that $C_k(\omega) = S_k(\omega).(CF)(\omega).).e^{-j\beta\omega}$; but all random variables C_k (respectively S_k) have the same probability density function, namely the same $\Gamma(\omega)$ and $\Gamma_1(\omega)$, whence (10).

Recall that for any random variable X, one put $\overline{X}$ instead of EX = mean .

Directly from the propositions 1 and 2, one then gets:

COROLLARY.

$$\overline{T} = \overline{C} + \overline{S}\ ;\ \overline{C} = N.\Gamma'(0).j \text{ and } \overline{S} = N.\Gamma_1'(0).j + \overline{F} + \alpha .$$

In the following, we deduce some explicit formulas for means and for variances. For means, the results are similar to some of those given in [1], in terms of Laplace transformations, but for variances the results are new. By a local convention, for the characteristic function $U(\omega)$, of a random variable U, put $\dot{U} = j.\frac{d}{d\omega}U(\omega)\big|_{\omega=0}$ and $\ddot{U} =$ $-\frac{d^2}{d\omega^2}U(\omega)\big|_{\omega=0}$. Thus, $\nu_1 = \overline{U} = \dot{U}$, $\nu_2 = \ddot{U}$ and Var $U = \ddot{U} - \dot{U}^2$,

PROPOSITION 3.

For any $1 \le k \le N$, the following relations hold :

$$\text{a) } \overline{C}_k = \overline{S}_k + \overline{CF} + \beta; \qquad (11)$$

$$\text{b) } \operatorname{Var} C_k = \operatorname{Var} S_k + \operatorname{Var} CF - 4\beta(\overline{S}_k + \overline{CF}). \qquad (12)$$

Proof. a) From (11), $\frac{d}{d\omega}\Gamma(\omega) = \frac{d}{d\omega}\Gamma_1(\omega).\,CF(\omega).e^{-j\beta\omega} +$

$+\,\Gamma_1(\omega).\,\frac{d}{d\omega}CF(\omega)\,.e^{-j\beta\omega} - j\beta\Gamma_1(\omega).CF(\omega)\,.e^{-j\beta\omega}$; multiplying by j and making $\omega = 0$, one gets:

$\dot{\Gamma} = \dot{\Gamma}_1.1.1 + 1.\dot{CF}.1 + \beta$ hence (11).

b) On the other hand, $\frac{d^2}{d\omega^2}\,\Gamma(\omega) = \frac{d^2}{d\omega^2}\,\Gamma_1(\omega).\,CF(\omega)\,.e^{-j\beta\omega} +$

$+\,\Gamma_1(\omega).\,\frac{d}{d\omega}CF(\omega)\,.e^{-j\beta\omega} - j\beta\frac{d}{d\omega}\,\Gamma_1(\omega)\,.\,CF(\omega)\,.e^{-j\beta\omega} +$

$+\,\frac{d}{d\omega}\Gamma_1(\omega)\,.\frac{d}{d\omega}CF(\omega)\,.e^{-j\beta\omega} + \Gamma_1(\omega).\,\frac{d^2}{d\omega^2}CF(\omega)\,.e^{-j\beta\omega} -$

$-j\,\beta\Gamma_1(\omega)\,.\,\frac{d}{d\omega}CF(\omega)\,.\,e^{-j\beta\omega} - j\,\beta\,\frac{d}{d\omega}\,\Gamma_1(\omega).\,CF(\omega)\,.\,e^{-j\beta\omega} -$

$-j\,\beta\Gamma_1(\omega)\,.\,\frac{d}{d\omega}CF(\omega)\,.\,e^{-j\beta\omega} - \beta^2\,.\,e^{-j\beta\omega}$ and making $\omega = 0$, $\ddot{\Gamma} = \ddot{\Gamma}_1 + 2\,\dot{\Gamma}_1.\dot{CF} - \quad -2\beta\,\dot{\Gamma}_1 + \ddot{CF} - 2\beta\,\dot{CF} + \beta^2$.

Therefore, by (11):

$$\operatorname{Var}\Gamma = \ddot{\Gamma} - \dot{\Gamma}^2 = \operatorname{Var}\Gamma_1 + \operatorname{Var} CF - 4\beta\,\dot{\Gamma}_1 - 4\beta\,\dot{CF}, \text{ whence (12).}$$

The distribution of the random variable CF is practically known (being the same with that of the size of a frame). It remains the problem to estimate the repartition of S_k. An admissible hypothesis is the following: any S_k is a random variable exponentially distributed, with a parameter $\lambda > 0$ which can be established by experiments. In this case, if the maximum contention window size of S_k's is M, then by putting

$$J = \int_0^M p_\lambda(t)\,dt$$

it follows that $\bar{S}_k = \frac{1}{I}\int_0^M t\, p_\lambda(t)\, dt$ and $\mathrm{Var} S_k = \frac{1}{I}\int_0^M (t-\bar{S}_k)^2 p_\lambda(t)\, dt$.

Recall that the probability density function of an exponentially distributed random variable is $p_\lambda(x) = \lambda . e^{-\lambda x}$ for $x \geq 0$ and $p_\lambda(x) = 0$ for $x < 0$. After a simple computation, it follows that, for any $1 \leq k \leq N$,

$$\bar{S}_k = \frac{e^{\lambda M} - \lambda M - 1}{\lambda e^{\lambda M} - \lambda} \approx \frac{M}{2} - \frac{\lambda M^2}{12} , \tag{13}$$

if the product λM is small ; a similar result gives Var S_k.

In this way, the formulas (11), (12) allow to estimate the main statistical indicators of C_k, S_k, whence one can estimate the behaviour of the mean $\bar{T}$ of the frame service time.

§ **3.** Statistical indicators for a full MAC – unity

As we have said, a fluid unity is made up of zero or more collision periods followed by a successful frame transmission. Consider now the more general case, that if a chain of p successive fluid unities; such a chain can be called a MAC – unity (fig. 2) and denoted by $\{T_1, \ldots , T_p\}$.

Fig.2.

We have denoted by S_k the number of idle slots before the k-th collision or the successful transmision, $1 \leq k \leq N$, in a fluid unity.

Denote by $L = T_1 + \ldots + T_p$, the length of the above considered MAC – unity. By a similar reasoning, by considering T_i like independent random variables which have the same distribution, one can show that the characteristic function $L(\omega)$ of L satisfies the functional relation

$$L(\omega) = S_t(\lambda - \lambda L(\omega)).T(\omega), \tag{14}$$

where S_t is the total number of idle slots in that MAC – unity, under the hypothesis that the number of transmission attempts is Poisson distributed with a parameter $\lambda > 0$.

Put $u = \lambda - \lambda L(\omega)$, hence $L(\omega) = S_t(u(\omega)).T(\omega)$; by derivating two times with respect to ω , one obtain:

$$\frac{dL}{d\omega} = \frac{d}{du}\, S_t(u(\omega)).(-\lambda\, \frac{dL}{d\omega}).T(\omega) + S_t(u(\omega)).\frac{dT}{d\omega} \text{ and}$$

$$\frac{d^2L}{d\omega^2} = \frac{d^2u}{dt^2}\, S_t(u(\omega)).\lambda^2\left(\frac{dL}{d\omega}\right)^2 T(\omega) - \lambda\, \frac{d}{du}\, S_t(u(\omega)).\, \frac{d^2L}{d\omega^2}.T(\omega) -$$

$$-\lambda\, \frac{d}{du}\, S_t(u(\omega)).\, \frac{dL}{d\omega}.\frac{dT}{d\omega} + \frac{d}{du}\, S_t(u(\omega)).\, (-\lambda\, \frac{dL}{d\omega}).\frac{dT}{d\omega} +$$

$$S_t(u(\omega)).\frac{d^2T}{d\omega^2}\, .$$

Put $Z = \frac{d}{du} S_t(u(\omega)) \big|_{\omega=0}$ and $W = \frac{d^2}{du^2} S_t(u(\omega)) \big|_{\omega=0}$. By making ω = 0 in the above relations and keeping into account that for any random variable X, one has $X(\omega) \big|_{\omega=0} = 1$, $\nu_1(X) = j.\frac{d}{d\omega} X(\omega) \big|_{\omega=0}$ and $\nu_2(X) = -\frac{d^2X}{d\omega^2} \big|_{\omega=0}$, one gets :

$$\nu_1(L) = Z(-\lambda\, \nu_1(\lambda)).1 + 1.\, \nu_1(T) \text{ and}$$

$$\nu_2(L) = -W.\lambda^2.(\nu_1(L))^2 - \lambda.Z.\, \nu_2(L) + \lambda.Z.\, \frac{1}{j}.\nu_1(L).\, \frac{1}{j}\, \nu_1(T) + Z +$$

$$+\lambda.\frac{1}{j}.\nu_1(L).\frac{1}{j}\, \nu_1(T) + \nu_2(T)\, ,$$

whence $(1 + \lambda Z).\, \nu_1(L) = \nu_1(T)$ and
$(1 + \lambda Z).\, \nu_2(L) = -W.\lambda^2.(\nu_1(L))^2 - \lambda.Z.\, \nu_1(L).\, \nu_1(T) + \nu_2(T)$ .Thus, we have proved :

PROPOSITION 4.

The mean and the variance of the length of a MAC – unity are given by :

$$\overline{L} = \frac{\overline{T}}{1+\lambda Z}\ ; \tag{15}$$

$$\text{Var } L = \frac{1}{1+\lambda Z}.\text{ Var } T - \frac{\lambda^2 \overline{T}^2}{(1+\lambda Z)^3}(1 + W + Z^2)\, . \tag{16}$$

These relations can be interpreted in terms of the physical reality.

R E F E R E N C E S

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