DESIGN OF WIND FARM LAYOUT FOR MAXIMUM WIND ENERGY CAPTURE

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The paper deals with the wind farm layout optimization problem (WFLOP). The objective of the wind farm layout optimization problem is to determine the optimal placement of wind turbines within the farm. In this approach, the expected power production is maximized, taking into account eventually different other supplementary technological and functional restrictions. Because the wake effects lead to significant losses of power produced by the wind turbines, minimizing them in order to maximize the expected power production is essential. In this paper a discrete representation of wind farm is used: farm area is decomposed into a set of cells, where each cell can contain only one wind turbine. Presently, this problem is usually solved using heuristic algorithms, trying to find good enough solutions in reasonable time. In this paper, some small cases are detailed investigated, using an exact solution method to solve this problem. Some new aspects regarding the multimodal character, the existence of multiple optimal solutions of objective function, and the exact gap between the lower and upper bounds of the objective function are outlined.

Keywords: wind farm layout, wake effects, optimization, multimodal solutions, lower and upper bounds of the objective function

1. Introduction

The most commonly used objective functions in literature for wind farm layout optimization are: expected power production or (annual) energy production [1-3], cost of generated power (energy) [4], levelized production cost [5], the annual profit obtained from the wind farm [6], net present value of the wind farm [7] or a combination of them [8]. Farm layout depends on the objective function and this could be influenced by some time/moment-dependent parameters, namely selling price for a kWh of electrical energy on the market, the equivalent discount rate, incentives and taxations etc. Therefore, the chosen objective function to be maximized in this study is the expected power production, which is independent of time-dependent costs.

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The discrete formulated wind farm layout optimization problem is NP hard, and exact optimization methods meet severe difficulties to solve this particular kind of problem. Therefore, the solution of this problem is dominated by heuristic search methods even for moderately sized instances. The heuristic methods most commonly used belong to the class of genetic algorithms [4], [8] [10]. Evolutionary algorithms and related techniques is another very attractive approach to wind farm layout problems [1], [11], [12]. The resolution of the problem with a greedy heuristic method is proposed in [6], and the Monte-Carlo method is used in [13]. All representatives heuristic methods have been used to solve the wind farm layout optimization problem: particle swarm optimization [14], ant colony search algorithm [15], viral based optimization algorithm [16], artificial neural network [17], simulated annealing [18]. These heuristic methods, however, cannot offer a guarantee of the solution quality, which an exact solution method can. In a wind farm, downstream turbines have lower wind speeds if they are located behind an upwind turbine. This effect is called wake effect [19]. This reduced and turbulent wind from an upwind turbine reduces the energy produced by the downwind turbines, decreasing the wind farms' overall energy output.

In this paper a discrete representation of wind farm is used. The corresponding optimization model is consequently an integer nonlinear discrete combinatorial problem. The main goal of this work is to find the exact solution. In order to find an exact solution, the preferred method to solve this problem was the complete solution enumeration. This method allows finding all feasible solutions and the optimal solution can be found by comparison. An additional advantage of this method is the possibility to discover multiple solutions, namely multiple optimal solutions. Due to the huge number of solutions, the computational effort is significant, and therefore this method is appropriate only for very small problems.

2. Wake Model

In order to simplify the wind field behavior, in this paper is used a simple wake model similar to the Jensen model as reported in [20], that presumes the linear expansion of the wake radius with the downstream spread distance. The initial free wind stream velocity is v_0, and v_x is the velocity in the wake at a distance x downstream of the turbine. Just behind the rotor the wake has the radius r_0 equal to the turbine radius. At the distance x downstream of the turbine the radius of the truncated cone-shaped wake, r_x, can be calculated using the simple approximate relation:

$$r_x = r_0 + \alpha x \tag{1}$$

The diameter of the turbine is considered perpendicular to the wind's direction. The wake spreading constant (wake decay coefficient) or entrainment constant α establishes how quickly the radius of the wake expands with distance and can be empirically calculated as $\alpha = 0.5/(\ln z/z_0)$, where z is the hub high of the wind turbine generating the wake, and z_0 is the surface roughness [21].

The wake decay coefficient has a great influence on the result of the wake loss calculation. Appropriate values for wake decay coefficients should be deduced from the landscape type. For open farmland, wake decay coefficient can be assumed to be about 0.075-0.080, while for open water is recommended 0.040 [22]. Under the assumption that the momentum is conserved inside the wake, the velocity in the wake at a distance x downstream of the turbine, v_x, is [20]:

$$v_x = v_0\left[1-\left(1-\sqrt{1-C_T}\right)\left(\frac{r_0}{r_0+\alpha x}\right)^2\right] \tag{2}$$

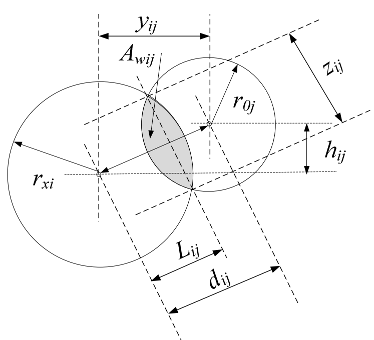
where C_T is the turbine thrust coefficient (specific for a wind turbine type and at a precise wind speed).

To estimate the average wind speed over the rotor, the momentum deficit must be averaged over the rotor area. In the general case when the wakes do not completely affect the downstream turbines, the velocity at the wind turbine j, v_j, can be expressed as:

$$v_j = v_0\left(1-\sqrt{\sum_{i=1}^{N}\left(1-\frac{v_i}{v_0}\right)^2 \frac{A_{wij}}{\pi r_{0j}^2}}\right) \tag{3}$$

A_{wij} is the part of the rotor area of the turbine j that is inside the wake produced by the upstream wind turbine I, and r_{0j} is the radius of the rotor of wind turbine j.

The calculation of the area A_{wij} is illustrated in Fig. 1 and is achieved concretely by using the relations (4) − (7).



r_{0i} is the radius of the rotor of the upwind turbine i;
h_{ij} is the difference in height between the center of wind turbine i and the center of wind turbine j (for wind turbine with different hub heights);
x_{ij}, y_{ij} are the distance between turbine i and turbine j in wind direction and in the direction perpendicular to the wind direction, respectively;
r_{xi}, is the radius of the wake produced by the turbine i, at the distance x_{ij} , downstream.

Fig. 1. Calculation of the area A_{wij} for the case of partial shadowing.

$$d_{ij} = \sqrt{y_{ij}^2 + h_{ij}^2} \tag{4}$$

$$L_{ij} = \frac{r_{xi}^2 + d_{ij}^2 - r_{0j}^2}{2d_{ij}} \tag{5}$$

$$z_{ij} = 2\sqrt{r_{xi}^2 - L_{ij}^2} \quad (6) \qquad A_{wij} = r_{0j}^2 \arccos(\frac{d_{ij} - L_{ij}}{r_{0j}}) + r_{xi}^2 \arccos(\frac{L_{ij}}{r_{xi}}) - \frac{d_{ij} z_{ij}}{2} \quad (7)$$

3. The Model of Generated Power and Energy

For a wind farm with N turbines and the wind distributed among M directions, the expected power generated by the farm, P_{aF}, can be is expressed as:

$$P_{aF} = \sum_{k=1}^{M} p_k \sum_{j=1}^{N} P_{Gj}(v_k) \quad (8)$$

where p_k is the annual frequency of the k-th wind direction, and $P_{Gj}(v_k)$ is the power generated by wind turbine j at the wind speed v_k. The average annual energy produced by the farm, E_{aF}, can be estimated to be approximately (for constant, known, annual average power of the farm):

$$E_{aF} = 8760 \sum_{k=1}^{M} p_k \sum_{j=1}^{N} P_{Gj}(v_k) \quad (9)$$

To calculate the total energy production is necessary to know the power curve. The used power curve was obtained with Wind Turbine Power Calculator for a Vestas V80/2 MW wind turbine. The wind turbine power output against wind speed, in tabular format, is given in Table 1.

Table 1

Dependency between generated active power and wind speed for a 2 MW wind turbine

Wind speed in [m/s]	Turbine power output [kW]	Wind speed in [m/s]	Turbine power output [kW]	Wind speed in [m/s]	Turbine power output [kW	Wind speed in [m/s]	Turbine power output [kW	Wind speed in [m/s]	Turbine power output [kW
1-3	0	6	261	9	957	12	1823	15	1998
4	44.1	7	437	10	1279	13	1945	16-25	2000
5	135	8	669	11	1590	14	1988	>25	0

In order to obtain generated power for intermediate points, a linear interpolation is used. To calculate the power that a wind turbine can generate it is usually used the well-known relation of proportionality of the turbine generated power with the cube (third power) of the wind speed and the power coefficient of the turbine, C_p. Unfortunately, the values obtainable with this formula are affected by important errors because the coefficient C_p varies significantly for different wind speeds and turbines. The above mentioned computation method of the wind turbine generated power using the power curve avoids this inconvenience.

4. Case Study and Results

In this study it is assumed that the turbines of the wind farm have the same performance characteristics. The parameters of the wind turbine are: hub height z=80 m, rotor diameter d=80 m, and the thrust coefficient C_T =0,88. The considered surface roughness of the wind farm is z_0=0.3m. The incoming wind is supposed to be unidirectional (predominant) and having a constant speed of 7,5 [m/s]. In order to find an exact solution, a model of a small wind farm site it was studied. The wind farm surface was divided into a 5x5 square grid. The possible positions of the turbines are allowed only in the center of these 25 cells. The length of the sides of each cell is equal to five rotor diameters, 5d (400 m). The number of the wind turbines considered in this study is N_t =10, 13, 16, 18, 22 and 25. Wind direction θ=0° corresponds to the positive direction of the Ox axe of the wind farm, and θ=90° positive direction of the Oy axe.

The values of the objective function, P_{max}, depending of the number of wind turbines and the wind directions are illustrated in Fig. 2. The first interesting important result is that for many situations concerning the turbines number and wind direction there are multiple optimal solutions (wind farm layout configurations). The number of multiple optimal solutions is different, having a large domain of variation, depending of turbines number and wind directions. Of course, for N_t= 25 we have an unique solution. For instance for N_t=10 and θ=0° we have 1 optimal solution, for θ=12°, θ=40° and θ=45° we have 401, 100 and 576 solutions, respectively. An example of multiple optimal solutions (configurations) for N_t=16 turbines and wind direction θ=45°is illustrated in Fig. 3 (a,b,c,d).

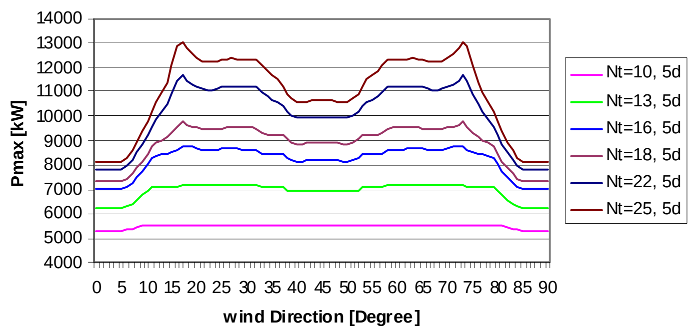


Fig. 2. Maximal wind farm power versus wind directions for different number of turbines

1	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	0	0	0	1
1	1	1	0	1

Fig. 3.a

1	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	0	0	0	0
1	1	1	1	1

Fig. 3.b

1	1	1	1	1
0	0	0	0	1
1	0	1	0	1
1	0	0	0	1
1	1	1	1	1

Fig. 3.c

1	0	1	1	1
1	0	0	0	1
1	0	1	0	1
1	0	0	0	1
1	1	1	1	1

Fig. 3.d

Fig. 3. Multiple optimal (best) configurations for N_t=16 turbines and wind direction θ=45°

The values of the maximal produced power have for lower values either one important level (L_{l1} for N_t=10 and 13) or two important levels (L_{l1} and L_{l2} for N_t=16, 18, 22 and 25). The values of the maximal produced power have for higher values either one level (L_{h1} for N_t=10 and 13) or two levels for (L_{h1} and L_{h2} for N_t=16, 18, 22 and 25). The angles ranges for wind directions corresponding to the four levels are given in table 2. For wind directions in level L_{h2} the maximal produced power values are the second lowest values. For wind directions in level L_{l2} the maximal produced power values are the first highest values.

To appreciate the gap between the lower and upper bound of the objective function it was calculated and the worse solution, P_{min} (the minimum value of the objective function) for the wind farm layout problem. The results for P_{min} have similar properties to those of the maximal wind farm power.

Table 2

Wind directions corresponding to the four levels of the maximal produced power

N_t	L_{h1} [°]	L_{h2} [°]	L_{l1} [°]	L_{l2} [°]
10	9-38 & 52-81	-	0-7 & 83-90	-
13	17-31 & 59-73	-	0-9 & 81-90	-
16	16-32 & 58-74	17-19	0-8 & 82-90	40-50
18	16-32 & 58-74	17-19	0-8 & 82-90	40-50
22	16-32 & 58-74	16-18	0-8 & 82-90	39-51
25	15-32 & 58-75	16-18	0-8 & 82-90	39-51

The percentage difference between the lower and higher value of the objective function is defined as $dP_2 = 100 \cdot (P_{\max} - P_{\min}) / P_{\max}$ and is plotted against wind directions for different number of turbines in Fig. 4.

Obvious, for N_t= 25 P_{min}= P_{max} and dP_2=0. The greatest values of dP_2 occur for the wind directions corresponding to the lowest values of the maximal wind farm power production. The greater the number of the wind turbines, the lower the percentage difference between the lower and higher value of the objective function. This difference has very important values.

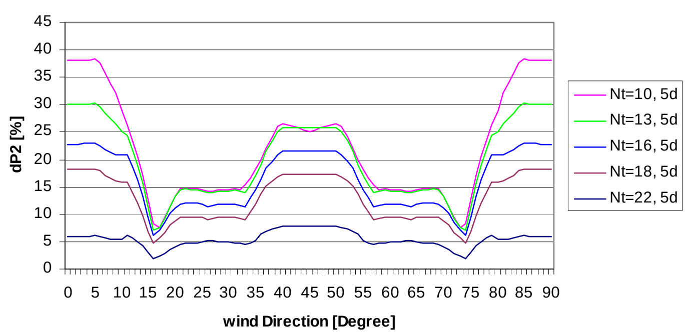


Fig. 4. Percentage difference between the lower and higher value of the objective function against wind directions for different number of turbines

5. Conclusions

This paper reveals interesting new aspects concerning the wind farm layout optimization problem. The optimization problem is formulated as an integer (boolean) nonlinear discrete combinatorial problem. The objective of the algorithm is to optimize the expected power production for a constant, predominant wind direction with a known average value. Using an exact solution method to solve this problem on a small dimension study case, important questions related of wind farm layout optimization problem are outlined: the multimodal character of objective function, the existence of multiple optimal solutions and exact value of the gap between the best and worst value of the objective function in dependence of wind direction and the number of turbines. An important research development in the future is the discovery of an appropriate method to find the exact solution and eventually of all multiple optimal solutions for big size problems of this type.

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