BOOLEAN GAMES WITH CURRENCY

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In this paper, we argue that a game-theoretic approach is suitable for modeling cooperation in Multi-Agent Systems. We briefly introduce the formal setting, and describe two different classes of games: cooperative games with transferable payoff and Cooperative Boolean Games. We discuss the capabilities and limitations of these approaches. We also introduce a new type of games, Boolean Games with Currency and give a computational characterization for a solution concept for coalitional stability: the core. We show that the core of a Boolean Game with Currency is always non-empty, and we prove the core membership problem to be co-NP complete.

Keywords: Multi-agent systems, coalition formation, coalition stability, transferable payoff, boolean games, game theory.

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1. Introduction

The success of a Multi-Agent System (MAS) is strongly affected by the way decision-making is approached. In traditional semantic systems, agents are treated as inanimate entities that interact with the environment in a strictly reactive way, by responding to external stimuli with particular actions, in order to achieve some goals. In frameworks that support agent cooperation such as [3], the emphasis falls on the knowledge representation means, and on developing a suitable agent platform. Semantic learning methods such as [11] or [10], focus on achieving a certain more-or-less fixed behavior, using machine learning techniques that do not require an in-depth understanding of the underlying processes. All these approaches ignore the fact that, in many situations, the decision to act in a certain way is influenced by the decisions of other agents. To illustrate this, consider the following example: agents 1, 2 and 3 are travelling together, and must decide on the transportation means. The possible options are (a) airplane, (b) bus, and (t) train. The decision is made by a simple voting procedure. If there is a tie, the fastest form of transportation wins. Agents have the following preferences (we use $x \succ_i y$ to denote that agent i prefers x to y): $a \succ_1 b \succ_1 t$, $b \succ_2 a \succ_2 t$, $t \succ_3 b \succ_3 a$. If each agent votes for his most preferred candidate, then the winner would be a, since there is a tie and it is assumed that airplanes are faster than buses or trains. This outcome is highly undesirable for agent 3, who least prefers travelling by air. Nevertheless, if 3 would observe the preferences of 1 and 2, then, he could change his vote from t to b, and therefore change the outcome of the voting procedure from a to b. Even if b is not 3's most desirable travelling means, for him it is better than a. In this latter case, 3

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behaves in a *strategic* way: his decision is based both on his goal, as well as on the others' decisions.

By *strategic interaction*, we refer to any situation where: the state of an agent is dependent both on the environment as well as on the states of other agents, each agent is aware of this fact and exploits it to his own advantage. In the following, we are interested in Multi-Agent Systems where any interactions are done in accordance with these properties. We also make the following assumptions on the characteristics of agents: (i) they are rational, (ii) self-interested and (iii) have limited resources.

Assumption (i) refers to agents that are fully aware of the actions available for them as well as their consequences and have well-defined preferences over all possible states-of-the world. By (ii) we understand that each agent has a certain preferred state-of-the world, and would take any possible action to achieve this state, regardless of the interests of other agents. Assumption (iii) states that any agent can only use some finite computational effort, in order to derive the proper actions he should undertake.

Game Theory (GT) is a powerful tool in the design of any kind of strategic interaction between agents in a MAS. It uses the following elements: a set of *actions* available for each agent, a set of *consequences* that result from performing some actions, a *consequence function* that, for each action, associates a proper consequence, and finally, for each agent, a preference relation over the set of consequences. GT distinguishes between two modeling approaches: non-cooperative, and cooperative. In the non-cooperative setting the emphasis falls on the actions available to the *individual*, as well as his preferences. The cooperative setting, still preservers the preferences of individuals but the emphasis now falls on what agents can *jointly* achieve.

In the following we give a brief illustration of the game-theoretic approach to modeling cooperation in Multi-Agent Systems, and introduce a new type of game, suitable for representing and reasoning about compromise. To our knowledge, there are no actual applications of game theory in semantic agent frameworks, although game theory is briefly mentioned in papers such as [8] or [7]. Finally, we note that our paper in an extended version of our previous work from [1]. The rest of the paper is structured as follows: in Section 2 we give a short introduction to cooperative games with transferable payoff and describe a solution concept for coalitional stability: the core. We also illustrate some limitations that characterize these types of games. In Section 3 we describe a particular class of cooperative games which are able to more accurately capture the preferences of users: Cooperative Boolean Games. In Section 4 we introduce a new type of game, Boolean Games with Currency, which combines the features of Cooperative Boolean Games, with the transferable payoff setting. In Section 5 we give some complexity bounds for the computation of the core and, finally in Section 6 we make some final remarks and sketch future directions concerning our approach.

2. **Cooperative Games**

In the following section, we restrict our attention to the subset of cooperative games (CG) with *transferable payoff.* They refer to situations where each consequence can be described by a certain payoff value which can be transferred between agents.

2.1. CG with transferable payoff

Definition 2.1. *A* ***coalitional game*** *with transferable payoff is a pair $\langle N, v\rangle$ where: N is a finite set of agents (or players) and $v : 2^N \to \mathbb{R}$ is a function that, for each non-empty coalition $S \subseteq N$, associates a certain worth $v(S)$. Traditionally, v is called the* characteristic function *of a coalitional game with transferable payoff.*

For each coalition S, $v(S)$ represents the total value that is jointly achieved by agents in S. As mentioned before, this value can be divided between the members of S. We model the actions available to the coalition S by the set of all distributions of the payoff $v(S)$, among its members. Let $\langle N, v\rangle$ be a coalitional game with transferable payoff. Given a N-dimensional vector of real numbers $x = (x_1, \ldots, x_N)$, we denote by $x(S)$ the sum $\sum_{i \in S} x_i$. Any vector x is a *S-feasible payoff profile* if $x(S) = v(S)$. If $S = N$, we say that S is the *grand coalition* and denote an N-feasible payoff profile as simply a *feasible payoff profile*.

Whenever a game is described using the pair $\langle N, v\rangle$, we say that it is in the **characteristic function** (cf) form. The detailed properties for the (cf) form are discussed extensively in [9].

Example 2.1. *Consider as a simple example, the following social game: Jim and Harry intend to go out, and can choose between a restaurant and a pub. Jim would like to have a drink at a restaurant, but is looking forward to spend more time with Harry, while Harry would really like to have some beers at the local pub. He is not particularly keen on talking to Jim, but if he brings a friend, he can get some free beers. This scenario can be modeled by a cooperative game with transferable payoff having $N = \{1, 2\}$, where players 1 and 2 are Jim and Harry, respectively, and the characteristic function v has the following definition: $v(\{1\}) = 1$, $v(\{2\}) = 3$ and $v(\{1, 2\}) = 4$.*

The value $v(\{1\}) = 1$ models Jim's relative small gain from going out alone to the restaurant, while $v(\{2\}) = 3$ and $v(\{1, 2\}) = 4$ show that Harry is happy going to the pub alone, but prefers taking Jim along, since together they can have more beers. The values $v(S)$ do not capture explicitly the possible outcomes of the game (going to the restaurant, or to the pub, etc.), as it would happen in a non-cooperative setting. They measure the collective gain of players, when they decide to cooperate.

Example 2.2. *Let us extend Example 2.1 by adding a third player: Tom. Like Jim, Tom is also indifferent about the location, and doesn't want to go out alone. But Tom finds Jim to be a really boring person, and prefers the company of Harry. For this scenario, v is: $v(\{1\}) = 1$, $v(\{2\}) = 3$, $v(\{3\}) = 1$, $v(\{1, 2\}) = 4$, $v(\{1, 3\}) = 0$, $v(\{2, 3\}) = 4$, $v(\{1, 2, 3\}) = 6$.*

One can observe that coalitions $\{1, 2\}$ and $\{2, 3\}$ achieve the same amount (since Harry is indifferent on his beer partner), and the grand coalition brings some improvement to the setting (both Tom and Jim can happily share the company of Harry).

2.2. The core of a CG with transferable payoff

How should a player decide whether to participate in a coalition or not? Obviously, the coalitions that achieve a higher value are more likely to be formed. But this criterion alone is not sufficient. The way the payoff is distributed is also important. Going back to Examples 2.1 and 2.2, let us assume that each value $v(S)$ can be transferred between the participants of S (for instance, $v(S)$ could account for the number of beers coalition S can obtain, at a restaurant or pub). The payoff profile $x = (2, 2, 2)$ for the grand coalition is a value distribution where Harry, Tom and Jim get equal shares of 2. But, if x is implemented, then Harry only gets 2 in the grand coalition, whereas alone, he could get more. Recall that $v(\{2\}) = 3$. Therefore, Harry can object to x, and make a threat of leaving the grand coalition if he doesn't get more. If this threat is implemented, then Tom and Jim, together, have no gain ($v(\{1, 3\}) = 0$), and each would rather go out alone. Now consider the payoff profile $x' = (1.2, 3.4, 1.2)$. This distribution makes everyone happy, and there is no (objective) deviation of some player, or some coalition. To see this, consider coalition $\{1, 2\}$ which achieves 4. There is no division of the value 4 that would give more to the players 1 and 2, than in x'.

The line of reasoning we have followed so far, takes us towards a general solution concept of *stability* of the grand coalition, *the core*, which we will formalise in what follows. The core is able to tell us whenever the grand coalition is stable (if it will form or not), and what are the payoffs for each player, that ensure stability.

Definition 2.2. *The core of a cooperative game with transferable payoff $\mathbb{G} = \langle N, v\rangle$ is the set of all feasible payoff profiles $x = (x_i)_{i\in N}$ such that there is no coalition S and no S-feasible payoff profile $y = (y_i)_{i\in N}$ that satisfies $y_i > x_i$ for all $i \in S$.*

$$core(\mathbb{G}) = \{(x_i)_{i\in N} |\ \nexists S \subseteq N, y \text{ such that } \forall i \in S.y_i > x_i\}$$

The core of the game described in Example 2.2 is:

$$\{(a, b, c) \quad | \quad a + b + c = 6 \text{ and } a \geq 1, b \geq 3, c \geq 1\}$$

There are two fundamental questions we are interested in, regarding the core of a given cooperative game: (i) is the core empty ($core(\mathbb{G}) = \emptyset$)? and (ii) for a given payoff profile x, is it the case that $x \in core(\mathbb{G})$? The first question helps us to establish whether a game is stable or not (with respect to our solution concept, the core), and the second question regards the payoff divisions that ensure the participation of each player to the grand coalition.

The computational complexity for the core emptiness problem was shown to be NP-complete in [2], and the core membership problem was established as co-NP-complete. More precisely, given a CG with transferable payoff $\mathbb{G}$ and a payoff profile x, it is NP-complete to establish if $x \notin core(\mathbb{G})$.

Returning to the cf-form of cooperative games, another remark, of a different nature, can be made. Given no restriction on the function v, the description of a cooperative game grows exponentially in the number of players $|N|$. In our Example 2.2, the number of values that have to be specified grow from 3 (in Example 2.1), to 7. In the general case, for a game with $|N|$ players requires specifying $2^{|N|-1}$ values for v, one for each coalition (and not including the empty coalition $\emptyset$). In order to address this issue, cooperative games with **compact representation** either introduce limitations on the form of v, or derive a representation of v that allows an efficient computation of values $v(S)$. Examples of compact representation games as well as the complexity bounds for solution concepts for stability can be found in [6]. The paper focuses on *graph games* and *marginal contribution nets*. In a graph game, the value of each coalition is encoded in a weighted undirected graph, where each node stands for a particular player, in the following way:

$$v(S) = \Sigma_{i,j \in S} \text{weight}(e_{(i,j)})$$

It is straightforward that, for any S, the value $v(S)$ can be computed in a tractable way, by adding the weights of all edges covered by nodes in S. In a marginal contribution network, a set of rules of the form $\{pattern\} \to value$ define possible values for possible coalitions. Patterns refer to the presence or absence of players. For instance, $\{a \wedge b\} \to 3$ states that any coalition having both a and b have worth 3. Whenever two or more rules are simultaneously matched, their values are combined. For instance, if we add to the previous rule, the following: $\{b\} \to 2$, then any coalition containing players $\{a, b\}$ will have a value of $2 + 3 = 5$. In the following, we restrict our attention to another class of games with compact representation: boolean games.

3. **Compact representations**

Cooperative Boolean Games are introduced in [4], which also proves some complexity bounds for different related solution concepts.

Let $\mathcal{V} = \{p, q, r, \ldots\}$ be a finite set of propositional variables, and $\mathcal{L}_\mathcal{V}$ be a propositional language consisting of formulas built using variables from $\mathcal{V}$, the negation operator $\neg$ and the connectives $\wedge$ and $\vee$. In a *Cooperative Boolean Game*, each player i controls a subset of variables $\theta_i \in \mathcal{V}$. $\theta_1, \theta_2, \ldots, \theta_n$ is a proper partition of $\mathcal{V}$ such that no two players can share control over a variable. The possible actions available to players consist of setting truth values to the variables they control. Each action has a certain cost. The player i's objective is expressed as a boolean formula $\gamma_i \in \mathcal{L}_\mathcal{V}$.

Definition 3.1. *A Cooperative Boolean Game (CBG) is a tuple* $\langle N, \mathcal{V}, (\theta)_{i \in N}, c, \gamma_1, \gamma_2, \ldots, \gamma_n \rangle$ *where:*

- $N = \{1, 2, \ldots, n\}$ *is the set of players;*
- $\mathcal{V}$ *is a set of propositional variables;*
- $\theta_1, \theta_2, \ldots, \theta_n$ *are the sets of variables controlled by each player;*
- $c : \mathcal{V} \to \mathbb{R}_+$ *is a cost function;*
- $\gamma_1, \gamma_2, \ldots, \gamma_n \in \mathcal{L}_\mathcal{V}$ *are the goals of each player.*

Example 3.1. *The game in cf-form described in Example 2.2 can be modeled as a Cooperative Boolean Game, in the following way:*

- $N = \{1, 2, 3\}$ *- the players, Jim, Harry and Tom*
- $\mathcal{V} = \{p_1, p_2, p_3, r_1, r_2, r_3\}$ *and the following partition: for all* $i \in \{1, 2, 3\}$ $\theta_i = \{p_i, r_i\}$ *model the player i's possible choices:* p_i *and* r_i *models player i's choice: going to the pub, or restaurant, respectively.*
- $c = 0$ *is a constant cost function;*
- $\gamma_1 = (p_1 \wedge p_2 \wedge \neg p_3) \vee (r_1 \wedge r_2 \wedge \neg r_3)$
- $\gamma_2 = p_2 \wedge \neg r_2$
- $\gamma_3 = (p_3 \wedge p_2) \vee (r_3 \wedge r_2)$

One can easily see that player 1, Jim, wants to go to the restaurant or the pub, but does not want Tom to be present. Player 2, Harry, simply wants to go to the pub, and the last player, Tom, is also indifferent on the location, as long as Harry is present. Also, notice that representing goals for n players can be achieved using n propositional formulae, which is a considerable improvement with respect to the transferable payoff setting. As we will further see, the formulae γ_i encode all necessary information for deriving stability issues.

A final remark is related to the representational properties of CBG. While, in Examples 2.1 and 2.2, real values where used to measure a certain joint satisfaction of players, in Example 3.1 the exact preferences of each individual are captured. The two modeling approaches, the CG with transferable payoff and CBG capture different features of our scenario, and as a result, there are inherent differences between them. Also, the setting is no longer one with transferable payoff, and, as we will further see, this affects the definition of the solution concepts we study.

In order to examine how cooperation is achieved in a CBG, we require the following definitions. A *valuation* is a possible *outcome* of a CBG. Formally, a valuation ξ is a subset of $\mathcal{V}$ with the interpretation that, whenever some variable v is a member of ξ, then v is set to true, and if $v \notin \xi$, then v is set to false. For any formula $\gamma \in \mathcal{L}_{\mathcal{V}}$, we write $\xi \models \gamma$ to designate the fact that, by assigning the value *true* for all variables in ξ, the formula γ is satisfied.

The total cost supported by player i under a valuation ξ is the sum of all costs of variables set to true by him: $c_i(\xi) = \sum_{v \in \xi \cap \theta_i} c(v)$. The total cost of all variables from $\mathcal{V}$ is $\mu = \sum_{v \in \mathcal{V}} c(v)$. Then, the utility of player i under valuation ξ is:

$$u_i(\xi) = \begin{cases} 1 + \mu - c_i(\xi) & \text{if } \xi \models \gamma_i \\ -c_i(\xi) & \text{otherwise} \end{cases}$$

Unlike the characteristic function v, which described the values of coalitions (groups of players), the u_i functions describe the individual gain of each player. Until this point, the setting of a CBG is strictly non-cooperative. The assumption made in the previous section, i.e. there is a certain payoff $v(S)$ that is achievable by a coalition, and transferable among its members, no longer holds. Here, in order to describe the effectiveness of a coalition, CBG's define a *preference relationship* of players over valuations. The preference relationship is naturally induced by the utility functions u_i: $\xi_1 \succeq_i \xi_2 \iff u_i(\xi_1) \geq u_i(\xi_2)$. We use $\succ$ to refer to the strict preference relation. $\succeq$ can be naturally extended to coalitions, in the following manner: $\xi_1 \succeq_C \xi_2 \iff \forall i \in C \quad \xi_1 \succeq_i \xi_2$. The definition of $\succeq$ has the following properties: (i) players will always prefer valuations ξ that satisfy their goal over the ones that do not satisfy it: a valuation ξ such that $\xi \not\models \gamma_i$ will produce a negative utility value for player i, and a positive value if $\xi \models \gamma_i$, (ii) if there are two valuations that satisfy the goal of some player, he will always prefer the one inducing smaller costs and (iii) if there are two valuations that do not satisfy the player's goal, he will again prefer the one producing a smaller cost.

Based on valuations and on the preference relation over coalitions $\succeq$, the following definition, due to [4] is natural:

Definition 3.2. *The core of a CBG is the set of valuations ξ such that, there is no coalition $S \subseteq N$ and ξ' such that ξ' and ξ make the same variable assignments for players in $N \setminus S$ and $\xi' \succeq_S \xi$.*

Going back to our example, in this setting the core is given by the set containing $\xi = \{p_2, p_3\}$. This valuation leaves player 1 unsatisfied, but here he cannot object, since there is no coalition and ξ' that can guarantee $\neg p_3$ for him, and prefer ξ' to ξ. Intuitively, this outcome corresponds to the natural fact that, nobody can stop player 3, Tom, from showing up at the pub.

4. **Games with currency**

In the following, we are enhancing Cooperative Boolean Games with the ability to transfer some shared value between players. Our proposal consists of Boolean Games with Currency. They combine the features of CBG with that of CG's with transferable payoff.

Definition 4.1. *A boolean game with currency (BGC) is a tuple*

$$\mathfrak{B} = \langle N, \mathcal{V}, (\mathcal{V}_i)_{i\in N}, (\gamma_i)_{i\in N}, \mu, c\rangle,$$

where:

- $N = \{1, 2, \ldots, n\}$ *is the set of players;*
- $\mathcal{V}$ *is a finite set of propositional variables;*
- $\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_n$ *is a partition of* $\mathcal{V}$*. Each* $\mathcal{V}_i$ *denotes the set of variables controlled by player* i*;*
- $\mu : N \to \mathbb{R}_{\geq 0}$*:* μ *assigns a value to an agent's goal formula; in the following, we use* μ_i *as a shorthand for the value* $\mu(i)$*.*
- $\gamma_1, \gamma_2, \ldots, \gamma_n \in \mathcal{L}_{\mathcal{V}}$ *are the goals of each player;*
- $c : N \times \mathcal{V} \times \mathbb{B} \to \mathbb{R}_{\geq 0}$ *is a function assigning for each player, variable and truth value, a cost of setting that particular truth value. We use* $c_i(w, t)$ *as a shorthand for* $c(i, w, t)$*;*

This new setting is motivated by the following insight. Again, consider Example 3.1 and the valuation $\xi = \{p_2, p_3\}$ which is in the core of the BGC game we defined. If Jim's goal satisfaction might be measured by a particular real value, and if this value could be transferred between players, then he could change the outcome of the game, by *tipping off* Tom and thus determining him not to come to the bar. This would be possible under the following assumption: Jim's happiness when going with Harry is much larger than Tom's, and therefore he can afford to *tip off* Tom, and make him earn more by staying at home (or going to the restaurant), than by coming to the pub to see Harry.

In order to model scenarios such as the above, consider the following definitions. For an arbitrary set of variables $V \in 2^{\mathcal{V}}$, let $\xi = (\xi_w)_{w\in V}$ be a V-valuation giving truth assignments for variables in V. If w is a variable in V, then either $\xi_w = \top$ (true) or $\xi_w = \bot$ (false). For all players i, we define a function testing the entailment of i's goal:

$$1_i(\xi) = \begin{cases} 1 & \text{if } \xi \models \gamma_i \\ 0 & \text{otherwise} \end{cases}$$

The utility of an agent can be defined in the spirit of [4]. If ξ is a V-valuation, then:

$$u_i(\xi) = 1_i(\xi) * \mu_i - \sum_{w\in V\cap\mathcal{V}_i} c_i(w, \xi_w)$$

Notice that, in our scenario, it might be the case that the utility of an agent is not positive when his goal is satisfied. Such a condition could be achieved only by adding restrictions to the value and cost functions μ and c. Therefore, it is not always the case that an agent prefers an outcome where his goal is satisfied. This gives us the intuition that, in settings such as ours, agents might attempt to maximise their utilities by means other than satisfying goals. One such mean is by participating in coalitions where some players have goals with great values. Agents contribute to these goals instead of their own, and, in certain settings, can achieve more value. This is the case for Tom, who could participate in forming the grand coalition by fulfilling Jim's goal instead of his own, under the condition that he can obtain a higher value this way.

Proposition 4.1. *Cooperative Boolean Games are a special case of Boolean Games with Currency.*

Proof. Consider a game $\mathfrak{B} = \langle N, \mathcal{V}, (\mathcal{V}_i)_{i\in N}, (\gamma_i)_{i\in N}, \mu, c\rangle$, with the following restrictions on μ and c:

- $\forall w \in \mathcal{V}, c_i(w, \bot) = 0$; only setting variables to true inflicts a cost on agents;
- $\forall w \in \mathcal{V}, c_i(w, \top) = c_j(w, \top)$; the costs for setting a variable to true are the same for all agents;
- $\mu_i > \sum_{w\in\mathcal{V}} c_i(w, \top)$; the value of each agent's goal is strictly larger than the sum of all the costs for variables he controls.

The last restriction ensures positive values for utilities u_i, if players satisfy their goals. Then u_i can induce a preference relationship over valuations ξ that obeys the properties discussed in Section 3. □

5. The core of a BGC

For a game $\mathfrak{B} = \langle N, \mathcal{V}, (\mathcal{V}_i)_{i\in N}, (\gamma_i)_{i\in N}, \mu, c\rangle$, and a coalition $S \subseteq N$, the *endowment* of each player $i \in S$ is:

$$1_i(\xi)\mu_i - \sum_{w\in\mathcal{V}\cap\mathcal{V}_i} c_i(w, \xi_w) \tag{1}$$

The endowment is computed by taking the value of the satisfied goal, if it is indeed satisfied, and subtracting the involved costs. Based on the endowment, the characteristic function v is defined as follows:

$$v(S) = \max_{\xi=(\xi_i)_{i\in V}} \sum_{i\in S} \left\{ 1_i(\xi)\mu_i - \sum_{w\in V\cap\nu_i} c_i(w, \xi_w) \right\} \tag{2}$$

Equation 2 describes the worth of a coalition S as being the maximum sum of all endowments, obtained by some valuation ξ_i.

The **cf form** $\langle N, v\rangle$ of a BGC $\mathfrak{B}$ is obtained by taking N to be the number of players of $\mathfrak{B}$, and by computing the characteristic function v according to Equation 2. The **core** of a BGC expressed in cf form $\langle N, v\rangle$ is the set of all feasible payoff profiles $(x_i)_{i\in N}$, for which there is no coalition S such that $v(S) > x(S)$. We say that $(x_i)_{i\in N}$ is group rational: no other coalition S has an incentive to deviate. Also, notice that this definition is not substantially different from the one given in Section 2.

Proposition 5.1. *Core membership for BGC is co-NP complete.*

(sketch). Core membership is a decision problem $\mathrm{MEM}(\mathfrak{B}, x)$ which, given a Boolean Game with Currency $\mathfrak{B}$ and a vector x of size N, asks whether x is in the core of $\mathfrak{B}$. Recall the definition of core membership, which requests that there is no coalition $S \subseteq N$ and payoff profile y such that S prefers y over x. Now, consider the complement of core membership problem $\overline{\mathrm{MEM}}(\mathfrak{B}, x)$ which asks if there exists a coalition S, and a payoff y such that $y(S) > x(S)$. An equivalent definition for this problem, is asking whether there exists a coalition $S \subseteq N$ such that $v(S) > x(S)$. If such an S exists, then there is also an S-feasible payoff profile y, which players in S will prefer over x.

In the following, we prove $\overline{\mathrm{MEM}}(\mathfrak{B}, x)$ to be NP-complete.

The membership $\overline{\mathrm{MEM}}(\mathfrak{B}, x) \in NP$ is straightforward: A procedure can build all coalitions S in nondeterministic polynomial time, and, for each S, checking whether $v(S) > x(S)$ holds can be done in deterministic polynomial time. The NP-hardness of $\overline{\mathrm{MEM}}(\mathfrak{B}, x)$ is due a reduction from the k-VERTEX-COVER(G) problem.

Let $G = (A, E)$ be a graph. k-VERTEX-COVER(G) asks if there is a subset $B \in A$ of vertices, with $|B| = k$, such that all edges from E are covered by at least one vertex. Starting from an instance of k-VERTEX-COVER(G), we build an instance of $\overline{\mathrm{MEM}}(\mathfrak{B}, x)$ in the following way: (i) for each vertex $a \in A$, we create a player $n_a \in N$. We add an additional player n_o to N; (ii) for each edge $e = (a, b) \in E$, we create two variables $p_a, p_b \in \mathcal{V}$, and subsequently assign their control to the corresponding players: $p_a \in \mathcal{V}_{n_a}$ and $p_b \in \mathcal{V}_{n_b}$ (each player controls one"side" of an edge). For the additional player, we have $\mathcal{V}_{n_o} = \emptyset$; (iii) we define the cost function c such that $\forall i$, $\forall p_e \in \mathcal{V}$, $c_i(p_e, \top) = 1$ and $c_i(p_e, \bot) = 0$; (iv) the goal of each player n_a is $\gamma_{n_a} = \bigwedge_{e=(a,b)} (p_a \vee p_b)$ (n_a's goal is satisfied when at least one "side" of each of his incident edges is set to $\top$). The goal for player n_0 is $\gamma_{n_0} = \bigwedge_{e=(a,b)\in E} (p_a \vee p_b)$ (it is satisfied when all edges have at least one side set to $\top$); (v) the goal value for each player n_a is $\mu_{n_a} = 0$. The goal for player n_0 is $\mu_{n_0} = 2|E|$; (vi) we build a payoff vector x such that $x_{n_o} = |E| - 1/2$ and $\forall n_a, x_{n_a} = \frac{1}{2(k+1/2)}$.

($\Longrightarrow$) Suppose W is a vertex cover of size k. Then, the coalition $S_W = \{n_0\} \cup \{n_a | a \in W\}$ will achieve the maximum value under any valuation $\xi = (\xi_i)_{i \in S_W}$ such that, $\forall e = (a, b)$ $\xi_{p_a} = 1$ and $\xi_{p_b} = 0$ (exactly one "side" of each edge is set to true). Under any such valuations, all players see their goals satisfied. Therefore:

$$v(S_W) = \sum_{i \in S_W} \mu_i - \sum_{i \in S_W} \sum_{w \in V \cap \nu_i} c_i(w, \top) = 2|E| - |E| = |E|$$

. Since, according to the above construction, the payoff profile x associated to S_W yields:

$$x(S_W) = x_{n_0} + k * \frac{1}{2(k+1/2)} = \tag{3}$$

$$|E| - 1/2 + \frac{k}{2*k+1} = |E| - \frac{1}{2k+1} < |E| = v(S_W) \tag{4}$$

then x together with S_W is a credible deviation to the grand coalition.

($\Longleftarrow$) is straightforward, and follows the same arguments described previously. □

Proposition 5.2. *Given a BGC in cf-form, the characteristic function v is super-additive, that is for every $S_1, S_2 \in N$ such that $S_1 \cap S_2 = \emptyset$, the following holds*

$$v(S_1 \cup S_2) \geq v(S_1) + v(S_2).$$

Proof. Let S_1, S_2 be two disjoint coalitions and ξ_1, ξ_2 be the solutions of Equation 2, giving the values $v(S_1)$ and $v(S_2)$, respectively. Let $V_1 = \cup_{i \in S_1} \mathcal{V}_i$ and $V_2 = \cup_{i \in S_2} \mathcal{V}_i$. It is straightforward from the definition of a BGC that variable sets V_1 and V_2 are disjoint (since no two players can have control over the same variable). Since ξ_1 and ξ_2 contain truth assignments for *every* variable in V_1 and V_2 respectively, then ξ_1 and ξ_2 cannot assign different values to the same variable. Then, for all goals γ_1 and γ_2 satisfied by ξ_1 and ξ_2 respectively, it is the case that $\xi_1 \cup \xi_2 \models \gamma_1$ and $\xi_1 \cup \xi_2 \models \gamma_2$. Therefore, each satisfied goal in S_1 or S_2 is satisfied also in $S_1 \cup S_2$ (possibly cheaper). As a result we have that

$$v(S_1 \cup S_2) \geq v(S_1) + v(S_2).$$

□

Proposition 5.3. *The core of a BGC is non-empty.*

Proof. This is a direct consequence of the property of super-additivity of BGC. First of all, notice that the definition of the core from Section 5 can be reformulated as:$core(\mathfrak{B}) = \{x \mid \forall S \subseteq N.x(S) \geq v(S)\}$ Now, since $\mathfrak{B}$ is super-additive, it follows that $\forall S \subset N$, $v(N) \geq v(S)$, since $N = S \cup N \setminus S$ and $v(N) \geq v(S) + v(N \setminus S)$. Then, any feasible payoff profile that offers a division of $v(N)$ is in the core of $\mathfrak{B}$. □

6. **Conclusions and Future Work**

The game-theoretic setting proves to be a suitable direction for studying agent interaction and cooperation. Unlike machine learning approaches, that merely mimic some more-or-less fixed behavior, game theory attempts to understand the nature of the cooperation process. Different modeling approaches fit on different scenarios. As seen in the paper, the non-transferable payoff setting has certain limitations when trying to capture particular preferences for users. On the other side, Cooperative Boolean Games lack features for modeling compromise and value division. Our proposal of BGC tries to address this issue, and to find a more general modeling perspective. The results we describe, the computational complexity for the core membership and core emptiness problems are inherently theoretic, but not without applicability. Although not quite optimistic (the core membership is shown to be co-NP-complete), the results suggest that, for small games, computing the core is possible, and could be implemented in systems that asist humans in decision-making. Also, iterative techniques, in the spirit of [4], for deriving a more efficient computation process should be researched. Following the line of [5], the goal value function from BGC's can be used to incentivise certain behavior within a group of agents. We are currently tackling these ideas, as well as attempting to produce an efficient implementation, based on our results.

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