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# OPTIMAL DESIGN FOR THE INTERIOR SHAPE OF A ANNULAR DIFFUSER WITH DIVERGENT CAREENING CONSIDERING THE MINIMUM WHOLE LOSS PRESSURE

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*În lucrare se prezintă un model matematic nou şi un program numeric scris în limbajul de programare Mapple 11, pentru proiectarea optimă a formei interioare a unui difuzor hidraulic inelar cu carenaj interior divergent.*

*Modelul matematic a fost determinat în condiţiile de respectare a valorilor minime a întregii căderi de presiune. Programul propus rezolvă numeric modelul matematic neliniar şi afişează rezultatele.*

*In this paper is presented a new mathematical model and a numerical programme in Mapple 11 programming language, for optimum design of the interior shape of a hydraulic annular diffuser with interior divergent careening.*

*The mathematical model was determined to respect the minimum values of the whole loss pressure. The proposed programme solves numericayl the nonlinear mathematical model and prints the results.*

**Keywords:** hydraulic diffuser, divergent careening, loss pressure, numerical simulation

## 1. Introduction

There are many engineering systems involving subsystems in which flexible structures are subjected to fluid flow. Examples are certain types of jet into the pumps, pistons and valves; oil and gas production systems etc.

This explains the interest in the study of fluid flow modelling on structures in annular geometries [1, 2, 3].

Since then, the research effort on this topic, specially applicable to hydraulic diffusers has been intensified and a number of interesting papers on the subject may be found in the proceedings of various symposia in this area.

Moreover, a great variety of experimental works is reported in the literature about optimization of hydraulic diffusers shape. [4, 5, 6, 7, 8, 9, 10, 11].

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Due to the complexity of the model and the limited knowledge of the physics of the device, design of hydraulic diffusers with divergent careening is a difficult as well as a challenging task.

Fluid flow processes can be analyzed both numerically and experimentally. Experimental methods can help in understanding the flow behavior and also can help in assessing available theoretical predictions.

The purpose of this work is to obtain a new mathematical model and a numerical programme in Mapple 11 programming language, for optimal design of the interior shape of a hydraulic annular diffuser with divergent careening.

## 2. The mathematical model

The hydraulic diffusers are circuit elements which made connections and permit the fluid flow between two sections with distinct areas.

If a symmetrical body is placed into the diffuser and its symmetry axis is coincidental with the symmetry axis of the diffuser an annular diffuser with an interior divergent careening, is obtained through the increasing of its cross section [12, 13]. The sketch of this hydraulic element is given in Fig. 1 and the three dimensional model is shown in Fig. 2.

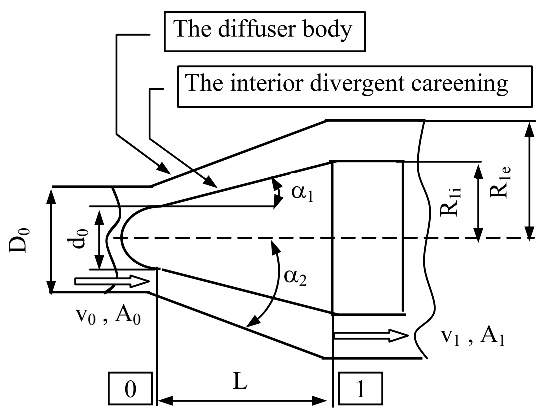


Fig. 1. The sketch of the diffuser with interior divergent careening

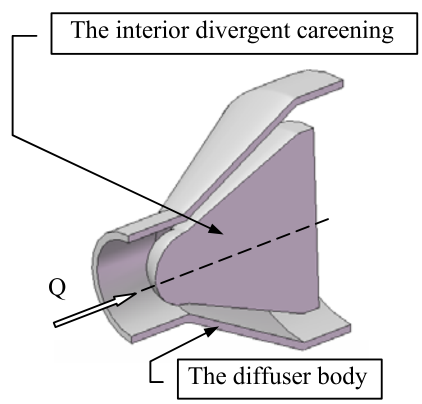


Fig. 2. The 3D model of the diffuser with interior divergent careening

The performance characteristics of an annular diffuser with an interior divergent careening depends on their geometry and the inlet conditions. The whole loss pressure into the diffuser Δp [Pa] is calculated with the relation [13, 14]:

$$\Delta p = \varsigma \cdot \rho \frac{v_0^2}{2} = 8 \cdot \varsigma \cdot \rho \cdot \left[ \frac{Q}{\pi (D_0^2 - d_0^2)} \right], \qquad (1)$$

where: $\rho$ [kg/m$^3$] - the density of fluid; $\zeta$ [dimensionless] - the Weissbach coefficient; $D_0$ [m] - the diameter of input pipe in diffuser; $d_0$ [m] - the diameter of spherical surface placed in front of the carrening body; Q [m$^3$/s]- the flow volume of fluid; $v_0$ [m/s]– the velocity of fluid to the input section.

The objective function $\Delta p \to \min$, with $\rho$ and $v_0$ constant, is equivalent with:

$$\varsigma \to \min \tag{2}$$

In practice, for this type of diffuser, the recommended constructive dimensions are given in [12]:

$$L = K_1 \cdot D_0, \quad K_1 = 0.5, ..., 1.0; \tag{3}$$

$$d_0 = K_2 \cdot D_0, \quad K_2 = 0.6, ..., 0.8; \tag{4}$$

$$n_1 = \frac{A_1}{A_0} \cdot, \quad n_1 = 2, ..., 4; \tag{5}$$

$$\alpha_1 = 8^\circ, ..., 16^\circ. \tag{6}$$

where: with $n_1$ [dimensionless] is denoted the divergence degree of diffuser.

The input annular section in the diffuser is given by the following relation:

$$A_0 = \frac{\pi \cdot (D_0^2 - d_0^2)}{4} = \frac{\pi \cdot D_0^2 (1 - K_2^2)}{4}, \tag{7}$$

and the interior and exterior rayons of output section from the diffuser are:

$$R_{1i} = \frac{d_0}{2} + L \cdot \sin\alpha_1 = \frac{D_0}{2}\left[K_2 + 2 \cdot K_1 \cdot \sin\alpha_1\right], \tag{8}$$

$$R_{1e} = \frac{D_0}{2} + L \cdot \sin\alpha_2 = \frac{D_0}{2}\left[1 + 2 \cdot K_1 \cdot \sin\alpha_2\right], \tag{9}$$

where: $\alpha_1$ [°] - the angle of divergent careening; $\alpha_2$ [°] - the interior divergent angle of diffuser, fig.1.

The interior rayon of the output section from the diffuser could be determined starting from the relation (5):

$$n_1 = \frac{A_1}{A_0} = \frac{\pi \cdot (R_{1e}^2 - R_{1i}^2)}{A_0}, \tag{10}$$

$$R_{1e} = \sqrt{R_{1i}^2 + \frac{n_1 \cdot A_0}{\pi}}, \tag{11}$$

From the equality of relations (9) and (11), and then by substitution of $R_{1i}$ [m] and $A_0$ [m$^2$], the divergent angle $\alpha_2$ is obtained:

$$\alpha_2 = \arcsin\left(\frac{\sqrt{[K_2 + 2 \cdot K_1 \cdot \sin\alpha_1]^2 + n_1(1 - K_2^2)} - 1}{2 \cdot K_1}\right), \tag{12}$$

The Weissbach coefficient is determined by substituting the previous relations in relation (1):

$$\varsigma=\frac{\left\{1+\frac{4\cdot K_1}{1-K_2{}^2}\left[tg^2\alpha_2-tg^2\alpha_1\right]+\frac{4\cdot K_1}{1+K_2}\left[tg\alpha_2-K_2\cdot tg\alpha_1\right]\right\}^2}{4\cdot\sqrt{K_1{}^3}}, \qquad (13)$$

The mathematical model given by the nonlinear objectiv function (13) must be minimized: $\varsigma=\varsigma\left(K_1,K_2,n_1,\alpha_1\right)\rightarrow$ min, with restrictions given by the relations (3), (4), (5) and (6).

## 3. The numerical programme

With the availability of fast computers programming, it is now possible to solve the equations of flows using modelling principles to give accurate results at much shorter time. This will enable the design optimization by which the number of costly experiments can be reduced.

To solve the mathematical model concerning the nonliniar optimisation [15, 16] an original programme for numerical calculus in Maple 11 programming language [17, 18] was designed. This programme determines through numerical calculation the minimum value of the Weissbach coefficient $\zeta$ and for the following parameters: $K_1$, $K_2$, $n_1$ and $\alpha_1$. After that the programme determines the optimal sizes of the interior shape of the hydraulic diffuser.

## 4. The numerical study

The input data for numerical study of simulation are given in Table 1.

*Table 1*

**The input data**

| Q | $K_1$ | $K_2$ | $N_1$ | $D_0$ | ρ | $\alpha_1$ |
|---|---|---|---|---|---|---|
| [$m^3$/s] | [dimensionless] | | | [m] | [kg/$m^3$] | [°] |
| 0.0001 | 0.5÷1 | 0.6÷0.8 | 2÷4 | 0,01 | $10^3$ | 8÷16 |

The optimum results of all parameters involved into the mathematical model, calculated with the numerical programme, are presented in Table 2.

*Table 2*

**The optimum results for output data**

| $\zeta_{opt}$ | $K_{1\ opt}$ | $K_{2\ opt}$ | $n_{1\ opt}$ | $\alpha_{1\ opt}$ |
|---|---|---|---|---|
| [dimensionless] | | | | [°] |
| 0.455 | 1 | 0.8 | 2 | 8 |

The optimal sizes of the diffuser are presented in Table 3.

*Table 3*

**The optimal sizes of the diffuser**

| $d_0$ | $R_{1i}$ | $R_{1e}$ | L | $A_0$ | $A_1$ | $\alpha_2$ | $K_3$ |
|---|---|---|---|---|---|---|---|
| [m] | | | | [m$^2$] | | [°] | [dimensionless] |
| 0.008 | 0.0054 | 0.0069 | 0.01 | $2.82\cdot10^{-5}$ | $5.65\cdot10^{-5}$ | 10°45′ | 1.35 |

Based on the optimal values calculated through numerical simulation, the minimum loss pressure is $\Delta p = 2850$ [Pa].

The input data are the optimum values, previous determinated:

$$\alpha_{1opt} = 8^\circ;\ \ n_{1opt} = 2;\ K_1 = 0.5,...1;\ K_2 = 0.6,...0.8, \tag{14}$$

Based on the input data, the numerical programme calculates the analytical relations for: $d_0$, L, $R_{1i}$, $R_{1e}$, $A_0$, $K_3$, $\alpha_2$, $A_1$, $\zeta$, $v_0$, $\Delta p$ then, the corresponding graphical representations are presented in Fig. 3 to Fig.10.

$$L = 0.01\cdot K_1, \tag{15}$$

$$d_0 = 0.01\cdot K_2, \tag{16}$$

$$A_0 = 7.854\cdot10^{-5}(1-K_2{}^2), \tag{17}$$

$$A_1 = \pi\cdot\left[5\cdot10^{-5} - 2.5\cdot10^{-5}\cdot K_2^2 + 1.392\cdot10^{-5}\cdot K_1K_2 + \right. \\ \left. + 0.19\cdot10^{-5}\cdot K_1^2 - \left(5\cdot10^{-3}\cdot K_2 + 1.392\cdot10^{-3}\cdot K_1\right)^2\right], \tag{18}$$

$$R_{1i} = 1.391\cdot10^{-3}K_1 + 5\cdot10^{-3}K_2, \tag{19}$$

$$R_{1e} = 0.2\cdot10^{-2}\sqrt{12.5 - 6.25\cdot K_2^2 + 3.478\cdot K_1K_2 + 0.484\cdot K_1^2}, \tag{20}$$

$$\alpha_2 = \arcsin\left(\frac{0.2\sqrt{12.5 - 6.25K_2^2 + 3.478K_1K_2 + 0.484K_1^2} - 250000}{K_1}\right), \tag{21}$$

$$v_0 = \frac{10^{-4}}{7.854\cdot10^{-5}\left(1-K_2^2\right)} \tag{22}$$

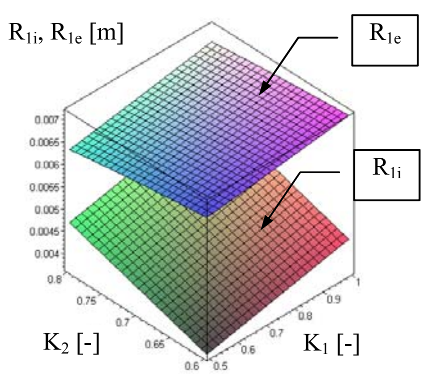


Fig. 3. The 3D variation of rayons $R_{1i} = f(K_1, K_2)$ and $R_{1e} = f(K_1, K_2)$

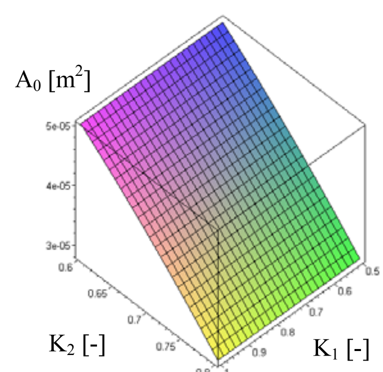


Fig. 4. The 3D variation of innelar area $A_0 = f(K_1, K_2)$

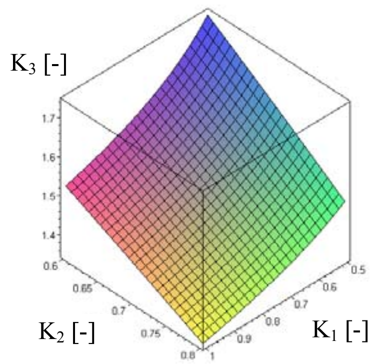


Fig. 5. The 3D variations of $K_3 = f(K_1, K_2)$

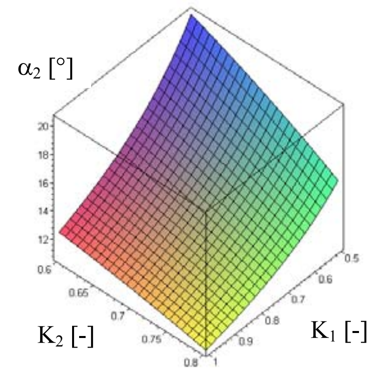


Fig. 6. The 3D variations of $\alpha_2 = f(K_1, K_2)$

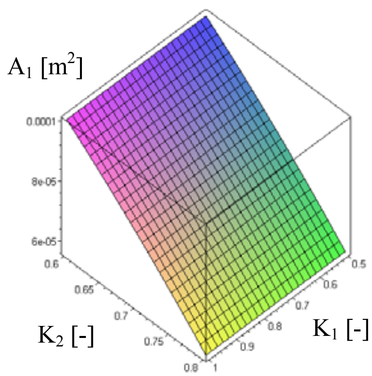


Fig. 7. The 3D variation of innelar areas $A_1 = f(K_1, K_2)$

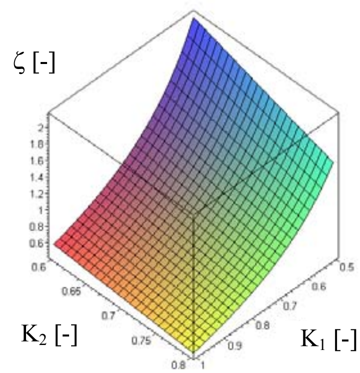


Fig. 8. The 3D variations of $\zeta = f(K_1, K_2)$

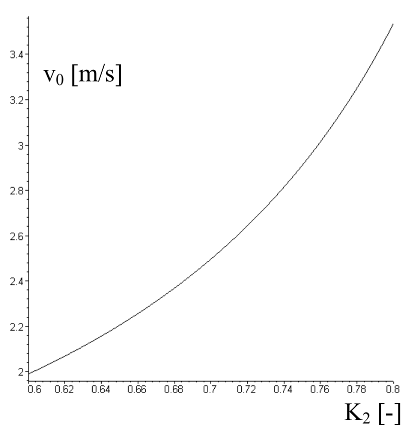


Fig. 9. The variation of velocity $v_0 = f(K_2)$

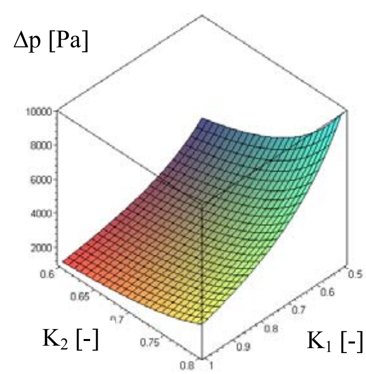


Fig. 10. The 3D variation of $\Delta p = f(K_1, K_2)$

The numerical programme also permits to find the analytical relations for all the sizes involved into the mathematical model, the function of variations and the corresponding plots.

## 5. Conclusions

In this paper a new mathematical model and a numerical programme in Mapple 11 programming language, for optimum design of the interior shape of a hydraulic annular diffuser with divergent careening is presented.

The predictions were made using the standard model and the predicted results have been validated with experiments. The proposed model was found to be a satisfactory physical model to obtain good predictions for different flow characteristics of a hydraulic annular diffuser with divergent careening.

The mathematical model was determined to respect the minimum values of the whole loss pressure owing to friction, which is the most important parameter of design from the exploitation point of view. The proposed numerical programme solves the nonlinear mathematical model and prints the graphical representations.

The present study indicates that the proposed procedure used in the paper is a powerful tool in the optimum design of the interior shape of a hydraulic annular diffuser with divergent careening.

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## R E F E R E N C E S

[1]. *S. Florea* ş.a., Analiza şi sinteza circuitelor hidraulice şi electrohidraulice, Institutul Politehnic Bucureşti, Facultatea de automatică, 1981.

[2]. *I. Mazilu, V. G. Marin,* Sisteme hidraulice automate, Editura Academiei R.S.R., Bucureşti, 1982.

[3]. *A. Oprean, C. Ispas, E. Ciobanu, A. Dorin, S. Medar, A. Olaru, D. Prodan*, Acţionări şi automatizări hidraulice. Modelare, simulare, încercare, Editura Tehnica, Bucureşti, 1989.

[4]. *S. Z. Shuja., M. A. Habib*, “Fluid flow and heat transfer characteristics in axisymmetric annular diffusers”, in Computers and Fluids, **vol. 25**, 1995, pp. 133-150.

[5]. *W. Shyy.*, “A numerical study of annular dump diffuser flows”, in Computer Methods in Applied Mechanics and engineering, **vol. 25**, 1985, pp. 47-65.

[6]. *D. E. Hobson, M. Jedwad*, “Investigations of the effect of eccentricity on the unsteady fluid forces on the centre body of an annular diffuser”, in Journal of Fluids and Structures, **vol. 4**, 1990, pp. 155-169.

[7]. *S. Patankar,* Numerical Heat Transfer and Fluid Flow, Hemisphere Publishing Corporation, New York, 1980.

[8]. *A. Ruprecht*, "Unsteady Flow Simulation in Hydraulic Machinery", in Task Quarterly, **vol. 6**, no. 1, 2002, pp. 187-208.

[9]. *S. Ubertini, U. Desider,* "Flow development and turbulence length scales within an annular gas turbine exhaust diffuser", in Experimental Thermal and Fluid Science*,* **vol. 22**, 2000, pp.55-70.

[10]. *S. Ubertini, U. Desider,* "Experimental performance analysis of an annular diffuser with and without struts", in Experimental Thermal and Fluid Science*,* **vol. 22**, 2000, pp.183-195.

[11]. *T.J. Viersma*, Analysis, synthesis and design of hydraulic servosystems and pipelines, Delft University of Technology, Faculty of Mechanical Engineering, Holland, 1990.

[12]. *I. E. Idelcik,* Îndrumător pentru calculul rezistenţelor hidraulice, Editura Tehnică, Bucureşti, 1984.

[13]. *P. G. Kiselev,* Indreptar pentru calcule hidraulice, Editura Tehnică, Bucureşti, 1988.

[14]. *A.J. Reynolds*, Curgeri turbulente în tehnică, Editura Tehnică, Bucureşti, 1982.

[15]. *A. Dinca A.,* ş.a., Calcul numeric şi aplicaţii, Reprografia Universităţii Craiova, 1984.

[16]. *G. I. Marciuc,* Metode de analiza numerică, Editura Academiei R.S.R., Bucureşti, 1983.

[17]. *** MAPLE 11 – Maplesoft, Waterloo Maple Inc., User Guide, 2006.

[18]. *M. Ţălu, Ş. Ţălu,* Calculul căderilor de presiune în conducte hidraulice. Regim de curgere stabilizat şi nestabilizat. Teorie, aplicaţii şi programe computaţionale. Editura Universitaria, Craiova, 2006.