ENTANGLED QUANTUM STATES, QUANTUM TELEPORTATION AND QUANTUM INFORMATION

Constantin P. Cristescu[1], Marcel Popescu [2]

Stările cuantice neseparabile (încâlcite) reprezintă elemente de importanţă fundamentală în informatica cuantică. Acest articol prezintă esenţa stărilor neseparabile, unele experimente care au demonstrat existenţa acestora, în principal pe baza dezbaterii de peste 70 de ani provocată de "paradoxul Einstein, Podolsky, Rosen (EPR)", precum şi principalele experimente care au atenuat considerabil disputa. Se descriu cele mai importante experimente de teleportare cuantică şi se prezintă unele implicaţii ale acesteia în transmiterea informaţiei.

The paper presents the essentials concerning the quantum entangled states as a fundamental resource for quantum informatics. The treatment roughly follows the debate of over 70 years generated by the Einstein, Podolsky, Rosen paper formulating what was later called "the EPR paradox", and presents the celebrated experiments that settled the debate. Recent experiments on the application of entangled states such as quantum teleportation and its relevance for quantum communications are discussed.

Keywords: Entangled states**,** EPR paradox, Bell inequalities, quantum teleportation.

1. Introduction

Recently, scientists using quantum theory identified remarkable new opportunities for informatics and computing. A fundamental resource in these applications is quantum entanglement [1], [2]. Schrödinger was the first to describe this phenomenon, to point out its non-local consequences and give it the name still in use today. In 1935 Einstein together with Podolsky and Rosen, published a paper suggesting that the non-local character implied by the entangled quantum states should be a consequence of the fact that quantum physics is not a complete science, and it must be completed with some « hidden variables » [3]. Some 30 years later, Bell demonstrated that no local theory which includes hidden variables can explain the non-local character of entangled quantum states [4]. Very sofisticated experiments followed and their results convincingly demonstrated the existence of non-locality giving full confirmation to the

[1] Professor, Department of Physics 1, POLITEHNICA University of Bucharest, Romania, cpcris@physics.pub.ro
[2] PhD student, Department of Physics 1, POLITEHNICA University of Bucharest, Romania

predictions of quantum physics [5]. Experimental validation was soon followed by the presentation of proposals for application of entangled states in communications: quantum criptography [6] and quantum teleportation [7]. A series of algorithms were proposed and developed which based on the principles of quantum physics can solve much quicker and more efficient problems such as searching in data bases or factorization of very large numbers [2].

The aim of this paper is the description of entangled states, the documentation and experimental proof of their existence and the introduction of their applications to quantum information processing. The first paragraph explains the existence and the meaning of entangled states. The next two present the so called EPR paradox and the Bell inequalities. In the fourth paragraph, some of the most important experiments demonstrating the correctness of the predictions of quantum physics related to entanglement are discussed. Theoretical basis and experimental realization of quantum teleportation as well as some aspects of its use in quantum communications are presented in the fifth paragraph.

2. Explaining the existence of entangled states

Consider a system of two components: the states of one generate a Hilbert space H_I and the states of the other, a Hilbert space H_II. The possible states of the whole system exist in a Hilbert space $\mathsf{H} = \mathsf{H}_\mathrm{I} \otimes \mathsf{H}_\mathrm{II}$, where $\otimes$ denotes the tensor product of the two spaces. If $\left|e_{\mathrm{I}j}\right\rangle$, j=1,2, ... ,dim(H_I) is an orthonormal basis in H_I and $\left|e_{\mathrm{IIk}}\right\rangle$, k=1,2, ... ,dim(H_II) is an orthonormal basis in H_II, then an arbitrary state of the component systems is written

$$\left|\psi_\mathrm{I}\right\rangle = \sum_j a_j \left|e_{\mathrm{I}j}\right\rangle \quad \left|\psi_\mathrm{II}\right\rangle = \sum_k b_k \left|e_{\mathrm{II}k}\right\rangle \tag{1}$$

respectively, and the states of the complex system are written

$$\left|\psi\right\rangle = \sum_{j,k} c_{jk} \left|e_{\mathrm{I}j}\right\rangle \otimes \left|e_{\mathrm{II}k}\right\rangle . \tag{2}$$

The coefficients of the expansions are complex numbers (some can be vanishing) and they satisfy the normalization condition, which for the c_{jk} is written

$$\sum_{j,k} \left|c_{jk}\right|^2 = 1 . \tag{3}$$

At first impression, one might think that the states (2) of the whole system could be written

$$\left|\psi\right\rangle = \left|\psi_\mathrm{I}\right\rangle \otimes \left|\psi_\mathrm{II}\right\rangle . \tag{4}$$

States of this kind are called *separable states* because they can be interpreted as telling that the first component is in state $\left|\psi_\mathrm{I}\right\rangle$ and the second in state $\left|\psi_\mathrm{II}\right\rangle$, i.e.

their states are independently identifiable. At a second look, we see that states of the type described by eq. (4) can be obtained only for special values of the coefficients c_{jk}, that can be written as products of the coefficients in the expansions (1). It means that the majority of states of the complex system are *non-separable states* also known as *entangled states*. In such a state, it is impossible to associate a subsystem with a certain state, because only the state of the whole system is determined. Consequently, only the observables of the whole system can have deffinite values.

We shall illustrate with the situation of *two state systems* relevant for the context of this paper, particularly electron systems. The projection of the electron spin on a direction can have only two values $\pm\hbar/2$, corresponding to parallel or anti-parallel orientation relativ to the respective direction. When the direction is the *z* axis, we call the two possibilities *spin up* and *spin down* states and denote them $|\uparrow\rangle$ and $|\downarrow\rangle$ respectively. Consider two electrons. According to (1), their individual states are written:

$$|\psi_1\rangle = a|\uparrow\rangle + b|\downarrow\rangle, \quad |\psi_2\rangle = c|\uparrow\rangle + d|\downarrow\rangle \tag{5}$$

and, according to (2) a state of the system composed of the two electrons can be

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right) \tag{6}$$

where, for brevity reasons, the tensor product is written $|\uparrow\rangle|\downarrow\rangle$ ($\otimes$ is omitted). It is easy to show that no values of the constants *a, b, c, d,* can be found such that the relationship $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$ holds. Consequently, (6) is an entangled state. It contains no information on the individual particles, it only indicates that the two particles are in opposite states. The important property of an entangled pair is that as soon as the state of one particle is known, by the projection resulting from a measurement, the state of the other particle is known instantly, no matter the distance between the particles at the moment of the measurement.

3. The EPR paradox

In their celebrated paper of 1935 entitled „Can Quantum-Mechanical Description of Physical Reality Be Considered Complete" [3], A. Einstein, B. Podolsky and N. Rosen present a thought experiment („Gedankenexperiment") whose conclusions, according to the authors' point of view, cast serious doubt on the uncertainty principle, suggesting that it is only a consequence of the fact that the description of reality given by quantum physics is not complete. According to Einstein, any acceptable theory should satisfy the following criteria:

1. The quantities involved in a theory should be characterized by *physical reality* in the sense that any attribute of a physical system that can be predicted accurately without disturbing the system is *an element of physical reality*. Furthermore, a complete description of the physical system under study must embody all the elements of physical reality that are associated with the system.

2. The theory must be *local*: in nature there exists no such thing as action at a distance (*spooky action at a distance* as Einstein called it).

The EPR invented a thought experiment that violates these criteria. It considers two systems that interact with each other for a time. The formalism of quantum physics can determine the state of the combined system after interaction but the states of the two sub-systems cannot be determined separately. However, if a measurement of a characteristic quantity (observable) is performed on one of the sub-systems then, from the known value of the respective quantity for the whole system, the value for the second sub-system is instantaneously obtained. The measurement of one observable on the first sub-system, instantaneously determines the state of the second one – the two sub-systems being completely separated physically. The result of the measurement of an observable can only be one of the eigenvalues of the corresponding operator. In the EPR thought experiment, two noncommuting observables are considered: the position X and the momentum P. As consequence of a measurement of X on the system I the system II is left in an eigenstate of the observable X. Alternatively, as consequence of a measurement of P on the system I the system II is left in an eigenstate of the observable P. Consequently, the two non-commuting observables X and P of system II acquire definite values as result of measurements performed on system I, although between the two systems no physical connection exists during the measurement or after.

As two non-commuting observables cannot be determined simultaneously, EPR consider that this can only be explained either if the two observables do not possess physical reality simultaneously or the quantum physics description of reality is incomplete. Moreover, considering that “it is unreasonable to suppose that the physical reality in one system depends on a measurement performed on another system”, the conclusion of the paper is that some additional “elements of physical reality” must exist that should remove the uncertainty of the two observables and the possibility of existence of non-locale effects [2]. The contradiction between these conclusions and the Einstein criteria is known as the EPR paradox.

A simplified thought experiment of the same type was proposed by Bohm, using a system of two spin 1/2 particles (electrons, for understanding convenience) in a zero total spin entangled state that can be written as in eq. (6), or also as

$$|\psi\rangle = \sqrt{\frac{1}{2}} \cdot (|+\rangle|-\rangle - |-\rangle|+\rangle) . \qquad (6')$$

Here, $|+\rangle$ describes the $+\hbar/2$ spin state and $|-\rangle$ describes the $-\hbar/2$ spin state with reference to the axis of a Stern-Gerlach apparatus. As discussed in the previous paragraph, $|\psi\rangle$ is an entangled state which cannot be factored as a (tensor) product of separate states associated to individual particles [5].

Suppose that the system disintegrates in two identical particles 1 and 2. After complete separation, one measures the projection of the spin of particle 1 on a certain, arbitrary direction taken as *z* axis. If the result is $+\hbar/2$ then instantaneously, the projection of the spin of particle 2 on the same axis is known to be equal to $-\hbar/2$, because the total spin is zero. This should be true, irrespective of the distance between the two particles at the moment of measurement. With probability 1, a subsequent measurement of the spin projection of particle 2 on the *z*-axis should give the result $-\hbar/2$. This shows that the spin of a particle can be determined (known) prior to its measurement, by measuring the spin of another particle. However, according to the relativity theory no information or action can be transmitted with a speed in excess of the light speed in vacuum. As no information on the result of the measurement on the first particle can instantaneously reach the second particle, it is inferred that the value of the spin of particle 2 should be predetermined by some *hidden variables* [8]. Apparently identical systems must be characterized by different values of the hidden variables. This hypothesis is not in contradiction with quantum physics: the predictions of quantum theory could be considered as averages over these parameters, the uncertainty present in the theory being the result of an incomplete knowledge of the intimate structure of the system.

4. Bell inequalities

Consider a source generating two spin 1/2 particles (electrons, for understanding convenience, not for experimental reality) in an entangled state $|\psi\rangle$ going in opposite directions. The set-up shown in Fig.1 consists of two Stern-Gerlach apparatuses that allow the measurement of the spin of the particles along two arbitrary directions $\vec{a}, \vec{b}$.

Mathematically, the "strange" behaviour of the entangled states can be described by calculation of the correlation coefficient for measurements performed in system I and measurements performed in system II.

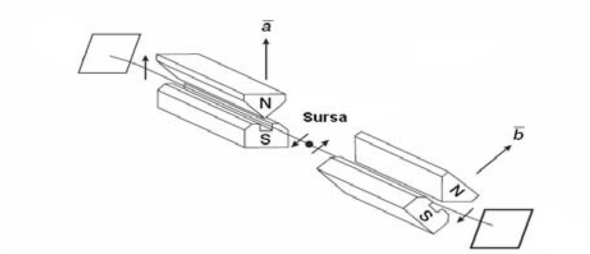

Fig. 1.Generaliztion of the Bohm experiment (after [31])

A measurement on particle 1 with the detector oriented along $\vec{a}$, can give with equal probability any of the two values $\pm\hbar/2$:

$$P_+^1 = P_-^1 = \frac{1}{2} \ . \tag{7}$$

Suppose that an effectively performed measurement gives the result $+\hbar/2$. The projection of the spin of particle 2 on the direction $\vec{a}$ instantaneously acquires the value $-\hbar/2$ that corresponds to the state $|-\rangle_{\vec{a}}$. In order to calculate the probabilities of results of measurements performed using the detector oriented along $\vec{b}$, we consider the direction of motion of the particles as *y*-axis. Using the rotation matrix for spin 1/2 particles [8], we can write:

$$\begin{aligned} |-\rangle_{\vec{a}} &= -\sin\left(\frac{\theta}{2}\right)\cdot|+\rangle_{\vec{b}} + \cos\left(\frac{\theta}{2}\right)\cdot|-\rangle_{\vec{b}} \\ |+\rangle_{\vec{a}} &= \cos\left(\frac{\theta}{2}\right)\cdot|+\rangle_{\vec{b}} + \sin\left(\frac{\theta}{2}\right)\cdot|-\rangle_{\vec{b}} \end{aligned} \ . \tag{8}$$

Using (8), we compute the following probabilities:

$$\begin{cases} P_+^2 = \sin^2\left(\dfrac{\theta}{2}\right) \\ P_-^2 = \cos^2\left(\dfrac{\theta}{2}\right) \end{cases} \Rightarrow \begin{cases} P_{++} = \dfrac{1}{2}\cdot\sin^2\left(\dfrac{\theta}{2}\right) \\ P_{+-} = \dfrac{1}{2}\cdot\cos^2\left(\dfrac{\theta}{2}\right) \end{cases} \tag{9}$$

$$\begin{cases} P_{--} = \dfrac{1}{2}\cdot\sin^2\left(\dfrac{\theta}{2}\right) \\ P_{-+} = \dfrac{1}{2}\cdot\cos^2\left(\dfrac{\theta}{2}\right) \end{cases} \tag{10}$$

In these equations θ is the angle between the two directions $\vec{a}$ and $\vec{b}$.

The correlation coefficient between the results of the two measurements is:

$$E_{QM}\left(\vec{a},\vec{b}\right) = P_{++} + P_{--} - P_{+-} - P_{-+} = -\cos(\theta) . \tag{11}$$

If $\vec{a}$ and $\vec{b}$ are in the same direction $E_{QM}\left(\vec{a},\vec{b}\right)=-1$, which corresponds to the situation where the spins of the two particles are equal in magnitude $(\hbar/2)$ but have opposite sign.

Bell computed a correlation coefficient based on a hidden variable hypothesis. Suppose there is a hidden variable λ that completely determine the values of a quantum variable obtained in an experiment. The result of a measurement may depend on $\vec{a}$ and $\vec{b}$ and on the uncontrollable parameters denoted collectively as λ. The result of the measurement on the first particle may depend on the setting $\vec{a}$ of the first instrument and on λ. Therefore we assume that there is a function which determines the result of the measurement on the first particle and a function which determines the result of the measurement on the second particle, respectively:

$$A\left(\vec{a},\lambda\right)=\pm 1;\ B\left(\vec{b},\lambda\right)=\pm 1; \tag{12}$$

and, due to the fact that the total spin is zero, the relationship

$$A\left(\vec{a},\lambda\right)=-B\left(\vec{b},\lambda\right) \tag{13}$$

has to be satisfied.

However, in accordance with Einstein's principle of locality (introduced in the second paragraph), we assume that the result of a measurement on the first particle does not depend on the setting $\vec{b}$ of the second instrument, and that the result of a measurement on the second particle does not depend on the setting $\vec{a}$ of the first instrument. Thus, we exclude functions of the form A($\vec{a}$, $\vec{b}$, λ) and B($\vec{a}$, $\vec{b}$, λ). Nothing need be assumed about the uncontrollable parameters λ. They may be associated with the particles, with the instruments, with the environment, or jointly with all of these. It makes no difference to the argument.

We study the correlation between the results of the measurements on the two particles. The uncontrollable parameters λ (hidden variables) are subject to some probability distribution $\rho(\lambda)\geq 0$ satisfying the normalization condition

$$\int \rho(\lambda)d\lambda=1\,. \tag{14}$$

For fixed settings of the instruments, the correlation function is

$$E\left(\vec{a},\vec{b}\right)=\int \rho(\lambda)\cdot A\left(\vec{a},\lambda\right)\cdot B\left(\vec{b},\lambda\right)\cdot d\lambda\,. \tag{15}$$

If $\vec{a}$ and $\vec{b}$ are in the same direction, using (13) and (14) we obtain

$$E\left(\vec{a},\vec{a}\right)=E_{QM}\left(\vec{a},\vec{a}\right)=-1\,. \tag{16}$$

Suppose now that the spin of the second particle is measured on a direction $\vec{c}$ different from $\vec{b}$. The correlation function for the new situation is given by the same eq. (15), and we can write

$$\begin{aligned} E(\vec{a},\vec{b}) - E(\vec{a},\vec{c}) &= \int \rho(\lambda)\left[A(\vec{a},\lambda)B(\vec{b},\lambda) - A(\vec{a},\lambda)B(\vec{c},\lambda)\right]d\lambda \\ &= -\int \rho(\lambda)A(\vec{a},\lambda)A(\vec{b},\lambda)\left[1 + A(\vec{b},\lambda)B(\vec{c},\lambda)\right]d\lambda \end{aligned} \tag{17}$$

where, for the final form we used eq. (14) and the relationship

$$[A(\vec{a},\lambda)]^2 = \left[A(\vec{b},\lambda)\right]^2 = 1\,. \tag{18}$$

Using again eq. (18), equation (17) becomes

$$\left|E(\vec{a},\vec{b}) - E(\vec{a},\vec{c})\right| \le \int \rho(\lambda)\left[1 + A(\vec{b},\lambda)B(\vec{c},\lambda)\right]d\lambda\,, \tag{19}$$

or, using (14) and (15),

$$\left|E(\vec{a},\vec{b}) - E(\vec{a},\vec{c})\right| \le 1 + E(\vec{b},\vec{c}). \tag{20}$$

This condition is one of Bell's inequalities that should be satisfied by a local hidden variables theory, irrespective of the directions $\vec{a}, \vec{b}$ and $\vec{c}$.

It is easy to find directions $\vec{a}, \vec{b}$ and $\vec{c}$ for which the quantum theory correlation $E_{QM}(\vec{a},\vec{b})$, violates inequality (20), and on this basis we can conclude that quantum physics is inherently a nonlocal theory. Let us consider that the angle between $\vec{a}$ and $\vec{b}$ is $\theta = \pi/3$, and the angle between $\vec{a}$ and $\vec{c}$ is $2\theta = 2\pi/3$, and the three directions are coplanar. In this case, the two sides of the inequality (20) for $E_{QM}(\vec{a},\vec{b})$ become

$$\begin{aligned} &\left|E_{QM}(\vec{a},\vec{b}) - E_{QM}(\vec{a},\vec{c})\right| = \cos(\pi/3) - \cos(2\pi/3) = 1 \\ &1 + E_{QM}(\vec{b},\vec{c}) = 1 - \cos(\pi/3) = 1/2 \end{aligned} \tag{21}$$

and consequently,

$$\left|E_{QM}(\vec{a},\vec{b}) - E_{QM}(\vec{a},\vec{c})\right| > 1 + E_{QM}(\vec{b},\vec{c}) \tag{22}$$

in violation of (20).

Another Bell inequality is obtained by choosing two settings $\vec{a}$ and $\vec{a}'$ for the first instrument and two settings $\vec{b}$ and $\vec{b}'$ for the second instrument. Then, going through the same steps as before the following condition is obtained:

$$-2 \le S \le 2 \tag{23}$$

where

$$S = E(\vec{a},\vec{b}) - E(\vec{a},\vec{b}') + E(\vec{a}',\vec{b}') + E(\vec{a}',\vec{b}). \tag{24}$$

For a hidden variables theory, inequality (23) should be satisfied irrespective of the chosen directions. Inequality (24) is usually known as the CHSH inequality because, this explicit form was first obtained by Clauser, Horne, Shimony and Holt [9]. The derivation of Bell's inequalities made no use of quantum physics.

We shall compute eq. (24) after replacing each E by the corresponding E_{QM} using (11), and considering the particular directions shown in Fig.2.

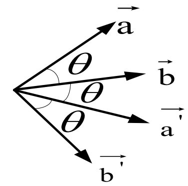

Fig.2. Detector settings for which quantum physics violates Bell inequality [5]

The result is

$$S = -\cos(\theta) + \cos(3 \cdot \theta) - \cos(\theta) - \cos(\theta) = -3 \cdot \cos(\theta) + \cos(3 \cdot \theta). \quad (25)$$

The graph of S shown in Fig.3, determines the angular ranges where

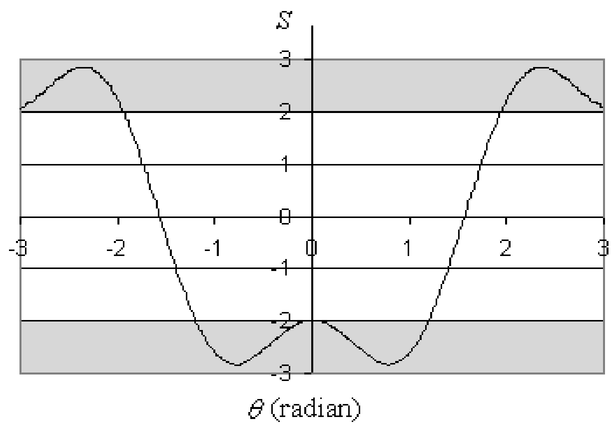

Fig.3. Graph of S(θ) for the detector settings shown in Fig.2

quantum physics violates the Bell inequality (23). These correspond to the segments of the curve that span the shaded zones. As we see, for a pure singlet (zero total spin) state the quantum theory predicts a maximal violation of this inequality i.e. $S_{min}=-2\sqrt{2}$ (for $\theta = \pi/4$) and $S_{max}=2\sqrt{2}$ (for $\theta = 3\pi/4$).

5. Experimental proofs

The most serious limitation of the experiments designed to test Bell's inequalities is the inefficiency of the detectors; many particles generated by the source go undetected. If the number of the undetected particles is very large, the magnitude of the experimental correlation function will be so small that it will automatically satisfy the Bell inequality. However, if the assumption that the detected particles are a statistically representative sample of the whole is justified, then we may compare the theoretical correlation coefficient (11) with experimental correlation given by

$$\mathrm{E}_{\exp}\left(\vec{\mathrm{a}},\vec{\mathrm{b}}\right)=\frac{N_{++}\left(\vec{\mathrm{a}},\vec{\mathrm{b}}\right)+N_{--}\left(\vec{\mathrm{a}},\vec{\mathrm{b}}\right)-N_{+-}\left(\vec{\mathrm{a}},\vec{\mathrm{b}}\right)-N_{-+}\left(\vec{\mathrm{a}},\vec{\mathrm{b}}\right)}{N}, \tag{26}$$

where $N=N_{++}\left(\vec{\mathrm{a}},\vec{\mathrm{b}}\right)+N_{--}\left(\vec{\mathrm{a}},\vec{\mathrm{b}}\right)+N_{+-}\left(\vec{\mathrm{a}},\vec{\mathrm{b}}\right)+N_{-+}\left(\vec{\mathrm{a}},\vec{\mathrm{b}}\right)$ is the total number of detected particles during the experiment. Here, N_{++} is the number of events for which both instruments recorded +1, N_{+-} is the number of events for which the first instrument recorded + 1 and the second instrument recorded − 1, etc.

The most available sources of pairs of entangled particles are photon sources. The photons are also two state systems. They have either right or left circular polarization, that can also be described by two linear polarizations with reciprocally rectangular directions.

In the first experiments the entangled photons are generated in a cascade deexcitation of certain ions. In the experiment of Freedman and Clauser [10] the source of entangled photons was a beam of calcium atoms emanating from a hot oven. The atoms were excited by strong UV light. In the deexcitation from the $4p^2\ {}^1S_0$ level to the level $4s^2\ {}^1S_0$ via level $4s4p\ {}^1P_1$, the atoms emit two correlated photons of visible light with wavelengths 551.3nm and 422.7nm. Because the initial and the final levels are both states of zero total angular momentum, and angular momentum is a conserved quantity, the emitted photon pair has zero angular momentum. This state is characterized by high symmetry and strong polarization correlation between the photons; the photons are polarization entangled. The emitted photons can have any available polarization state and, prior to a measurement no information exists on this state. However, when a polarization measurement is performed on one of the photons, the polarization of the other one is known with 100% precision. Because of the reduced probability of this cascade transition, the source has very reduced brightness and the measurements took a very long time. However, the results presented in the paper of Freedman and Clauser are highly statistically significant. The measured value of S agreed with the prediction of quantum physics exceeding the limit of 2 by more than 5 times the standard deviation of the experimental data.

The same type of source was used in the celebrated experiments performed by Aspect and his group [5, 11, 12]. He considerably improved on the brightness of the source by exciting the calcium atoms in a two-photon process using two lasers: a krypton laser at $\lambda = 406\text{nm}$ and a tunable dye laser at $\lambda = 581\text{nm}$. Unlike the previous experiments that used single channels, i.e. one polarizer in each leg, Aspect's set-up included a double channel arrangement. In order to understand the system we shall first write the ket of the entangled state when the photons are propagating along *z*-axis and the directions of polarization are two rectangular directions normal to *z* denoted *x* and *y* (the base states):

$$|\psi\rangle = \sqrt{\frac{1}{2}} \cdot (|x\rangle \otimes |x\rangle - |y\rangle \otimes |y\rangle) . \tag{27}$$

The kets $|x\rangle$ and $|y\rangle$ represent linearly polarized states.

A polarizing beam splitter selectively reflects the photons with a certain polarization (say x) in one direction and transmits the photons with the other polarization (y) in a different direction. It represents the optical equivalent of a Stern-Gerlach device. The two photon beams are collected by different detectors. One arrangement of this type is placed on each leg of the measuring system.

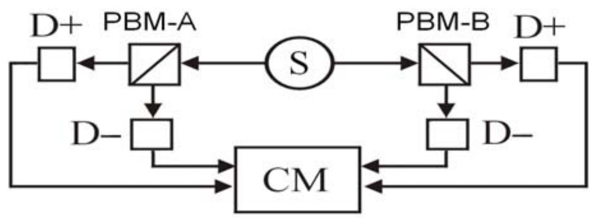

Fig.4. Sketch of the set-up for two channel detection

The set-up is outlined in Fig.4. The entangled photons generated at S, travel in opposite directions towards the polarizing beam-splitters (PBM) A and B and reach the four detectors by D_+ and D_- Emerging signals from each channel are detected and simultaneous detections counted by the coincidence monitor CM. Simultaneous detections are cassified as $++, --, +- \text{ or } -+$, and the corresponding counts accumulated. The experimental estimate for $E(\vec{a}, \vec{b})$ is then calculated according to (26) for each combination of the four directions $\vec{a}, \vec{a}', \vec{b}, \vec{b}'$, and finally S is computed according to (24). The directions $\vec{a}, \vec{a}', \vec{b}, \vec{b}'$ are chosen to correspond to maximal violation (see Fig.4). Their results violated Bell's inequality by more than 40 standard deviations.

In the last experiment of the series they addresed specifically the problem of locality. They tried to eliminate the possibility that the polarising devices in the two legs could send information to each other on the chosen setting (by some strange unknown mechanism!). This could be achieved if the measurement could be carried out in a time smaller than L/c, where L is the distance between the detectors in the two legs and c the speed of light [5]. In this experiment L=13m and L/c=43nsec. The polarizing beam splitters are replaced by acusto-optical deflectors that can rapidly change the direction of the light beam, and in front of each detector a polarizor with appropriate orientation is placed. The acusto-optical device could switch the direction of the beam towards one polarizer or the other in only 13 nanoseconds, much less than the 43nsec required by a luminal communication between the settings in the two legs. The results were again in good agreement with the predictions of quantum physics.

More recent experiments benefitted considerable improvements in the detector precision but perhaps the most important advance was in source of entangled fotons (usually called an "EPR source"). We shall only present the source based on the phenomenon of *spontaneous parametric down-conversion* in a nonlinear crystal [13, 14].

In optical spontaneous parametric down-conversion (SPDC) a pump laser beam is incident on a birefringent crystal, usually beta barium borate (BBO). The pump beam is intense enough so that nonlinear effects lead to spontaneous emission of a pair of highly correlated photons. The phase space characteristics of the photons in the two beams known as signal (1) and idler (2) are determined by conservation of energy and momentum:

$$\omega_1 + \omega_2 = \omega_p, \quad \vec{k}_1 + \vec{k}_2 = \vec{k}_p \tag{28}$$

where ω_i is the frequency and $\vec{k}_i$ the corresponding wave-number vector. The emerging photons may propagate in different directions or collinearly. The down conversion is called type I or type II depending on whether the photons in the pair have parallel or orthogonal polarization. The frequency and propagation directions are determined by the orientation of the nonlinear crystal. In contrast to type I SPDC, the spin entanglement in type II SPDC depends on the lengths of the nonlinear crystal and the direction of detection. The first demonstration of Bell's inequality violation using type II SPDC in a simple experiment was achieved in 1994 [15]. In more recent experiments [16] the beam used to pump the BBO crystal is the violet (405nm) light emitted by a continuous wave (CW) GaN laser diode with 18.6 mW optical power.

An important progress was recorded when it was demonstrated that entangled photons can be transmitted through regular optical fibers used in present day communication systems. In Europe the initial experiments were carried out in Vienna, first, with a connection across the Danube river, 600m long [17], and later with an 8km long connection across the city [18]. Recently, transmission of entangled states by optical fibers on 100km distance was reported [19]. This success opens wide possibilities for entangled state communications.

6. Quantum teleportation

By teleportation an object is supposed to disappear at one place and immediately reappear at some distant location. An object to be teleported must be fully characterized. To make a copy of that object at a distant place all that is needed is to send the complete information so that it can be used for reconstructing the object. In classical physics the required information can be obtained in principle by arbitrarily accurate measurements. However, even the simplest objects consist of particles such as electrons, atoms and molecules. Their individual properties are characterized by quantum states and, according to Heisenberg's uncertainty principle these cannot be measured with arbitrary precision. Moreover, when a measurement is performed, the state vector "collapses" to the eigenvector that corresponds to the eigenvalue observed. Accordingly, it looks like the laws of quantum physics prohibits teleportation.

It seems very surprising that in 1993, Bennett *et al.* [7] have suggested that it is possible to transfer the quantum state of a particle onto another particle, a process known as quantum teleportation. The central idea of Bennett's proposal is using entangled quantum states. Four years later, Zeilinger *et al.* [20] performed the first experimental demonstration of quantum teleportation. Using a pair of entangled photons, they were able to transfer a quantum property (the polarization state) from one photon to another. Recent experiments have demonstrated the possibility of transfer of a quantum property from an atom or ion to another. In letters published in the same issue of Nature of 2004, two groups of scientists, one from Austria [21] and the other from United States [22] reported teleportation of calcium ions and beryllium ions, respectively using entangled pairs of ions.

Apart from details on the specific generation and detection processes, the main idea is the same and is based on the quantum treatment of two state systems. So far we used various notations that were considered more illustrative for the specific situation discussed. Now we shall introduce the qubit notation, widely used in quantum informatics. Irrespective of the particular two level system involved, the two base states will be denoted

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{29}$$

such that a general state of the system will be written

$$|\psi\rangle = a|0\rangle + b|1\rangle \tag{30}$$

where the amplitudes a and b are complex numbers satisfying $|a|^2 + |b|^2 = 1$.

For a two qubits system, the basis states form the set $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ where the simplified notation $|\mathrm{i}\rangle \otimes |\mathrm{j}\rangle = |\mathrm{i}\rangle|\mathrm{j}\rangle = |\mathrm{ij}\rangle$ is used. The vectors

$$\begin{aligned} |\Phi^+\rangle &= \frac{1}{\sqrt{2}} \cdot [|00\rangle + |11\rangle], \quad |\Phi^-\rangle = \frac{1}{\sqrt{2}} \cdot [|00\rangle - |11\rangle], \\ |\Psi^+\rangle &= \frac{1}{\sqrt{2}} \cdot [|01\rangle + |10\rangle], \quad |\Psi^-\rangle = \frac{1}{\sqrt{2}} \cdot [|01\rangle - |10\rangle], \end{aligned} \tag{31}$$

form an orthonormal basis of a two-qubit system and is known as the "Bell basis".

Each member of this set is called a "Bell vector" or a "Bell state". We observe that all Bell states are entangled.

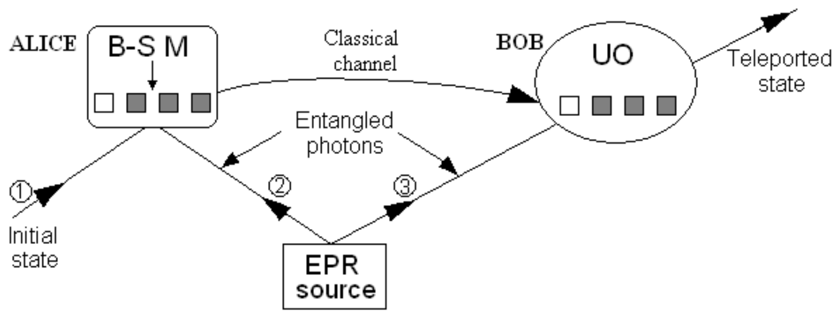

Fig.5. Sketch of the teleportation arrangement (after [20])

The essentials of quantum teleportation will be explained using [20] and [23, 24]. As usual, we consider two corresponding partners Alice and Bob. Alice has some particle in a quantum state $|\psi\rangle$ and she wants Bob, at a distant location to have a particle in that state. As discussed, the *projection postulate* prevents her from getting complete information on that state. The strategy to be adopted was formulated by Bennett *et al*. [7]. They suggested that two communication channels, a classical one and a quantum mechanical one (also known as *EPR channel)* should be involved. An essential role in their teleportation scheme is played by an ancillary pair of entangled particles which will be initially shared by Alice and Bob (via the nonclassical EPR channel). This pair is generated by a type II SPDC EPR source. As shown in Fig.5, one of the two particles (say 2) goes to Alice and the other one (3) to Bob. Now, besides her particle in state $|\psi\rangle_1 = a|0\rangle + b|1\rangle$, Alice has particle 2 entangled with Bob's particle. Alice performs a specific measurement on her two particles which projects them into one of the four Bell states (31) (known as a Bell-state measurement), particularly the $|\Psi^-\rangle$ state, further denoted $|\Psi^-\rangle_{12}$.

Indeed, before Alice's measurement, the complete state of the three particles is

$$|\Psi\rangle_{123} = \frac{a}{\sqrt{2}}\left(|0\rangle_1|0\rangle_2|1\rangle_3 - |0\rangle_1|1\rangle_2|0\rangle_3\right) + \frac{b}{\sqrt{2}}\left(|1\rangle_1|0\rangle_2|1\rangle_3 - |1\rangle_1|1\rangle_2|0\rangle_3\right) \qquad (32)$$

Expressing each product of two kets with indices 1 or 2 in terms of the Bell bases, (32) can be rewritten

$$\left|\Psi\right\rangle_{123} = \frac{1}{2}\begin{pmatrix} -\left|\Psi^-\right\rangle_{12}\left(a\left|0\right\rangle_3 + b\left|1\right\rangle_3\right) + \left|\Psi^+\right\rangle_{12}\left(-a\left|0\right\rangle_3 + b\left|1\right\rangle_3\right) + \\ \left|\Phi^-\right\rangle_{12}\left(a\left|1\right\rangle_3 + b\left|0\right\rangle_3\right) + \left|\Phi^+\right\rangle_{12}\left(a\left|1\right\rangle_3 - b\left|0\right\rangle_3\right) \end{pmatrix}. \quad (33)$$

From this equation we conclude that, regardless of the unknown state $\left|\psi\right\rangle_1$, Alice's Bell state measurement can produce any of the four Bell states with equal probability. Quantum physics predicts that once the particles 1 and 2 are projected onto one of the Bell states, particle 3 is instantaneously projected into one of the pure states in eq. (33). Particularly, if the obtained Bell state is $\left|\Psi^-\right\rangle_{12}$, particle 3 is instantaneously projected into the initial state of particle 1. This is explained as follows. Because we observe particles 1 and 2 in the state $\left|\Psi^-\right\rangle_{12}$, we know that regardless what the state of particle 1 is, particle 2 must be in the opposite state, i.e., in the state orthogonal to the state of particle 1. Suppose we initially prepared the particles 2 and 3 in the entangled state $\left|\Psi^-\right\rangle_{23}$ which means that particle 2 is also orthogonal to particle 3. This is only possible if particle 3 is in the same state as the initial state of particle 1. Therefore, the final state of particle 3 is $\left|\psi\right\rangle_3 = a\left|0\right\rangle + b\left|1\right\rangle$. If one of the other three Bell states is obtained in Alice's Bell-state measurement, Bob has to perform a unitary operation on his particle to obtain the original state $\left|\psi\right\rangle_3 = a\left|0\right\rangle + b\left|1\right\rangle$. Bob can perform this operation only after the result of the Bell measurement performed by Alice reaches him. This information is transmitted by Alice to Bob via a classical communication channel.

Accordingly, we can conclude that information transmitted by quantum teleportation cannot exceed the speed of light. However, the debate on the possibility of superluminal communications is still going on. At present, it remains at the thoretical level and very bold suggestions that allegedly could settle the problem have been put forward [25, 26]. A very recent experiment performed by the Geneva group was intended to add strong evidence in support of the non-local character of entanglement [27]. They tried to measure *the speed of spooky action at a distance* using the Earth motion in a way reminiscent of Michelson-Morley experiment. Using a space basis across the Geneva lake 18km long, they demonstrated that, in order to maintain an explanation based on spooky action at a distance one would have to assume that the spooky action is transmitted at speeds that cannot be below 10,000 times the speed of light.

It should be noted that during the Bell state measurement, particle 1 loses its identity because it becomes entangled with particle 2. Consequently, the state $\left|\psi\right\rangle_1$ is destroyed on Alice's side during teleportation. This is consistent with the so called *no cloning theorem* [28], because particle 3 is not a clone of particle1 but is really the result of teleportation.

The distance between the locations of Alice and Bob is of no consequence because, as discussed, it was experimentally demonstrated that quantum entanglement survives over very long distance.

The unknown initial state of particle 1 can be even quantum mechanically undefined before the moment of the Bell-state measurement. This is the case when particle 1 itself is one of the members of an entangled pair and therefore has no well-defined properties of its own. This ultimately leads to the phenomenon of *entanglement swapping* [29, 30]. Consider two pairs of entangled photons that are generated by two independent EPR sources. A Bell-state measurement on two of the photons - one from each of the two pairs - results in the remaining two photons becoming entangled, even though they have never interacted with each other in the past. One can indeed interpret this entanglement swapping as the teleportation of a completely undefined quantum state. A sketch of entanglement swapping is shown in Fig.6.

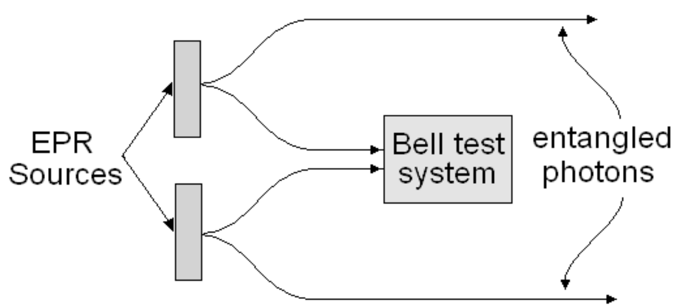

Fig.6. Entanglement swapping

Performing a Bell-state analysis on two photons is a very difficult experimental problem. The usual procedure is based on two photon interference at a standard beam splitter. We remember that the essential feature of quantum interference is that an interference pattern can be formed when there is only one particle in the apparatus at any one time. In the case of a Young type experiment, a necessary condition for quantum interference is that the experiment must be performed such that there is no way of knowing, not even in principle, which of the two slits the particle passed through on its way to the screen. The same condition has to be fulfilled to obtain one particle interference with a beamsplitter. There should be no possibility to decide on whether the particle was reflected or transmitted. Suppose that two photons, the Alice's particle and one of the entangled ancillaries are simultaneously incident on the beam splitter. There are four different possibilities: both photons are reflected, both photons are transmitted, the upper photon is reflected, the lower one transmitted, and the upper one is transmitted and the lower one is reflected. A detailed analysis of the interference results can serve to identify the Bell state [31, 32].

The previous discussion was concentrated on polarization entangled states. The *energy-time* or *time bin entanglement* plays a very important role in information transmission. Two particles exhibit energy-time entanglement when they are emitted at the same time in an energy conserving process. The energy and the time of creation of each particle are uncertain, but the sum of their energies and the difference between their emission times, which is nearly zero, are well-determined [33]. The SPDC process can be used for generation of energy-time entangled photons.

To observe energy-time entanglement a non-local interference setup is used. Each photon is transmitted to an unbalanced Mach-Zehnder interferometer, both with the same length difference between the long and the short arm, ΔL (Franson type arrangement [34]). There will be no single photon interference when ΔL is longer than the coherence length of each single photon. Alternatively, when both photons pass through the long arm or the short arm, they are indistinguishable and exhibit interference fringes. When one photon passes through the long arm and the other through the short arm they can be distinguished because one photon clearly arrives before the other at the detector. Thus, when both photons are detected simultaneously, an interference pattern is observed [35, 36].

Entangled photon pairs in the $1.5\,\mu\text{m}$ telecom band, where silica fiber has its minimum loss, are clearly advantageous for quantum communications operating over existing optical fiber networks. Polarization entanglement is unsuitable for transmission use due to the polarization mode dispersion that occurs in optical fibers, which limits the transmission distance [37, 38].

7. Conclusion

The paper presents an up to date review on some of the most difficult aspects of quantum physics, particularly the evolution of knowledge from a pure philosophical discussion on the characteristics of a reasonable scientific theory to experimental results with tremendous impact on established technologies such as computing, safe communications, and information processing.

REFERENCES

[1] *G. Chen, D. Church, B. G. Englert, C. Henkel, B. Rohwedder, M. Scully, S. Zubairy*, Quantum Computing Devices, Principles, Design and Analysis, Chapman & Hall/CRC, 2007
[2] *M. Nakahara, T. Ohmi,* Quantum Computing From Linear Algebra to Physical Realizations, Taylor & Francis Group, 2008
[3] *A. Einstein, B. Podolsky, N. Rosen*, Phys. Rev. **47**, 777 (1935)
[4] http://www.drchinese.com/David/Bell_Compact.pdf
[5] *A. Aspect, J. Dalibard, G. Roger,* Phys. Rev. Lett. **49**, 1804 (1982)
[6] *A. Eckert,* Phys. Rev. Lett. **67,** 661-663 (1991)

[7] *C. H. Bennet, G. Brassard, C. Crépeau, R. Josza, A. Peres, W. K. Wootters,* Phys. Rev. Lett. **70**, 1895 (1993)
[8] *L. Ballentine,* Quantum Mechanics: A Modern Development, World Scientific Publishing Co. Pte. Ltd., 1998
[9] *J. F. Clauser, M. A. Horne, A. Shimony, R. A. Holt*, Phys. Rev. Lett. **23,** 880-84 (1969)
[10] *S. J. Freedman, J. F. Clauser*, Phys. Rev. Lett. **28,** 938 (1972)
[11] *A. Aspect, P. Grangier, P. Roger*, Phys. Rev. Lett. **47**, 460 (1981)
[12] *A. Aspect, P. Grangier, P. Roger,* Phys. Rev. Lett. **49**, 91 (1982)
[13] *P.G. Kwiat, K. Mattle, H. Weinfurter, and A. Zeilinger*, Phys. Rev. Lett., **75**, 4337 (1995)
[14] *C. Kurtsiefer, M. Oberparleiter, and H. Weinfurter*, Phys. Rev.A, **64**, 23802 (2001)
[15] *Y. H. Shih, A. V. Sergienko, M. H. Rubin*, Phys. Rev. A **50,** 23 (1994)
[16] *M. Aspelmeyer, A. Zeilinger, H. R. Böhm, A. Fedrizzi, S. Gasparoni, M. Lindenthal, G. Molina-Terriza, A. Poppe, K. Resch, R. Ursin, P. Walther, T.D. Jennewein*, **"**Advanced Quantum Communications Experiments with Entangled Photons**",** in Quantum communications and cryptography, A. V. Sergienko, editor**,** Taylor & Francis Group, 2006
[17] *M. Aspelmeyer, H.R. Böhm, T. Gyatso, T. Jennewein, R. Kaltenbaek, M. Lindenthal, G. Molina-Terriza, A. Poppe, K. Resch, M. Taraba, R. Ursin, P.Walther, A. Zeilinger*, Science, 301, 621 (2003)
[18] *K.J. Resch, M. Lindenthal, B. Blauensteiner, H.R. Böhm, A. Fedrizzi, C. Kurtsiefer, A. Poppe, T. Schmitt-Manderbach, M. Taraba, R. Ursin, P. Walther, H. Weier, H. Weinfurter, A. Zeilinger*, *Opt. Express*, **13**, 202 (2005)
[19] *N. Gisin, S. Iblisdir, W. Tittel, H. Zbinden*, "Quantum Communications with Optical Fibers", in Quantum communications and cryptography, A. V. Sergienko, editor**,** Taylor & Francis Group, 2006
[20] *D. Bouwmeester, Jian-Wei Pan, K. Mattle, M. Eibl, H. Weinfurter, A. Zeilinger*, NATURE, **390,** 11 Dec. 1997, p.575
[21]. *M. Riebe, H. Häffner, C. H. Roos, W. Hänsel, J. Benhelm, G. P. T. Lancaster, T. W. Körber, C. Becher, F. Schmidt-Kaler, D. F. V. James, B. Blatt*, NATURE, **429,** 17 June 2004, p.734
[22] *M. D. Barrett, J. Chiaverini, J. Britton, W. M. Itano, J. D. Jost, E. Knill, C. Langer, D. Leibfried, R. Ozerl, D. J. Wineland*, NATURE, **429,** 17 June 2004, p.737
[23] *I. Marcikic, H. de Riedmatten, W. Tittel, H. Zbinden, N. Gisin*, NATURE **421**, 509 (2003)
[24] *R. Ursin, T. Jennewein, M. Aspelmeyer, R. Kaltenbaek, M. Lindenthal, P. Walther, and A. Zeilinger*, NATURE **430**, 849 (2004)
[25] *P. Mittelstaedt*, Euro. Phys Journ. B **13**, 353 (2000)
[26] *D. Greenberger*, Physica Scripta, **T76**, 57 (1998)
[27] *D. Salart, A. Baas, C. Branciard, N. Gisin, H. Zbinden*, NATURE, **454,** 861 (2008)
[28] *W. K. Wootters, W. H. Zurek*, NATURE, **299**, 802 (1982)
[29] *M. Zukowski, A. Zeilinger, M. A. Horne*, A. Ekert, Phys. Rev. Lett. **71,** 4287–4290 (1993).
[30] J-*W. Pan, D. Bouwmeester, H. Weinfurter, A. Zeilinger*, Phys. Rev. Lett. **80**, 3891 (1998)
[31] *A.D. Aczel, Entanglemet:* The Greatest Mystery in Physics, Four Walls Eight Windows, 2001
[32] Y-*H. Kim, S. P. Kulik, Y. Shih*, Phys. Rev. Lett. **86**, 1370 (2001)
[33] *S. P. Walborn, W. A. T. Nogueira, S. Padua, C. H. Monken*, Europhys. Lett. **62**, 161 (2003)
[34] *J.D. Franson,* Phys. Rev. Lett., **62**, 2205 (1989)
[35] *J. Brendel, N. Gisin, W. Tittel, H. Zbinden*, Phys. Rev. Lett. **82**, 2594 (1999)
[36] *O. Zhang, H. Takesue, S. W. Nam, C. Langrock, X. Xie, B. Baek, M. M. Fejer, Y. Yamamoto*, Optics Express, **16**, 5776 (2008)
[37] *A. Cabello, A. Rossi, G. Valone, F. De Martini*, Phys. Rev. Lett. **102,** 040401 (2009)
[38] *T. Hohjo, H. Takesue, K. Inoue*, Optics Express, **15**, 1679 (2007)