DETERMINATION OF INTERACTION FORCES OF ROLLING STOCK IN CURVES WITH CONTINUOUS AND DISCONTINUOUS VARIATION OF CURVATURE

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This study addresses the issue of vehicle - track interaction dynamic effects, in horizontal-transversal direction related to traffic, in a curve where there may be a curvature discontinuity. This situation leads to the emergence of significant transverse forces (attack shock) that may even cause the derailment of the vehicle. The paper presents a mathematical and mechanical model for the study of the "attack shock". The associated formulas have been particularized for the case of a tank wagon. The influences of the track unevennesses, the constructive factors and the impact of the vehicle on the dynamic force, are highlighted. The considered mechanical model has been validated through test data..

Keywords: arrows, lateral oscillations, elbow discontinuous.

1. Introduction

The case of a vehicle in a curve on a runway with seamless tracks is considered. Generally, due to environmental factors and stresses, the track can deform resulting in a so-called discontinuous elbow. Thereby, between the ends of two adjacent rails there is a shock angle which can generate, at the passage of the vehicle, transverse dynamic shock forces. These forces may lead to the increase of the directing force *Y* of the driving axle's attacking wheel. This force can lead to the increase of ratio *Y/Q, Q* being the load on the wheel, resulting in the increase of the permissible limit value $(Y/Q)_{\lim}$ and thus leading to derailing. The value of this limit ratio is set at 1.2 for tapered wheel sets, as is the case of the tank wagon considered below.

In order to elucidate the various factors determining the dynamic forces, the paper is limited to a particular case study, namely the movement at different speeds (between 5 and 50 kilometers per hour) of a two-axle tank wagon type Zes series, in empty state, with the wheelbase of 4 m, running on a circular curve with radius $R \geq 400$ m without coupling and without rail canting, on which there is a

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discontinuous elbow with a shock angle of δ =1°. The implementation of a discontinuous elbow with a shock angle δ =2° has been also tried out, but in this case, the curve has led to the scraping of the track, even at low speeds (20 kilometers per hour]), therefore it was not possible to take into account the obtained values [1, 2]. The two-axle tank wagon was chosen as study vehicle, because this type of wagon was lately at the origin of most railway incidents. On the other hand, it allows the accurate determination through calculation of the influences of masses, as well as of the pole of inertia (accelerations) which is located always in the middle of the wheelbase and thus allowing an easy assessment of the transverse dynamic forces through theoretical calculation, in order to compare calculations with test data and for the extrapolation of the findings. [9].

2. Theoretical assessment of the dynamic forces on the theoretical curve

The measuring equipment and the analysis for determining dynamic performance and driving quality analysis consists of: system measuring equipment; data acquisition and analysis software experimental in frequency and time domains; analytical model with 17 degrees of freedom that shows movement of the railway vehicle.

For a detailed analysis of the dynamic interaction between the main components of the vehicle are useful information on response induced excitations wheelset to tread in the functional model will be introduced three acceleration sensors for signal acquisition axle.

The measuring system was developed on analog input channels that can be configured for CCLD transducers type (Constant Current Line Drive) or direct input voltage.

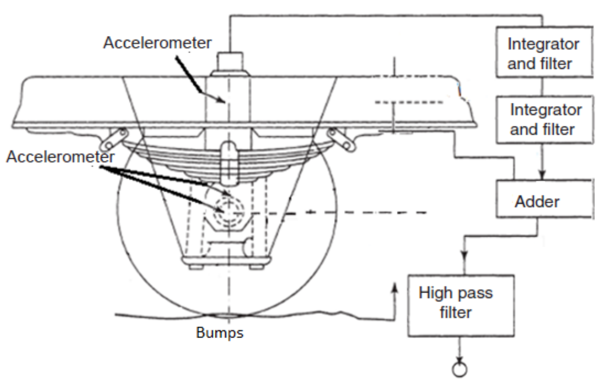

Fig. 1. The system of collecting data from accelerometers (schematic)

With the vehicle traveling at speed V in kilometers per hour over a curve of radius R in m with a rail canting h in mm, under centrifugal force F_c, the suspended part will tilt by angle φ_c and the center of mass is moved towards the interior line of the track by clearance $o = \delta + h_c \varphi_c$ (Figure 2) [9], where δ is the displacement of vehicle's suspended part on the suspender braces, and h_c is the height of the center of mass, measured in the spring eyes plane.

The displacement δ of the vehicle chassis on the frame results from the simple pendulum formula $\delta = \left(\frac{F_n}{W_n}\right) \cdot \lambda$, where λ is the effective length of the suspender braces (vertical projection), F_n is the unbalanced centrifugal force, of value $F_n = F_c - G_c(\varphi_o - \varphi_c)$, and W_n is the orthogonal component on the chassis of the weight G_c of the suspended part $W_n = G_c + F_c(\varphi_o - \varphi_c)$.

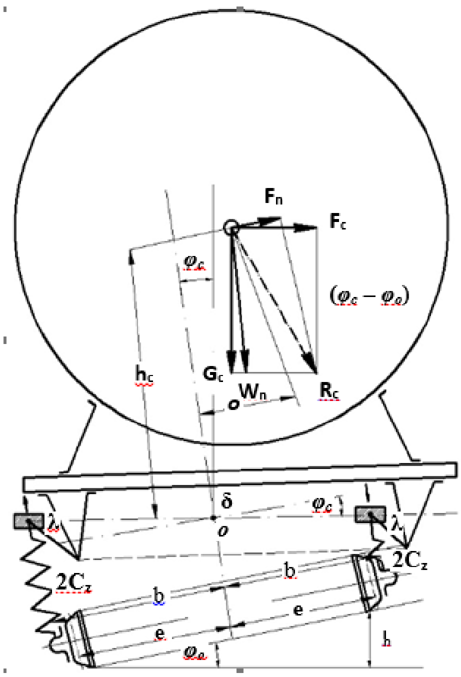

Fig. 2. The transverse forces acting on the frame of the vehicle in a curve

From the equation of force moments acting on the suspended part, with respect to the axis of rotation o, the following relation is obtained:

$$F_c(h_c + \lambda) - G_c(h_c + \lambda) - b^2 \varphi_c \Sigma C_z = 0\,, \qquad (1)$$

Denoting $A = \frac{b^2 \varphi_c \Sigma C_z}{G_c(h_c + \lambda)}$, the constant corresponding to a geometric and elastic characteristic of the vehicle, as it depends on the rigidity C_z N/mm of each spring and on the transverse gap of the spring 2b, and given that 2e is the axles' gauge (2e = 1.500 mm), equation (1) yields the tilting angle of the chassis φ_e, shown in relation:

$$\varphi_e = \frac{1}{A-1}\left(\frac{V^2}{127 \cdot R} - \frac{h}{2e}\right), \tag{2}$$

and the unbalanced centrifugal force in relation:

$$F_n = G_c \frac{A}{A-1} \cdot \left(\frac{V^2}{127 \cdot R} - \frac{h}{2e}\right), \tag{3}$$

as well as the unbalanced centrifugal acceleration in relation:

$$\gamma_n = \frac{F_n}{G_c} \cdot g = \frac{A}{A-1}\left(\frac{V^2}{3{,}6^2} \cdot \frac{1}{R} - g\frac{h}{2e}\right) \quad m \cdot s^{-2}, \tag{4}$$

In the case of the studied wagon there are available values obtained through relations:

$$C_i = \frac{10^3}{8{,}4992} = 1176.58 \quad N/mm; \quad G_c = 75200 \quad N;$$

$$h_c = 1.839 \quad m; \qquad \lambda = 0.117 \cos 70^o; \qquad A = 33.306$$

On the implemented track, the rail canting h being nil, the unbalanced centrifugal force is expressed by relation:

$$\gamma_n = \frac{A}{A-1} \cdot \left(\frac{V}{3{,}6}\right)^2 \cdot \frac{1}{R}, \tag{5}$$

and the quasi-static force H_{cs} acting between wheel and rail, in the case of vehicles [1] with steerable axles will be given by relation:

$$H_{cs} = \frac{1}{2} m_c \gamma_n, \tag{6}$$

where the mass of the box m_c = 7665.647 kg.

3. Theoretical assessment of the dynamic forces on the discontinuous curve

The discontinuous elbow occurring next to a joint, has the form shown in Figure 3.

By measuring the consecutive deflections f and f_d, at the middle of chord C, the angle δ composed by the tangents to the two circles of radius R, next to the joint can be determined, so that the deflection measured in the middle of the chord, by assimilating the circle with a parabola [2, 11], is given by relation:

$$f = \frac{\left(\frac{C}{2}\right)^2}{2R} = \frac{C^2}{8R}, \tag{7}$$

Equally, the deflection denoted f_o in Figure 3 [2] shall be expressed in value by means of relations:

$$f_o = \frac{\left(\frac{C}{2} + x\right)^2}{2R} = \frac{C^2}{8R} + \frac{C \cdot x}{2R} + \frac{x^2}{2R}, \tag{8}$$

and:

$$f_d = f_o - \frac{x^2}{2R} = f + \frac{C \cdot x}{2R}, \tag{9}$$

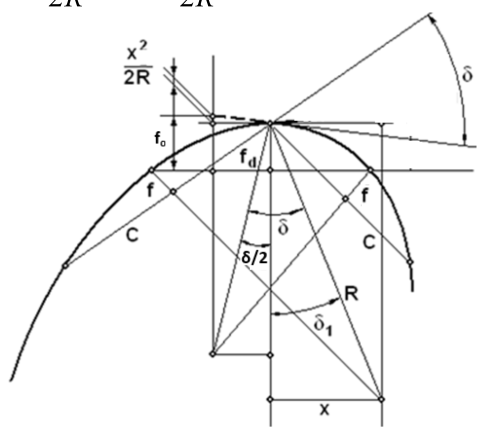

Fig. 3. Determination of angle of shock

The vehicle rolling over curves having a constant radius R will be in equilibrium under the action of the unbalanced centrifugal force F_n, the runway opposing to it, on each axle, the reactions $H_{cs} = \left(\frac{1}{2}\right) m_c \cdot \gamma_n$ [4]. The compression of the structure and axle elements and as well that of the transverse structural elements between the vehicle's chassis and axle (connecting plates) is usually of elastic nature [7].

The respective rigidities have been determined experimentally. Thereby, rigidity C_c of the track and of the wheels for main railway lines [10] can be accounted as $C_c \approx 500$ N/mm and the total stiffness of the axle guards for one axle can be assumed as $C_v = 65$ N/mm. As they are installed serially, they are added together in accordance with relation:

$$C_\Sigma = \frac{C_v \cdot C_c}{C_v + C_c} = \frac{500 \cdot 65}{565} = 575 \quad N/mm, \tag{10}$$

Under these circumstances $H_{cs} = C_\Sigma \cdot y_{cs}$ and the vehicle can be considered as a simple harmonic oscillator where $y_{cs} = \frac{H_{cs}}{C_\Sigma}$ is the static deformation of the spring [6].

If the vehicle moves forward with a constant tangential speed v, at the moment when it reaches the tip of the elbow with the outer wheel of the first axle, the rail will be attacked at a $v \cdot \sin\delta = v \cdot \delta$ speed called attack speed, having a perpendicular direction on the attacked rail [8].

The dynamic force $H_d = c_y \cdot y_d$ acting through the first axle of the vehicle during the shock process, in accordance with the type of oscillations [9], will increase quickly from zero, up to a maximum, and then it falls just as quickly, overlapping the constant value of H_{cs}.

During the shock process, not the entire mass m of the vehicle is involved, but only part of it, the so-called reduced mass. The value of the reduced mass m_r involved in the shock is determined based on the following considerations [9].

With the dynamic shock force H_d acting on the offset vehicle, compared to the center of mass o, at distances $x = a$ and $z = h_c$, the vehicle being considered as a body suspended on springs, the following dynamic equilibrium conditions need to be observed [10]. The total acceleration at the point of application H_d shall result from relation:

$$\ddot{y}_d = \ddot{y}_o + x\ddot{\psi} + z\ddot{\varphi} = \frac{H_d}{m}\left(1 + \frac{x^2}{i_z^2} + \frac{z^2}{i_x^2}\right), \tag{11}$$

of which results the expression of the reduced mass:

$$m_{rc} = \frac{m_c}{1 + \dfrac{x^2}{i_z^2} + \dfrac{z^2}{i_x^2}}, \tag{12}$$

For this case, the equation of the maximal shock force can be deduced by applying the energy conservation theorem [5]. The attack speed component $v\delta$ perpendicular to the rail will transfer to mass m_r, following this direction, the kinetic energy $(1/2).m_r.(v\delta)^2$, energy absorbed elastically by the spring between the mass and rail, with an overall stiffness C_Σ.

At the moment in time, when the compression of the spring is at its maximum value y_{dmax}, the kinetic energy becomes nil, being fully transformed into potential energy equal to $(1/2)c_y y^2_{d\max}$. In this case the energy balance will be expressed by means of relation:

$$\frac{1}{2}c_y y_{d\max}^2 = \frac{1}{2}m_r(v\sin\delta)^2, \tag{13}$$

being mentioned that this energy equation is valid in the case when the vehicle was not jerked before reaching the elbow, so it has not stored any mechanical work as effect of an additional acceleration. The maximum shock force $H_{\max} = C_\Sigma \cdot y_{d\max}$ is resulting from equation (13) and through this formula, the expression of the maximum shock force is deduced, by means of equation:

$$H_{d\max} = \left(\frac{V}{3{,}6}\cdot \sin\delta\right)\sqrt{C_{\Sigma}\cdot m_r}\ , \tag{14}$$

which reveals the sinusoidal evolution of the shock force (Figure 4).

The maximum force applied to the outer line of the track shall be $H_{\max} = H_{cs} + H_{d\max}$, and the minimum force applied th rough tensile yield to the interior line of the track shall be $H_{max} = H_{dmax} - H_{cs}$.

If the vehicle accumulates an additional acceleration γ_s, caused by the track irregularities [13], before attacking the discontinuity, this results in an additional acceleration of the vehicle, acceleration leading to the performing by mass m_r of an additional mechanical work over the distance y_{dmax}, equal to the maximum elongation of the spring.

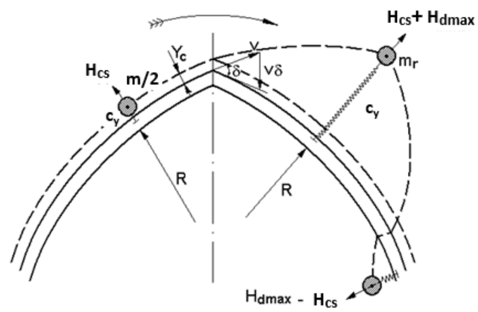

Fig. 4. Illustration of the shock phenomenon

The value of this additional mechanical work is $m_r\gamma_s y_{dmax}$. In this case, the energy balance shall be expressed by means of relation:

$$\frac{1}{2}c_y y_{d\max} = \frac{1}{2}m_r(v\cdot\sin\delta)^2 + m_r\gamma_s y_{d\max}, \tag{15}$$

which results in the expression:

$$H_{d\max} = c_y y_{d\max} + m_r\gamma_s + \sqrt{(m_r\gamma_s)^2 + c_y m_r\left(\frac{V}{3{,}6}\cdot\sin\delta\right)^2}\ , \tag{16}$$

of the maximum shock force $H_{d\max}$ [10].

It turns out that in the case when the discontinuous elbow disappears, that is for $\sin\delta = 0$, the energy accumulated by the vehicle and caused by the additional accelerations, is transferred to the track as double value, namely $H_{d\max} = m_r\gamma_s$.

In the case of the vehicle used for the experiment, the calculation data are set as follows: $x = 1.883$ m; $i_z = 1.656$ m; $z = 1.839$ m; $i_x = 0.675$ m.

The mass of the suspended part of the wagon is the one expressed by relation:

$$m_c = \frac{G_c}{g} = \frac{75{,}200}{9{,}81} = 7665.647 \quad kg, \tag{17}$$

which results immediately in the reduced mass of the box (with the accelerations pole in the middle) [3] by means of relation:

$$m_{rc} = \frac{m_c}{1+(x/i_z)^2+(z/i_x)^2} = 789.008 \quad kg, \tag{18}$$

taking into account that the weight of the axle (with the accelerations pole on the axis of the axle) is m_o = 1646.278 kg and the wagon's reduced mass is $m_r = m_{rc} + m_o$ = 2435.5257 kg.

4. Calculation of the dynamic forces and their comparison with the experimental results:

a) **On the continues curve:** Over the continuous curve, in the absence of rail canting, the unbalanced centrifuge acceleration is determined using relation:

$$\gamma_n = \frac{A}{A-1} \cdot \left(\frac{V}{3,6}\right)^2 \cdot \frac{1}{R}, \tag{19}$$

and the quasi-static force H_{cs} by relation:

$$H_{cs} = \frac{1}{2} m_c \gamma_n , \tag{20}$$

The values obtained through measurements and sequential determinations are shown in tabular form in Table. 1.

Table 1

$V\,[km/h]$	$\gamma_n\,[m\cdot s^{-2}]$	$H_{cs}\,[kN]$	$\ddot{y}_c\,[m s^{-2}]$	$\ddot{y}_o\,[m\cdot s^{-2}]$	$H_{d\max}\,[kN]$	$H_{\max}\,[kN]$
10	0.0198872	0.0762241	0.140	0.191	0.6509702	0.7271943
20	0.0795489	0.3048968	0.211	0.300	1.0210628	1.3259596
30	0.1789850	0.6860179	0.211	0.436	1.4688504	2.1548683
40	0.3181956	1.2195875	0.127	0.245	0.8267170	2.0463045
50	0.4971806	1.9053195	0.211	0.327	1.1099618	3.0152813

Admitting the existence of differences in level or even in curvature, these lead to additional unbalanced centripetal accelerations γ_s, ie, for the difference in level shown in relation:

$$\gamma_n = \frac{A}{A-1} \cdot \left(\frac{V^2}{3,6^2} \cdot \frac{1}{R} - g\frac{h}{2e}\right) , \tag{21}$$

that is, for the difference in level expressed in relation

$$\gamma_{sn} = \frac{A}{A-1} \cdot \frac{g}{2e}\left(\Delta h_1 - \Delta h_o\right) , \tag{22}$$

or, for the difference in curvature:

$$\gamma_{sn} = \frac{A}{A-1} \cdot \left(\frac{V}{3{,}6} \right)^2 \cdot \left(\frac{1}{R_1} - \frac{1}{R_o} \right), \tag{23}$$

if the distance to which these differences in level, and the curvature be incomparably greater than the vehicle wheelbase.

The level variation effects have uncertainties tasks during the ascension cranes attacking wheel, so that in calculations we consider transverse acceleration measured on the box and multiplied by the reduced mass m_{rc} = 789.008 kg.

Using at this point the transverse acceleration multiplied by the reduced mass m_o = 1646.278 kg, relation (6) shall yield, for $\delta = 0$, the expression of the shock force in its final form (23) and $H_{d\max} = (m_{rc}\ddot{y}_c + m_o\ddot{y}_o) \cdot 2$ namely the maximum dynamic force produced by the irregularities of the track.

Consequently, for the maximum force [12] on the rail will be $H_{\max} = H_{cs} + H_{d\max}$.

b) On the discontinues curve δ = 1°: Without considering track irregularities, unmatched centripetal acceleration is determined by relationship:

$$\gamma_n = \frac{A}{A-1} \cdot \left(\frac{V}{3{,}6} \right)^2 \cdot \frac{1}{R}, \tag{24}$$

and quasi-static force H_{cs} by following equation (20):

H_{dmax} maximum dynamic force which is acting on the rail horizontal direction transverse is determined in two versions:

- Considering only the isolated effect of the shock of attack produced on the discontinuous elbow with angle shock $\delta = 1^{\circ}$ by relationship:

$$H_{d\max} = \left(\frac{V}{3{,}6} \cdot \sin\delta \right) \sqrt{C_y \cdot m_r}, \tag{25}$$

The maximum force on the rail being (see Table 2): $H_{\max} = H_{cs} + H_{d\max}$,

- Taking in consideration, as at the solid curve, dynamic actions caused by the accumulation of additional acceleration caused by track irregularities after the relationship:

$$H_{d\max} = (\ddot{y}_c \cdot m_{rc} + \ddot{y}_o \cdot m_{ro}) + \sqrt{(\ddot{y}_c \cdot m_{rc} + \ddot{y}_o \cdot m_{ro})^2 + c_y m_r \left(\frac{V}{3{,}6} \sin\delta \right)^2}, \tag{26}$$

Table 2

V $[km/h]$	γ_n $[m \cdot s^{-2}]$	H_{cs} $[kN]$	$H_{d\max}$ $[kN]$	$H_{\max}$ $[kN]$
10	0.0166172	0.0636907	6.6383845	6.7020745
20	0.0664691	0.2547643	13.276782	13.531546
30	0.1495555	0.5732198	19.915180	20.488393
40	0.2658765	1.0190576	26.553579	27.572636
50	0.415432	1.5922775	33.191963	34.784240

The maximum force on rail track is also (see Table 3): $H_{\max} = H_{cs} + H_{d\max}$.
For $R = 478.7$ m; $\sin\delta = 0.017452$; $m_c = 7665.647$ kg; $c_y = 7700$ kN/m; $m_r = m_{rc} + m_{ro} = 7.89008 + 16.46278 = 2435.287$ kg we have:

Table 3

$V\ [km/h]$	$\ddot{y}_c\ [m \cdot s^{-2}]$	$\ddot{y}_o\ [m \cdot s^{-2}]$	$H_{d\max}\ [kN]$	$H_{\max}\ [kN]$
10	0.134	0.327	7.3136146	7.3773053
20	0.178	0.409	14.115468	14.370232
30	0.282	0.709	21.353319	21.926538
40	0.312	0.763	28.098320	29.117377
50	0.357	0.818	34.860210	36.452487

At the same time were carried determinations about the variation of horizontal forces measured in rail in rapport with to the wagon's speed and rail status. (see Table 4).

Table 4

	Horizontal force Σ $(R_A+R_C)^*$ [kN]				
Speed	10	20	30	40	50
δ = 0° axle I	14.2	9.8	6.5	9.8	8.7
δ = 1° axle I (A)	10 – 10.4	9.4	21	28.4	28.4
δ = 1° axle I (C)	10.6	14.6		27.2	

* Horizontally reaction from axle to rail:
A – external rail path; and C – inner rail path.

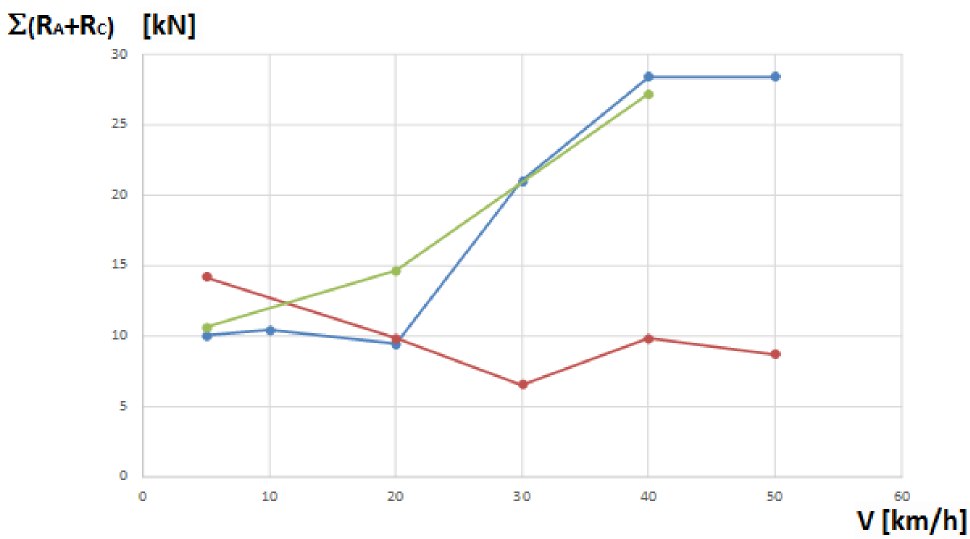

Fig. 4. Variation chart of horizontal forces measured in rail in rapport with to the wagon's speed and rail status.

5. Conclusions

Without considering the track irregularities, the unbalanced centripetal acceleration is determined bearing in mind that the maximum dynamic force H_{dmax} acting on the rail in horizontal transverse direction, is to be determined as two variants: one variant considering only the isolated effect of the attack shock exerted on the discontinuous curve, at attack angle $\delta = 1°$, respectively a second variant, taking into account, as in the case of the continuous curve, the dynamic actions caused by the accumulation of additional accelerations caused by track irregularities and by the influence of the quasi-static force H_{cs}.

By examining the numeric values obtained by successive experimental determinations, it turns out that the influence of the track irregularities on the maximum shock force in the case of this wagon is at most +2 kN. Although this influence would be computed as negative related to the movement direction of the maximum shock force H_{dmax}, it would not represent more than -1.7 kN. For example, the transverse ribbed would be flush with monolithic concrete beams to generate more resistance of the ballast.

It also follows that the variations obtained for the maximum force H_{max} applied to the track is consistent with the measured values, except for the value obtained for V = 50 km/h, where the difference of 36.45 – 28.4 kN, according to the aforementioned, cannot be explained but as consequence of the scraping of the track.

The calculation formulas applied in this paper for determining the maximum shock force can be also used to extrapolate the calculations for two-axle vehicles with higher center of gravity (in loaded state), having larger masses and larger wheelbases, with the purpose of limiting the maximum values applied to the track.

The relations in this paper are not applicable to heavy vehicles on bogies, because at these vehicles the pole of inertia no longer coincides with the center of the bogie, what significantly alter the effects of the unsprung masses on the shock force. New experimental and theoretical investigations are required for such cases.

Acknowledgement

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