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SQUEEZE OF A BINGHAM FLUID BETWEEN A MULTIPLE-DISK STACK

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This paper presents an original theoretical model for squeeze flow of a Bingham fluid between a multiple-disk stack. The volume of fluid and the speed of the squeeze process are constant. Low and high squeeze speeds were considered in accordance with Covey-Stanmore theory adapted for the constant volume fluid conditions. The case for low speed was validated experimentally using a stack of polyethylene discs filled-up in central zone with a given volume of Bingham paste.

Keywords: squeeze, Bingham fluid, multiple-disk stack, damper.

1. Introduction

The Bingham plastic model has been first used in rheodynamic lubrication to describe the behaviour of greases [1]. The Bingham fluid model is characterized by a threshold shear stress and a viscosity. If the stress tensor applied does not reach the yield stress of the fluid, the material behaves like a plastic. If the threshold limit is reached the fluid flow and behave like a Newtonian fluid.

Many studies proved the damping capacity of Bingham fluids for thin or thick layers according with the applications. If this capacity is wished to be extended to shock protection application, thicker layers must be used due to higher absorbed energy. Damping capacity can be improved if a multilayer structure is used due to multi-layer shearing surfaces. Damping solutions based on multilayer structures and Bingham fluids can be found for shock absorbers [3-4] and vibration dampers [2] applications. The squeeze flow of a Bingham fluid is used to obtain the damping effect. However, the mechanism is based on shear effects. None of them studied the damping capacity for relatively high vertical displacement. However, there is no theoretical model yet proposed for squeeze effect with a Bingham fluid with a multilayer structure.

An original model for squeeze flow of a Bingham fluid between a set of multiple-disks stacked together is proposed in this paper. The fluid flow occurs between consecutive discs. The volume of fluid is constant and the flow is studied for constant speed squeeze conditions. An original experimental analysis is also

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presented and the experimental data are compared with theoretically predicted results.

2. Analytical model

A finite volume of Bingham fluid placed in a central reservoir is axisymmetrically squeezed out between the parallel discs of the stack (Fig.1). The squeeze is produced with constant speed by a flat, rigid disc indenter of radius R_e. At the initial moment the fluid can be found only in central zone, in the interstice of thickness h_0, and inside the delimited area with radius R_0. The interstice is defined as the disc volume delimited by two consecutive solid discs. The discs are flat, impermeable, solid and non-deformable with thickness δ. During compression the fluid flows radially from central reservoir through the interstice between the discs until it reaches the boundaries of the disks. The discs are equally spaced during the squeeze process.

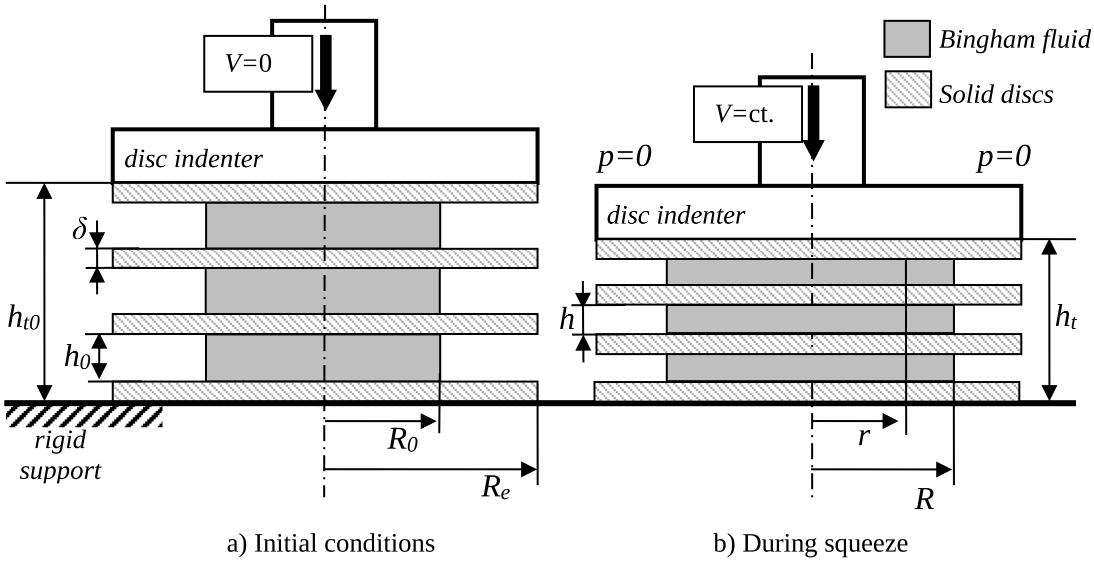

Fig.1 Geometry of the model

All the assumptions used for rheodynamic lubrication are accepted [1]. The pressure is constant across the thickness of the fluid layer. No-slip boundary conditions are used. The total thickness of the stack built of *(n+1)* discs with interstices thickness *h* is:

$$h_t = n \cdot h + (n+1)\delta \tag{1}$$

Before compression, the total thickness is:

$$h_{t0} = n \cdot h_0 + (n+1)\delta \tag{2}$$

The Covey-Stanmore model [5] describes the squeeze flow of a Bingham fluid between two parallel plates in the case of a parallel-plate plastometer. The fluid is considered incompressible and the flow is laminar. The diameter of the fluid sample subjected to compression is always less than that of the plates, and correspondingly the fluid volume is constant and the whole volume of the sample is always subjected to shearing stresses. According to Covey-Stanmore model [5] two regimes of squeeze flow are defined, function of the plasticity number $S_0 = R_0 V\eta/(h_0^2\tau_0)$: for $S_0 < 0.05$ the resistance to deformation is generated mainly by yield stress component and for $S_0 > 10$ the flow is dominated by viscous effects. First regime can be considered for low squeeze speed and the second regime is applicable for high speed squeeze.

According to the Covey-Stanmore model [5] the expression of force for $S_0 < 0.05$ is:

$$F_{<0.05} = \frac{2\pi\tau_0 R^3}{3h} + \frac{4\pi}{7h^2}\sqrt{2\tau_0\eta V R^7} \qquad (3)$$

Assuming that during squeeze, the total volume $W_0 = n\pi h_0 R_0^2$ and the volume of fluid between two consecutive discs is constant, i.e. there is no flow of the fluid outside the discs, the radial extent of the fluid at a given interstice thickness, *h*, is:

$$R = R_0\sqrt{h_0 / h} \qquad (4)$$

Using eq. (4) to replace *R*, equation (3) can be rewritten as:

$$F_{<0.05} = \frac{2\pi\tau_0 {R_0}^3 h_0^{1.5}}{3h^{2.5}} + \frac{4\pi}{7h^2}\sqrt{2\tau_0\eta V R_0^7 h_0^{3.5} / h^{7.5}} \qquad (5)$$

In the case of $S_0 > 10$ the expression of force is [5]:

$$F_{>10} = \frac{\pi\tau_0 R^3}{h} + \frac{3\pi\eta V R^4}{2h^3} \qquad (6)$$

Same as above, using eq. (4) the force from (6) is rewritten:

$$F_{>10} = \frac{\pi\tau_0 R_0^3 h_0^{1.5}}{h^{2.5}} + \frac{3\pi\eta V R_0^4 h_0^2}{2h^5} \qquad (7)$$

Further, the force can be expressed as a function of total volume W_0:
$(S_0 < 0.05)$

$$F_{<0.05} = \frac{2\tau_0 W_0 R_0\sqrt{h_0}}{3nh^{2.5}} + \frac{4W_0}{7h^2}\sqrt{2\tau_0\eta V R_0^3 h_0^{1.5} / h^{7.5}} \qquad (8)$$

$(S_0 > 10)$

$$F_{>10} = \frac{\tau_0 W_0 R_0 \sqrt{h_0}}{nh^{2.5}} + \frac{\eta W_0 V R_0^2 h_0}{2nh^5} \qquad (9)$$

The damping effect is based on the dissipative force generated by fluid viscous flow inside the multiple-disk stack. A high total volume of fluid generates a high force to damp the squeeze. Mahmoodi [4] considered the total volume of fluid to determine the amount of energy damped. The absorbed energy is proportional with the total volume of fluid. In order to underline the important damping potential of the high speed squeeze regime, a theoretic analysis was made. If the expression of force for high speed squeeze flow regime $S_0 > 10$ is divided by the expressions of force for low speed squeeze regime $S_0 < 0.05$, can be shown that the efficiency of the multiple-disk stack increases with the squeeze speed. The expressions of force (6) and (8) were rewritten dimensionless:

$(S_0 < 0.05)$

$$\overline{F}_{<0.05} = \frac{2f}{3H^{2.5}} + \frac{4f}{7}\sqrt{\frac{2S_0}{H^{7.5}}} \qquad (10)$$

$(S_0 > 10)$

$$\overline{F}_{>10} = \frac{f}{H^{2.5}} + \frac{3S_0 f}{2H^5} \qquad (11)$$

were $\overline{F} = F/\pi\tau_0 R_0^2$, $H = h/h_0$, $f = R_0/h_0$.

The ratio between these two expressions is:

$$\Re = \frac{\overline{F}_{<0.05}}{\overline{F}_{>10}} \qquad (12)$$

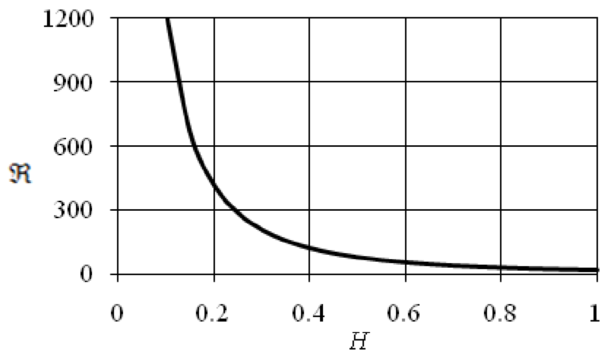

Fig.2 Variation of $\Re$ as a function of dimensionless thickness H for the S_0 limit values 0.05 and 10

The variation of $\Re$ as a function of dimensionless thickness *H* is presented in Fig.2. As can be seen, the force generated for a flow regime with high plasticity number exceeds two orders of magnitude for low thickness compared with the low plasticity number flow regime. This proves the potential of this high speed regime. Also this graph can be used to extrapolate the variation of the dissipative force using one flow regime to the other.

3. Experiments

Experimental validation of the theoretical model was made only for low plasticity number $S_0 < 0.05$. An experimental model built using four discs of polyethylene with thickness $\delta = 0.1mm$ and toothpaste as Bingham fluid, was used. An equal quantity of toothpaste ($1000\ mm^3$) was placed in the center of each solid disc with radius $R_e = 33mm$. The toothpaste was disposed uniformly using a syringe with a micrometric screw on an area with the radius $R_0 = 16.5mm$. The initial thickness of each of the three fluid layers was determined approximately as $h_0 \cong 1.17mm$. After that, the discs were stacked and an additional disc was used to complete the structure on top. The initial total thickness of the stack is $h_{t0} = 3.91\,mm$.

The paste rheological behaviour was determined using a rheometer. It was found that it can be approximated with a Bingham fluid having the yield stress $\tau_0 = 350\,Pa$ and a viscosity $\eta = 0.33\,Pa \cdot s$.

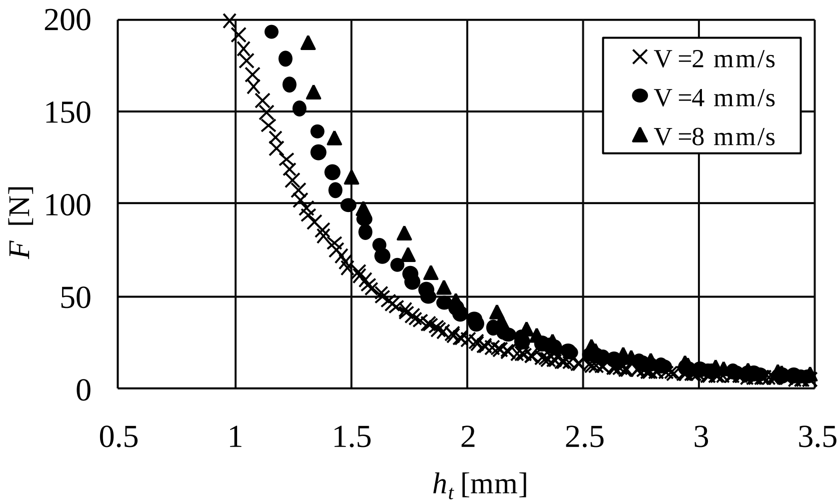

Fig.3 Experimental results obtained for disc of polyethylene and paste

The test rig used was CETR-UMT2 universal tribometer. Constant squeeze speed was used with one solid disc indenter against a rigid support. During squeeze, the force generated was measured simultaneously with vertical displacement and squeeze speed. The precision of the displacement measurement is 1 *μm*. The squeeze velocity can be varied between 0.001 -10 *mm/s*. The squeeze speeds used were 2, 4 and 8 *mm/s*. The force sensor is capable to measure up to 200 *N* with a resolution of 0.02 *N*.

In Fig.3 are presented the results for the experimental model presented above. Force increase continuous during squeeze with the decrease of stack thickness. The force reached 200 *N* for a thickness above 1 *mm*. Flow outside did not occurred before the maximum force was reach. Experimental results show a low variation of force with squeeze speed.

4. Results and validation of theoretical model

The experimental results were also used for validation of the proposed theoretical model. The plasticity number was calculated for 2 *mm/s* and 4 *mm/s* and was found S_0=0.023, and S_0=0.046, respectively. Both values are below the model limit S_0=0.05 so that the force can be calculated with equation (5). The predicted force values for squeeze with 2 *mm/s* and 4 *mm/s*, is depicted with solid and dashed line, respectively, in Fig. 4 For squeeze speed *V=8mm/s*, the plasticity number exceeds the limit of the model (S_0>0.05) and, therefore, was skipped from the comparison.

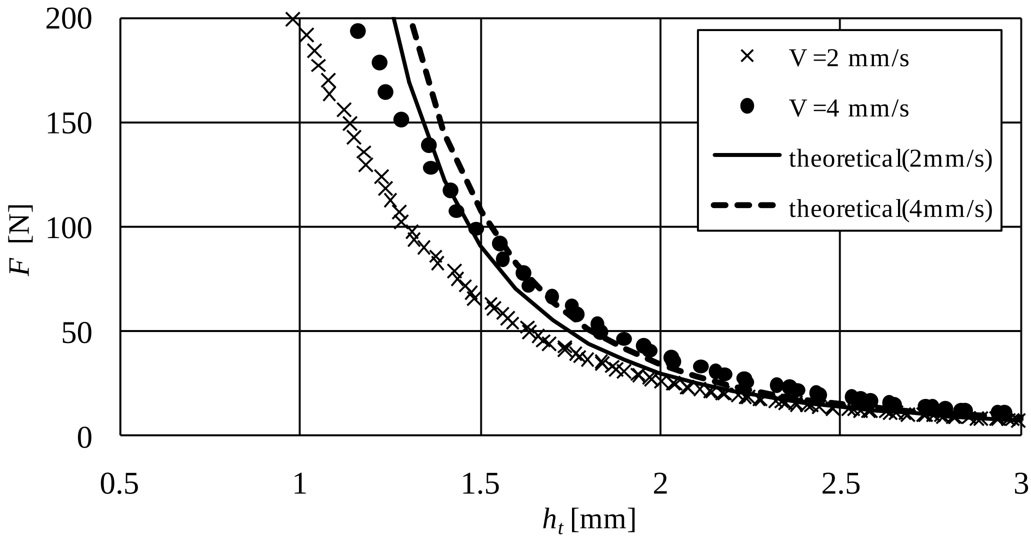

Fig.4 Comparison between experimental results and the force theoretically predicted using the model for squeeze with 2 *mm/s* and 4 *mm/s*

The comparison between experimental data and predicted results shows better correlation at high thickness (h_t>2 *mm* if *V*=2 *mm/s* and h_t>1.5 *mm* if *V*=4 *mm/s).* Increasing the compression, greater forces are generated and the differences between experiments and theory increase also. Errors about 35% were obtained at maximum load, for *V*=4 *mm/s*, but the effect is more acute for *V*=2 *mm/s*. Possible sources of errors are:

- the misalignment of the discs;
- the differences between the volumes of the paste in each layer (the accuracy of the syringe dosing volume was about 0.1ml);
- the deviation of the rheological behaviour of the paste from the ideal Bingham model.

7. Conclusions

An original theoretical model for this experiment of finite volume squeeze of a Bingham fluid between multiple-disk stack structure was developed. The structure is made of equally spaced discs and the flow occurs radially outwardly inside the interstice. The damping effect depends on the total volume of fluid. The multiple-disk stack increases the dissipated energy for a given squeeze speed.

An original experimental analysis was performed for the validation of the model. High forces were generated for low compression speed and for relative thin layers. This reveals a great potential for squeeze damping due to high resistance force. The comparison between experimental and theoretical data shows good correlation.

Further experiments should be performed for other materials and fluids. This approach could be extended for other fluids with different rheological models. The influence on performances of the central reservoir volume should be also evaluated.

List of notations

Latin alphabet notations

- F force;
- $F_{<0.05}$ force for $S_0 < 0.05$;
- $F_{>10}$ force for $S_0 > 10$;
- η dynamic viscosity;
- h fluid thickness;
- h_0 initial fluid thickness;
- h_t multiple-disk stack thickness;
- h_{t0} multiple-disk stack initial thickness;
- n number of discs;
- p pressure;

Greek alphabet notations

- δ disc thickness.
- τ_0 threshold/yield stress.

Dimensionless notations

- $f = R_0 / h_0$ reservoir size factor.
- $\overline{F} = F / \pi\tau_0 R_0^2$ dimensionless force
- $H = h / h_0$ dimensionless fluid thickness
- $\Re$ force ratio.

r radial coordinate;
R external extension of fluid reservoir;
R_e external radius of the discs;
R_0 radius of the fluid reservoir;
S_0 plasticity number, $S_0 = R_0 V \eta / (h_0^2 \tau_0)$;
V squeeze speed;
W_0 total volume of fluid.

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R E F E R E N C E S

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