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KINEMATICS AND KINETOSTATICS ANALYSIS OF THE 3-DOF PARALLEL CUBE-MANIPULATOR

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Lucrarea prezentă stabileşte relaţii matriceale pentru cinematica şi cinetostatica Cub-manipulatorului cu triplă acţionare prismatică. Cele trei elemente active ale manipulatorului sunt orientate în sistemul cartezian astfel încât direcţiile de acţionare să fie ortogonale două câte două. Trei picioare identice conectate la platforma mobilă sunt localizate în trei plane perpendiculare, De aceea acest tip de mecanism este denumit Cub-manipulator. Cunoscând mişcarea de translaţie a platformei, se dezvoltă mai întâi o problemă de cinematică inversă pentru a determina poziţiile, vitezele şi acceleraţiile manipulatorului. Utilizând principiul lucrului mecanic virtual, se rezolvă în continuare analiza cinetostatică a manipulatorului. În partea finală a lucrării se stabilesc relaţii matriceale şi se reprezintă grafice pentru forţele celor trei sisteme active.

Recursive matrix relations for kinematics and kinetostatics of a 3-DOF parallel Cube- manipulator having three prismatic actuators are established in this paper. The concurrent actuators are arranged according to the Cartesian coordinate system with fixed orientation, which means that the actuating directions are normal to each other. Three identical legs connecting to the moving platform are located on three planes being perpendicular to each other too. For such reason this type of mechanism is called Cube-manipulator. Knowing the translation motion of the platform, we develop first the inverse kinematics problem and determine the positions, velocities and accelerations of the manipulator. Further, the principle of virtual work is used in the kinetostatics analysis. Some matrix equations offer compact expressions and graphs for the forces of the three actuators

Key-words: kinematics, kinetostatics, dynamics, parallel manipulator, virtual work

1. Introduction

Parallel manipulators are closed-loop structures presenting very good performances in terms of accuracy, rigidity and ability to manipulate large loads. Generally, the mechanism of the manipulator has two platforms: one of them is attached to the fixed reference frame and the other one can have arbitrary motions

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in its workspace. Some movable legs, made up as serial robots, connect the moving platform to the fixed platform. Typically, a parallel mechanism is said to be *symmetrical* if it satisfies the following conditions: the number of legs is equal to the number of degrees of freedom of the moving platform, one actuator, which can be mounted at or near the fixed base, controls every limb and the location while the number of actuated joints in all the limbs are the same (Tsai [1]).

The last few years have witnessed an important development in the use of robots in the industrial words, mainly due to their flexibility. However, the mechanical architecture of the most common robots does not seem adapted to certain tasks. Other types of architectures [2] have therefore recently been studied, and are being more and more regularly used within the industrial world such as machine tools [3] and industrial robots [4].

Parallel manipulators attracted to the attention of many researches that consider them as valuable alternative design for robotic mechanisms [5], [6], [7]. As stated by a number of authors [1], conventional serial kinematical machines have already reached their dynamic performance limits, which are bounded by high stiffness of the machine components required to support sequential joints, links and actuators. Thus, while having good operating characteristics: large workspace, high flexibility and manoeuvrability, serial robots have disadvantages of low precision and low powers. Also, they are generally operated at low speed to avoid excessive vibrations and deflections.

In the past two decades, some studies have led to the identification of several mechanical architectures [8], [9] with potential applications in parallel manipulators. Most of the parallel mechanisms studied to date consist of six legs with six degrees of freedom, and are popular in the industrial applications, where the high load capability and multi-DOF are needed. The spatial parallel mechanisms with less than 6-DOF have increasingly attracted the researchers and some of them have been used in the structure design of robotic manipulators and in the development of high precision machine tools. The Hexapod machine tools, for example, are one of the successful applications.

The parallel robots are spatial mechanisms with supplementary characteristics, compared with the serial architecture manipulators such as: more rigid structure, important dynamic charge capacity, high orientation accuracy, stabile functioning as well as good control of velocity and acceleration limits. On the other hand, parallel kinematics machines offer essential advantages over their serial counterparts: lower moving masses, higher natural frequencies, simpler modular mechanical construction and possibility to locate actuators on the fixed base. Even then, these parallel mechanisms also suffer the problem of lower mobility, which limits their applications in some fields where high dexterity is needed, e.g. parallel kinematics machines [10]. However, most existing parallel manipulators have limited and complicated workspace with singularities and highly non-isotropic input-output relations [11].

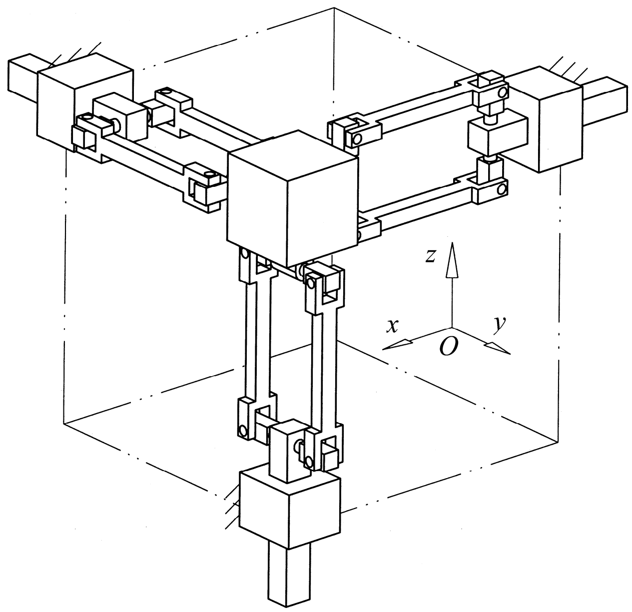


Fig. 1 Parallel Cube-manipulator

Recently, many efforts have been assigned to the kinematics and dynamics analysis of fully parallel manipulators. Theses devices can be found in many technical applications in which it is desired a high-speed orientation of a rigid body in space. Accuracy and precision in the execution of the task are essential since the robot is intended to operate on fragile objects; any error in the positioning of the tool could lead to expensive damages. Research in the field of parallel manipulators began with the most known application in the flight simulator with six degrees of freedom, which is in fact the Stewart-Gough platform (Stewart [12]; Merlet [13]; Parenti-Castelli and Di Gregorio [14]). The Star parallel manipulator (Hervé and Sparacino [15]) and the Delta parallel robot (Clavel [16]; Staicu and Carp-Ciocardia [17]; Tsai and Stamper [18]) equipped with three motors, which have a parallel setting, train on the effector in a three-degrees-of-freedom general translation motion.

The kinematics and the dynamics of parallel robots have been studied extensively during the last two decades. When good dynamic performance and precise positioning under high load are required, the dynamic model is important for their control. The analysis of parallel manipulators is usually implemented trough analytical methods in classical mechanics [19], in which projection and resolution of equations on the reference axes are written in a considerable number of cumbersome, scalar relations and the solutions are rendered by large scale computation together with time consuming computer codes. Geng [20] developed Lagrange's equations of motion under some simplifying assumptions regarding the geometry and inertia distribution of the manipulator. Dasgupta and Mruthyunjaya [21] used the Newton-Euler approach to develop closed-form dynamic equations of Stewart platform, considering all dynamic and gravity effects as well as viscous friction at joints. In recent years, several new kinematical structures have been proposed that possess higher isotropy [22], [23], [24].

The objective of this paper is to analyse the kinematics and kinetostatics of the 3-DOF parallel Cube-manipulator, which is well adapted to the applications of precision assembly machines. In design, the three actuators are arranged according to the Cartesian coordinate space, which means that the actuating directions are normal to each other and the joints connecting to the moving platform are located on three planes being perpendicular to each other too. For such reason this type of mechanism is called Cube-manipulator. The prototype of this robot [25], [26] have some technological advantages such as: symmetrical design, regular workspace shape properties with a bounded velocity amplification factor and low inertia effects.

In the present paper we focus our attention on a recursive matrix method, which is adopted to derive the kinematics model and the inverse dynamics equations of the spatial parallel Cube-manipulator, which has three translation degrees of freedom (fig. 1).

2. Inverse kinematics analysis

The mechanism input of the manipulator is made up of three actuated orthogonal prismatic joints. The output body is connected to the prismatic joints through a set of three identical kinematical chains (fig. 2).

The architecture of one of the three parallel closed chains of the Cube-manipulator consists in an active prismatic system, a passive revolute joint, an intermediate mechanism with four revolute links that connect four bars, which are parallel two by two, ending with a passive revolute link connected to the moving platform. Inside each chain, the parallelogram mechanism is used and oriented in a manner that the end-effector is restricted to *translation* movement only. The arrangement of the joints in the chains has been defined to eliminate any constraint singularity in the Cartesian workspace (Chablat and Wenger [27]; Liu et al. [28]).

We develop the inverse kinematics problem and determine the velocities and accelerations of the manipulator, supposing that the translation motion of the moving platform is known. Let us locate a fixed reference frame $Ox_0y_0z_0(T_0)$ at the intersection point of three axes of actuated prismatic joints, about which the three-degrees-of-freedom manipulator moves. It has three legs of known dimensions and masses. To simplify the graphical image of the kinematical scheme of the mechanism, in the follows we will represent the intermediate reference systems by only two axes, so as one proceeds in most of books [1], [7], [11], [13]. The z_k axis is represented, of course, for each component element T_k. We mention that the relative rotation or relative translation with $\varphi_{k,k-1}$ angle or $\lambda_{k,k-1}$ displacement of T_k body most be always pointing about or along the direction z_k.

The first element **1** of leg A is one of the three active *sliders* of the upside-down robot. It is a homogenous rod of length $A_1A_2 = l_1$ and mass m_1, moving horizontally along the z_1^A axis with a displacement λ_{10}^A.

The centre of the transmission rod $A_3A_6 = l_2$ is denoted as A_2. This link **2** is connected to the frame $x_2^A y_2^A z_2^A$ (called T_2^A) and it has a relative rotation about z_2^A axis with the angle φ_{21}^A, so that $\omega_{21}^A = \dot{\varphi}_{21}^A$ and $\varepsilon_{21}^A = \ddot{\varphi}_{21}^A$. It has the mass m_2 and the central tensor of inertia $\hat{J}_2$. Further one, two identical and parallel bars A_3A_4 (**3**) and A_6A_7 with same length l_3 rotate about the T_2^A frame with the angle $\varphi_{32}^A = \varphi_{62}^A$. They have also the same mass m_3 and the same tensor of inertia $\hat{J}_3$. The four-bar parallelogram is closed by an element T_4^A (**4**) of length l_4, which is identical with T_2^A. Its tensor of inertia is $\hat{J}_4$. This element rotates with the relative angle $\varphi_{43}^A = \varphi_{32}^A$.

The centre A_5 of the interval between the two revolute joints A_4 and A_5 connects the moving platform attached at the frame $x_5^A y_5^A z_5^A (T_5^A)$. The platform of the robot **5** can be a cube of masse m_5, central tensor of inertia $\hat{J}_5$ and side dimension l, which rotate relatively by an angle φ_{54}^A with respect to the neighbouring body T_4^A. Finally, another central and principal reference system $x_G y_G z_G$ is located at the centre G of the cubic moving platform. The angle α gives the initial orientation of the three upper arms about their guide-ways.

The mobility of the constrained mechanism is generally given by the Grübler criterion. Due to the special arrangement of the four-bar parallelograms and the three prismatic joints at points A_1, B_1, C_1, the mechanism has three translation degrees of freedom. This unique characteristic is useful in many applications, such as a $x-y-z$ positioning device.

The three concurrent displacements $\lambda_{10}^{A}, \lambda_{10}^{B}, \lambda_{10}^{C}$ of the prismatic actuators A_1, B_1, C_1 are the joint variables that give the *input vector* $\vec{\lambda}_{10} = [\lambda_{10}^{A} \;\; \lambda_{10}^{B} \;\; \lambda_{10}^{C}]^T$ of the instantaneous position of the mechanism. But, the objective of the inverse geometric problem is to find the vector $\vec{\lambda}_{10}$ and the position of the robot with the given three absolute coordinates of the center G of the platform: x_0^G, y_0^G, z_0^G.

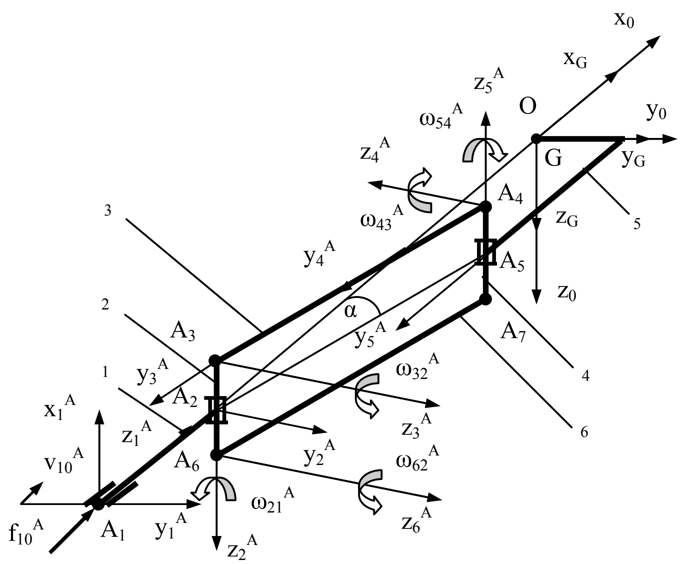


Fig. 2 Kinematical scheme of leg A of the upside-down mechanism

Pursuing the three legs A, B and C, we obtains the following transformation matrices

$$\begin{aligned}
&a_{10} = a_1,\ a_{21} = a_{21}^{\varphi} a_{\alpha} a_2,\ a_{32} = a_{32}^{\varphi} a_3 \\
&a_{43} = a_{32}^{\varphi} a_4,\ a_{54} = a_{54}^{\varphi} a_{\alpha} a_2\ , a_{62} = a_{32} \\
&b_{10} = a_5,\ b_{21} = b_{21}^{\varphi} a_{\alpha} a_2,\ b_{32} = b_{32}^{\varphi} a_3 \\
&b_{43} = b_{32}^{\varphi} a_4,\ b_{54} = b_{54}^{\varphi} a_{\alpha} a_2\ , b_{62} = b_{32} \\
&c_{10} = a_6,\ c_{21} = c_{21}^{\varphi} a_{\alpha} a_2,\ c_{32} = c_{32}^{\varphi} a_3 \\
&c_{43} = c_{32}^{\varphi} a_4,\ c_{54} = c_{54}^{\varphi} a_{\alpha} a_2\ , c_{62} = c_{32}
\end{aligned} \tag{1}$$

where we denoted [29]:

$$a_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, a_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}, a_3 = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$a_4 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, a_5 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, a_6 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$a_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad a^{\varphi}_{k,k-1} = \begin{bmatrix} \cos\varphi^A_{k,k-1} & \sin\varphi^A_{k,k-1} & 0 \\ -\sin\varphi^A_{k,k-1} & \cos\varphi^A_{k,k-1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$a_{k0} = \prod_{j=1}^{k} a_{k-j+1,k-j} \quad (k = 1,2,...,5) \ .$$

The translation conditions for the platform are expressed by the following identities

$$\overset{\circ}{a}{}^T_{50} a_{50} = \overset{\circ}{b}{}^T_{50} b_{50} = \overset{\circ}{c}{}^T_{50} c_{50} = R = I \, , \tag{3}$$

with the notations

$$\overset{\circ}{a}_{50} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \overset{\circ}{b}_{50} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \overset{\circ}{c}_{50} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

where $R = I$ represent the diagonal identity matrix. From these relations, one obtains the following relations between angles

$$\varphi^A_{54} = \varphi^A_{21} \, , \varphi^B_{54} = \varphi^B_{21} , \varphi^C_{54} = \varphi^C_{21} \, . \tag{5}$$

For the inverse geometric analysis, the position of an end-point $P(x^P_0, y^P_0, z^P_0)$ is treated as known and the goal is to find the joint variables $\lambda^A_{10}, \lambda^B_{10}, \lambda^C_{10}$ that yield the given location of the tool. If the aim is to generate a sequence of points to move the tool along an arc, care must be taken to avoid branch switching during motion, which may cause inefficient or impossible manipulator motions. Moreover, leg singularities may occur at which the manipulator loses degrees of freedom and the joint variables become linearly dependent.

Supposing, for example, that the *rectilinear motion* of the mass center G of the platform is expressed by the following relations

$$\vec{r}^{\,G}_0 = [x^G_0 \quad y^G_0 \quad z^G_0]$$

$$x_0^G = x_0^{G*}(1-\cos\frac{2\pi}{3}t)$$
$$y_0^G = y_0^{G*}(1-\cos\frac{2\pi}{3}t) \qquad (6)$$
$$z_0^G = z_0^{G*}(1-\cos\frac{2\pi}{3}t),$$

the inputs $\lambda_{10}^A, \lambda_{10}^B, \lambda_{10}^C$ of the manipulators and the variables $\varphi_{21}^A, \varphi_{32}^A, \varphi_{21}^B, \varphi_{32}^B, \varphi_{21}^C, \varphi_{32}^C$ will be given by the following geometrical conditions

$$\vec{r}_{10}^{\,A} + \sum_{k=1}^{4} a_{k0}^T \vec{r}_{k+1,k}^{\,A} + a_{50}^T \vec{r}_5^{\,GA} =$$
$$= \vec{r}_{10}^{\,B} + \sum_{k=1}^{4} b_{k0}^T \vec{r}_{k+1,k}^{\,B} + b_{50}^T \vec{r}_5^{\,GB} = \qquad (7)$$
$$= \vec{r}_{10}^{\,C} + \sum_{k=1}^{4} c_{k0}^T \vec{r}_{k+1,k}^{\,C} + c_{50}^T \vec{r}_5^{\,GC} = \vec{r}_0^{\,G} \,,$$

where, for example, one denoted

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \vec{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \tilde{u}_3 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$\vec{r}_{10}^{\,A} = (\lambda_{10}^A - l_1 - l_3\cos\alpha - \frac{l}{2})a_{10}^T\vec{u},\ \ \vec{r}_{21}^{\,A} = l_1\vec{u}_3,\ \ \vec{r}_{32}^{\,A} = -\frac{l_2}{2}\vec{u}_3 \qquad (8)$$
$$\vec{r}_{43}^{\,A} = -l_3\vec{u}_2, \vec{r}_{54}^{\,A} = \frac{l_2}{2}\vec{u}_1,\ \ \vec{r}_5^{\,GA} = [\,l_3\sin\alpha \quad -\frac{l}{2} \quad 0\,]^T.$$

Actually, these equations means that there is only one inverse geometric solution for the manipulator

$$\sin\varphi_{32}^A = -\frac{z_0^G}{l_3}\ ,\ \sin(\varphi_{21}^A+\alpha) = \frac{y_0^G + l_3\sin\alpha}{l_3\cos\varphi_{32}^A}$$
$$\lambda_{10}^A = x_0^G + l_3\cos\alpha - l_3\cos(\varphi_{21}^A+\alpha)\cos\varphi_{32}^A$$
$$\sin\varphi_{32}^B = -\frac{x_0^G}{l_3}\ ,\ \sin(\varphi_{21}^B+\alpha) = \frac{z_0^G + l_3\sin\alpha}{l_3\cos\varphi_{32}^B} \qquad (9)$$
$$\lambda_{10}^B = y_0^G + l_3\cos\alpha - l_3\cos(\varphi_{21}^B+\alpha)\cos\varphi_{32}^B$$
$$\sin\varphi_{32}^C = -\frac{y_0^G}{l_3}\ ,\ \sin(\varphi_{21}^C+\alpha) = \frac{x_0^G + l_3\sin\alpha}{l_3\cos\varphi_{32}^C}$$
$$\lambda_{10}^C = z_0^G + l_3\cos\alpha - l_3\cos(\varphi_{21}^C+\alpha)\cos\varphi_{32}^C.$$

The motions of the component elements of each leg (for example the leg *A*) are characterized by the following skew symmetric matrices [30]:

$$\tilde{\omega}_{10}^{A} = \tilde{0}$$
$$\tilde{\omega}_{k0}^{A} = a_{k,k-1}\tilde{\omega}_{k-1,0}^{A} a_{k,k-1}^{T} + \omega_{k,k-1}^{A}\tilde{u}_3 ,\ (k = 2,...,5)\,, \qquad (10)$$

which are *associated* to the absolute angular velocities given by the recurrence relations

$$\vec{\omega}_{10}^{A} = \vec{0}$$
$$\vec{\omega}_{k0}^{A} = a_{k,k-1}\vec{\omega}_{k-1,0}^{A} + \omega_{k,k-1}^{A}\vec{u}_3 ,\ \omega_{k,k-1}^{A} = \dot{\varphi}_{k,k-1}^{A} \,. \qquad (11)$$

Following relations give the velocities $\vec{v}_{k0}^{A}$ of the joints A_k

$$\vec{v}_{10}^{A} = \dot{\lambda}_{10}^{A}\vec{u}_3 ,\ \vec{v}_{k0}^{A} = a_{k,k-1}\left\{\vec{v}_{k-1,0}^{A} + \tilde{\omega}_{k-1,0}^{A}\vec{r}_{k,k-1}^{A}\right\},. \qquad (12)$$

If the other two kinematical chains of the manipulator are pursued, analogous relations can be easily obtained. Equations of geometric constraints (3) and (7) can be differentiated with respect to time to obtain the following *matrix conditions of connectivity* [31]

$$\omega_{21}^{A}\vec{u}_i^{T} a_{20}^{T}\vec{u}_3 + \omega_{54}^{A}\vec{u}_i^{T} a_{50}^{T}\vec{u}_3 = 0$$
$$v_{10}^{A}\vec{u}_i^{T} a_{10}^{T}\vec{u}_3 + l_3\omega_{21}^{A}\vec{u}_i^{T} a_{20}^{T}\tilde{u}_3 a_{32}^{T}\vec{u}_2 + l_3\omega_{32}^{A}\vec{u}_i^{T} a_{30}^{T}\tilde{u}_3\vec{u}_2 = \vec{u}_i^{T}\dot{\vec{r}}_0^{G} ,\quad (i = 1, 2, 3)\,, \qquad (13)$$

where $\tilde{u}_1, \tilde{u}_2, \tilde{u}_3$ are skew symmetric matrices associate to three orthogonal unit vectors $\vec{u}_1, \vec{u}_2, \vec{u}_3$ .From these equations, relative velocities $v_{10}^{A}, \omega_{21}^{A}, \omega_{32}^{A}$ and $\omega_{54}^{A} = \omega_{21}^{A}$ result as functions of the translation velocity of the platform. The relations (13) give the *complete* Jacobian matrix of the manipulator. This matrix is a fundamental element for the analysis of the robot workspace and the particular configurations of singularities where the manipulator becomes uncontrollable.

Rearranging, above nine equations (9) of the Cube-manipulator can immediately written as follows

$$(x_0^{G} + l_3\cos\alpha - \lambda_{10}^{A})^2 + (y_0^{G} + l_3\sin\alpha)^2 + z_0^{G2} = l_3^2$$
$$(y_0^{G} + l_3\cos\alpha - \lambda_{10}^{B})^2 + (z_0^{G} + l_3\sin\alpha)^2 + x_0^{G2} = l_3^2 \qquad (14)$$
$$(z_0^{G} + l_3\cos\alpha - \lambda_{10}^{C})^2 + (x_0^{G} + l_3\sin\alpha)^2 + y_0^{G2} = l_3^2 ,$$

where the "zero" position $\vec{r}_0^{\,0G} = [0 \quad 0 \quad 0]^T$ corresponds to the joints variables $\vec{\lambda}_{10}^{\,0} = [0 \quad 0 \quad 0]^T$. The derivative with respect to time of conditions (14) leads to the matrix equation

$$J_1\dot{\vec{\lambda}}_{10} = J_2\dot{\vec{r}}_0^{G} \,. \qquad (15)$$

Matrices J_1 and J_2 are, respectively, the inverse and forward Jacobian of the manipulator and can be expressed as

$$J_1 = diag\,\{\alpha_1 \quad \alpha_2 \quad \alpha_3\}$$

$$J_2 = \begin{bmatrix} \alpha_1 & \beta_2 & z_0^G \\ x_0^G & \alpha_2 & \beta_3 \\ \beta_1 & y_0^G & \alpha_3 \end{bmatrix}, \tag{16}$$

with

$$\alpha_1 = x_0^G + l_3 \cos\alpha - \lambda_{10}^A, \alpha_2 = y_0^G + l_3 \cos\alpha - \lambda_{10}^B, \alpha_3 = z_0^G + l_3 \cos\alpha - \lambda_{10}^C$$
$$\beta_1 = x_0^G + l_3 \sin\alpha, \beta_2 = y_0^G + l_3 \sin\alpha, \beta_3 = z_0^G + l_3 \sin\alpha. \tag{17}$$

The three kinds of singularities of the three closed-loop kinematical chains can be determined through the analysis of two Jacobian matrices J_1 and J_2 [34].

Let us assume that the manipulator has a virtual motion determined by the velocities $v_{10a}^{Av} = 1$, $v_{10a}^{Bv} = 0$, $v_{10a}^{Cv} = 0$. The characteristic virtual velocities are expressed as functions of the position of the mechanism by the kinematical constraints equations of two independent loops $A - B$ and $B - C$ determined by the three legs:

$$\vec{u}_i^T a_{50}^T \vec{v}_{50a}^{Av} = \vec{u}_i^T b_{50}^T \vec{v}_{50a}^{Bv} = \vec{u}_i^T c_{50}^T \vec{v}_{50a}^{Cv}, \; (i = 1, 2, 3)$$
$$\omega_{54a}^{Av} = \omega_{21a}^{Av}, \omega_{54a}^{Bv} = \omega_{21a}^{Bv}, \omega_{54a}^{Cv} = \omega_{21a}^{Cv}. \tag{18}$$

Some other relations of connectivity can be obtained if one considers successively that $v_{10b}^{Bv} = 1, v_{10b}^{Cv} = 0, v_{10b}^{Av} = 0$ and $v_{10c}^{Cv} = 1, v_{10c}^{Av} = 0, v_{10c}^{Bv} = 0$.

As for the relative accelerations γ_{10}^A, ε_{21}^A, ε_{32}^A and $\varepsilon_{54}^A = \varepsilon_{21}^A$ of the manipulator, the derivatives of the relations (13) give other following *conditions of connectivity*

$$\varepsilon_{21}^A \vec{u}_i^T a_{20}^T \vec{u}_3 + \varepsilon_{54}^A \vec{u}_i^T a_{50}^T \vec{u}_3 = 0$$
$$\gamma_{10}^A \vec{u}_i^T a_{10}^T \vec{u}_3 + l_3 \varepsilon_{21}^A \vec{u}_i^T a_{20}^T \tilde{u}_3 a_{32}^T \vec{u}_2 + l_3 \varepsilon_{32}^A \vec{u}_i^T a_{30}^T \tilde{u}_3 \vec{u}_2 = \vec{u}_i^T \ddot{\vec{r}}_0^G -$$
$$- l_3 \omega_{21}^A \omega_{21}^A \vec{u}_i^T a_{20}^T \tilde{u}_3 \tilde{u}_3 a_{32}^T \vec{u}_2 - l_3 \omega_{32}^A \omega_{32}^A \vec{u}_i^T a_{30}^T \tilde{u}_3 \tilde{u}_3 \vec{u}_2 -$$
$$- 2 l_3 \omega_{21}^A \omega_{32}^A \vec{u}_i^T a_{20}^T \tilde{u}_3 a_{32}^T \tilde{u}_3 \vec{u}_2, \; (i = 1, 2, 3) \tag{19}$$

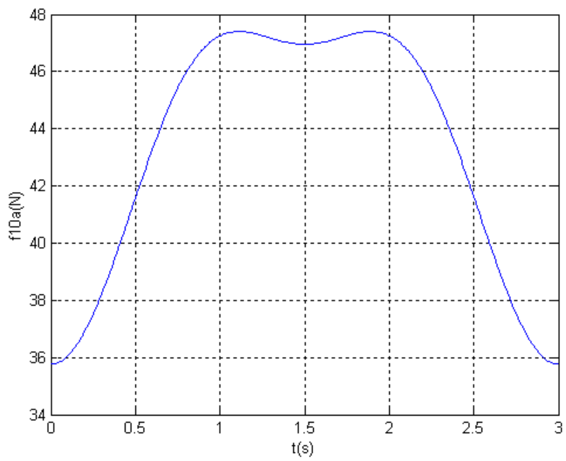
The angular accelerations $\vec{\varepsilon}_{k0}^A$ and the accelerations $\vec{\gamma}_{k0}^A$ of joints are given by some relations, obtained by deriving the relations (11) and (12):

$$\vec{\varepsilon}_{10}^A = \vec{0}$$
$$\vec{\varepsilon}_{k0}^A = a_{k,k-1} \vec{\varepsilon}_{k-1,0}^A + \varepsilon_{k,k-1}^A \vec{u}_3 + \omega_{k,k-1}^A a_{k,k-1} \tilde{\omega}_{k-1,0}^A a_{k,k-1}^T \vec{u}_3$$
$$\tilde{\omega}_{k0}^A \tilde{\omega}_{k0}^A + \tilde{\varepsilon}_{k0}^A = a_{k,k-1} \left(\tilde{\omega}_{k-1,0}^A \tilde{\omega}_{k-1,0}^A + \tilde{\varepsilon}_{k-1,0}^A \right) a_{k,k-1}^T + \omega_{k,k-1}^A \omega_{k,k-1}^A \tilde{u}_3 \tilde{u}_3 +$$
$$+ \varepsilon_{k,k-1}^A \tilde{u}_3 + 2 \omega_{k,k-1}^A a_{k,k-1} \tilde{\omega}_{k-1,0}^A a_{k,k-1}^T \tilde{u}_3 \tag{20}$$
$$\vec{\gamma}_{10}^A = \ddot{\lambda}_{10}^A \vec{u}_3$$
$$\vec{\gamma}_{k0}^A = a_{k,k-1} \left[\vec{\gamma}_{k-1,0}^A + \left(\tilde{\omega}_{k-1,0}^A \tilde{\omega}_{k-1,0}^A + \tilde{\varepsilon}_{k-1,0}^A \right) \vec{r}_{k,k-1}^A \right]$$

The relations (13), (19) represent the *inverse kinematics model* of the parallel Cube-manipulator.

3. Equations of motion

In the context of the real-time control, neglecting the frictions forces and considering the gravitational effects, the relevant objective of the dynamics is to determine the input forces, which must be exerted by the actuators in order to produce a given trajectory of the effector.



Fig, 3 Force f_{10}^{A} of first actuator

There are three methods, which can provide the same results concerning these actuating forces. The first one is using the Newton-Euler classic procedure [32], the second one applies the Lagrange's equations and multipliers formalism [20] and the third one is based on the principle of virtual work [1], [11], [29].

Within the kinematics problem, in the present paper one applies the principle of virtual work in order to establish some recursive matrix relations for the forces of the three active systems.

Three independent mechanical systems A_1, B_1, C_1 that generate three spatial forces $\vec{f}_{10}^{A} = f_{10}^{A}\vec{u}_3$, $\vec{f}_{10}^{B} = f_{10}^{B}\vec{u}_3$, $\vec{f}_{10}^{C} = f_{10}^{C}\vec{u}_3$, which are concurrent in O and oriented along the axes z_1^A, z_1^B, z_1^C, control the motion of the three sliders of the manipulator and the displacement of the moving platform.

The force of inertia

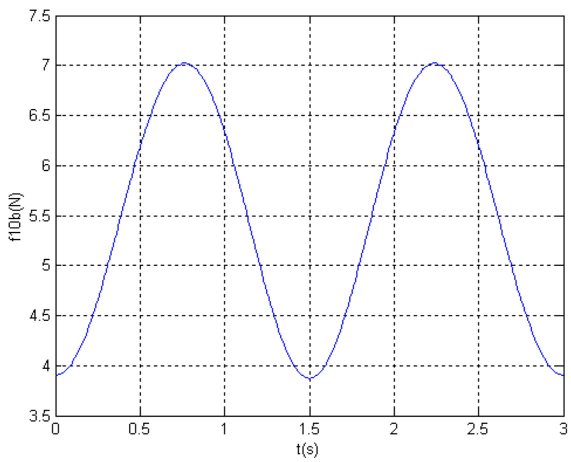
$$\vec{f}_{k0}^{\,inA} = -m_k^A\left[\vec{\gamma}_{k0}^{\,A} + \left(\tilde{\omega}_{k0}^{A}\tilde{\omega}_{k0}^{A} + \tilde{\varepsilon}_{k0}^{A}\right)\vec{r}_k^{\,CA}\right] \tag{21}$$

and the resultant moment

$$\vec{m}_{k0}^{inA} = -[m_k^A \tilde{r}_k^{CA} \vec{\gamma}_{k0}^A + \hat{J}_k^A \vec{\varepsilon}_{k0}^A + \tilde{\omega}_{k0}^A \hat{J}_k^A \vec{\omega}_{k0}^A] \qquad (22)$$

of the forces of inertia of an arbitrary rigid body T_k are determined with respect to the joint's center A_k. On the other hand, the wrench of two vectors $\vec{f}_k^*$ and $\vec{m}_k^*$ evaluates the influence of the action of the weight $m_k \vec{g}$ and of other external and internal forces applied to the same element T_k of the manipulator, for example

$$\vec{f}_k^{*A} = 9.81 m_k^A a_{k0} \vec{u}_3$$
$$\vec{m}_k^{*A} = 9.81 m_k^A \tilde{r}_k^{CA} a_{k0} \vec{u}_3 \quad (k = 1, 2, ..., 5). \qquad (23)$$



Fig, 4 Force f_{10}^B of second actuator

Knowing the position and kinematics state of each link as well as the external forces acting on the robot, in that follow one apply the principle of virtual work for the kinetostatics problem. The active forces required in a given motion of the moving platform will easily be computed using a recursive procedure.

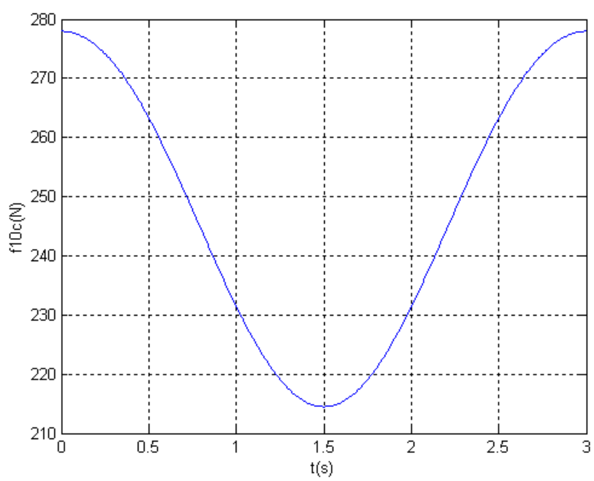
The fundamental principle of the virtual work states that a mechanism is under dynamic equilibrium if and only if the virtual work developed by all external, internal and inertia forces vanish during any general virtual displacement, which is compatible with the constraints imposed on the mechanism. Assuming that frictional forces at the joints are negligible, the virtual work produced by the forces of constraint at the joints is zero. Applying *the fundamental equations of the parallel robots dynamics* established by Staicu [33], [35] the following compact matrix relation results

$$\begin{aligned} f_{10}^{A} &= \vec{u}_3^{T}\Big[\vec{F}_1^{A} + \omega_{54a}^{Av}\vec{M}_5^{A} + \\ &+ \omega_{21a}^{Av}\vec{M}_2^{A} + \omega_{32a}^{Av}\left(\vec{M}_3^{A} + \vec{M}_4^{A} + \vec{M}_6^{A}\right) + \\ &+ \omega_{21a}^{Bv}\vec{M}_2^{B} + \omega_{32a}^{Bv}\left(\vec{M}_3^{B} + \vec{M}_4^{B} + \vec{M}_6^{B}\right) + \\ &+ \omega_{21a}^{Cv}\vec{M}_2^{C} + \omega_{32a}^{Cv}\left(\vec{M}_3^{C} + \vec{M}_4^{C} + \vec{M}_6^{C}\right)\Big], \end{aligned} \tag{24}$$

where one denoted:

$$\begin{aligned} \vec{F}_k^{A} &= \vec{F}_{k0}^{A} + a_{k+1,k}^{T}\vec{F}_{k+1} \\ \vec{M}_k^{A} &= \vec{M}_{k0}^{A} + a_{k+1,k}^{T}\vec{M}_{k+1} + \tilde{r}_{k+1,k}^{T}a_{k+1,k}^{T}\vec{F}_{k+1} \\ \vec{F}_{k0}^{A} &= -\vec{f}_{k0}^{\,inA} - \vec{f}_k^{\,*A} \\ \vec{M}_{k0}^{A} &= -\vec{m}_{k0}^{\,inA} - \vec{m}_k^{\,*A} \quad (k = 1,2,\ldots,6). \end{aligned} \tag{25}$$

The relations (23) and (24) represent the *inverse dynamics model* of the parallel Cube-manipulator.



Fig, 5 Force f_{10}^{C} of third actuator

As application let us consider a manipulator which has the following characteristics:

$$l = 0.20\,m,\ l_1 = 0.15\,m,\ l_2 = 0.08\,m,\ l_3 = 0.85\,m$$

$$l_4 = l_2,\ \alpha = \frac{\pi}{36}$$

$$m_1 = 0.35\,kg,\ m_2 = 0.2\,kg,\ m_3 = 2.5\,kg$$

$$m_4 = m_2,\ m_5 = 15\,kg,\ m_6 = m_3.$$

Considering the kinematical parameters

$$x_0^{G*} = 0.10\,m\,,\; y_0^{G*} = 0.05\,m\,,\; z_0^{G*} = -0.15\,m,\;\; \Delta t = 3\,s$$

and the analitical equations (6), in a MATLAB simulation program we obtains the graphs of forces f_{10}^{A} (fig.3), f_{10}^{B} (fig.4), f_{10}^{C} (fig.5) of the three actuators.

4. Conclusions

Most of dynamical models based on the Lagrange formalism neglect the weight of intermediate bodies and take into consideration only the active forces or moments and the wrench of applied forces on the moving platform. The number of relations given by this approach is equal to the total number of the position variables and Lagrange multipliers inclusive. The commonly known Newton-Euler method, which takes into account the free-body-diagrams of the mechanism, leads to a large number of equations with unknowns among which are also the connecting forces in the joints.

Within the inverse kinematics analysis some exact relations that give in real-time the position, velocity and acceleration of each element of the parallel robot have been established in present paper. The dynamics model takes into consideration the masses and forces of inertia introduced by all component elements of the parallel mechanism. The new approach based on the principle of virtual work can eliminate all forces of internal joints and establishes a direct determination of the time-history evolution of forces and powers required by the actuators. The recursive matrix relations (23) and (24) represent the explicit equations of the dynamics simulation and can easily be transformed in a model for the automatic command the parallel Cube-manipulator.

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