U.P.B. Sci Bull. Sci. Bull. Series D, Vol. 81, Iss.1,2019 ISSN 1454-2358

ALGORITHM FOR AXONOMETRIC DRAWING OF CYLINDER INTERSECTING A CONE*

Hu ZHIGANG[1], Chen SHAOKUN[2], Hu CHEN[3],
Ning XIN[1], Su JIANXIU[1 *]

In terms of the general theory of conics, we derive the general and standard form of equation of the ellipse, the axonometric projection of circular base for cylinder, cone and their intersecting combination, hence the directions, end points and lengths of principal diameters of the ellipse are determined. We also deliver the related equations for tangent points and directions of tangent lines of cylinder and cone axonometric drawing by which the straight-line segments of the outlines can be drawn, another conclusion is made that one of the two principal directions of the ellipse is identified with the direction of tangent line of the cylinder. The axonometric drawing for a combination of cylinder intersecting a cone can be drawn with the results of discussions above along with the parametric equations of intersection.

Key words: axonometric drawing; Quadric; principal direction; cone; cylinder.

1. Introduction

Cylinders and cones are elementary and frequently used quadrics in engineering design. The combination of cylinder intersecting a cone is always seen in geometric modeling. Pictorial drawing is such an engineering drawing only preceded by Multiview drawings that it is widely used in engineering design, manufacture, sell and use processes of industrial products[1-5]. Axonometric drawings play a main role in engineering pictorial representations, converse construction and many other fields [6-10]. Many discussions with respect to methods of axonometric drawing construction or transformation algorithms have been made before [11-13]. For constructing a cone, a cylinder or their intersecting combinations of axonometric drawing, determination of the contour is prior to many other problems resolved. From analytical geometry and descriptive geometry, we know that the contour comprises tangent lines (limiting elements) and elliptical arcs. We can use a method called quadric envelop line to form the contours[14], that is, a pencil of element curves are selected first which are

[1] School of Mechanical and Electrical Engineering, Henan Institute of Science and Technology, Xinxiang, 453003, China
[2] Department of Mechanical Engineering, Zhumadian Technician College, Zhumadian, China
[3] Xinxiang Branch of the Boiler & Pressure Vessel Safety Inspection Institute of Henan Province, Xinxiang, 453003, China
* Corresponding author: SU JIANXIU, Email: dlutsu2004@126.com.

parallel one another on a quadric surface, determine their analytical equation. Secondly, we draw an envelope line tangent to each of the element curve pencil of the quadric, at the tangent point, there is an identical tangency vector for them, it can be derived the analytical equation of the envelope line. As for cylinders and cones, each of their element curve pencil is a circle or an ellipse which takes the shape of an ellipse in axonometric projection drawings, and the envelop line is straight line. Taking this method is not appropriate; it would lead to inefficient calculation for drawing. Another choice could also be considered with the principle of descriptive geometry by which cone vertex, tangent lines (limiting elements) and tangent points on ellipse(s) of cones and cylinders must be determined, then the contours can be drawn [14]. This method is simple and computational efficiency, but it doesn't work when the axes of cylinders and cones are tilted with respect to picture planes or they are in general orientation. In this paper, an approach will be delivered based on general quadrics theory and Feng Jinhua method [15] to perform the axonometric drawing for an object of cylinder intersecting a cone. It can be determined the tangent lines, tangent points and principal directions of conic on general quadrics theory, and intersections of cylinder intersecting a cone can be computed by the parametric intersection equation given in reference [16-17].

2. Cylinder and cone axonometric transformation

In 3D space, a O'X'Y'Z' Cartesian coordinate system is set up, a combination of a cone intersecting of a cylinder is positioned in it. The cone vertex at the origin O',Z' axis is the cone axis, half cone angle denoted α, so the cone is in canonical position and orientation. The cylinder is positioned and defined by O' being its base point, Y' axis and radius r, shown in Fig.1. Here we will discuss the process of cylinder and cone axonometric transformation.

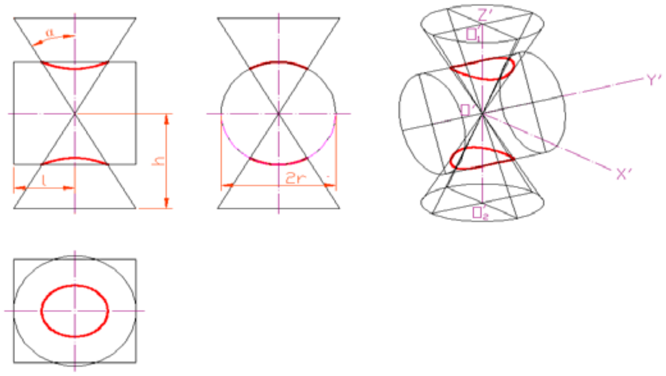

Fig.1 The coordinate system and original position of cone and cylinder

3. Cone axonometric transformation

Axonometric transformation matrix. In O'X'Y'Z' coordinate system, the following steps are to transform cone from canonical position to an axonometric projection in 2D space coordinate system OXY relative to O'X'Y'Z' with its origin at O'Y'Z' plane.

1) revolve the cone about Z' by θ.

2) revolve the cone about Y' by φ.

3) Projecting the cone to O'Y'Z' plane (X'=0), the cone axonometric projection is obtained.

4) in OXY coordinate system, translate the cone projection to the start point P_0 (x_0,y_0), then translate *dx* and *dy* along X and Y axes to the final position, respectively, see Fig. 2.

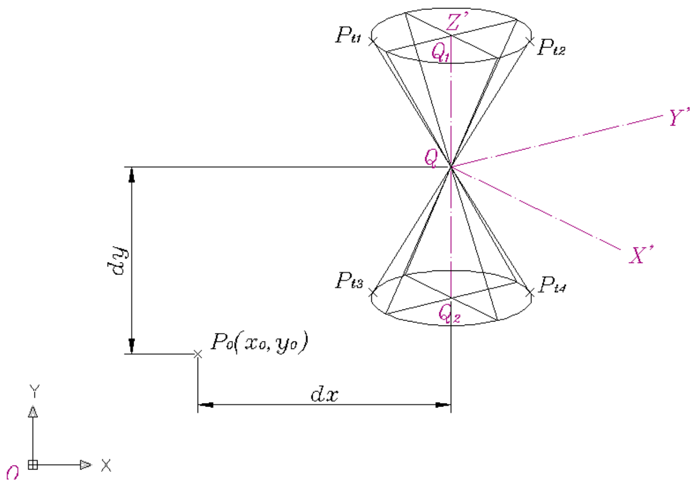

Fig.2 Start point of drawing P_0 (x_0,y_0) and displacement

The axonometric projection transformation from O'X'Y'Z' to OXY for cone is represented as T_{con}

$$T_{con} = \begin{bmatrix} \sin\theta & -\cos\theta\sin\varphi & 0 & 0 \\ \cos\theta & \sin\theta\sin\varphi & 0 & 0 \\ 0 & \cos\varphi & 0 & 0 \\ x_0 + dx & y_0 + dy & 0 & 1 \end{bmatrix} \tag{1}$$

Equations of coordinates are

$$\begin{cases} x = x'\sin\theta + y'\cos\theta + x_0 + dx \\ y = -x'\cos\theta\sin\varphi + y'\sin\theta\sin\varphi + z'\cos\varphi + y_0 + dy \end{cases} \tag{2}$$

Derivation of axonometric projection equations for cone base. In O’X’Y’Z’, equations of cone base are denoted

$$\begin{cases} x'^2 + y'^2 = h\left(\tan\alpha\right)^2 \\ z' = \pm h \end{cases} \tag{3}$$

where h is the distance from cone vertex to base plane, as shown in Fig.1, equations of cone axis are

$$\begin{cases} x' = 0 \\ y' = 0 \end{cases} \tag{4}$$

Base centers are represented as O’$_1$（0,0,h）and O’$_2$（0,0,-h）, also shown as in Fig.1. From Eq.(2) along with $z' = \pm h$, substituted into Eq. (4), it can be obtained Eq.(5).

$$\begin{cases} x' = x\sin\theta - y\dfrac{\cos\theta}{\sin\varphi} \pm h\dfrac{\cos\theta}{\tan\varphi} + (y_0 + dy)\dfrac{\cos\theta}{\sin\varphi} - (x_0 + dx)\sin\theta \\ y' = x\cos\theta + y\dfrac{\cos\theta}{\sin\varphi} \mp h\dfrac{\sin\theta}{\tan\varphi} - (y_0 + dy)\dfrac{\sin\theta}{\sin\varphi} - (x_0 + dx)\cos\theta \end{cases} \tag{5}$$

Substitute Eq.(5) into Eq.(3) and make an arrangement, the general circular base equation is obtained in OXY, see Eq.(6).

$$F(x, y) = a_{11}x^2 + a_{22}y^2 + 2a_{13}x + 2a_{23}y + a_{33} \tag{6}$$

Where

$$\begin{cases} a_{11} = 1 \\ a_{22} = \dfrac{1}{\left(\sin\varphi\right)^2} \\ a_{13} = -(x_0 + dx) \\ a_{23} = \dfrac{y_0 + dy \pm h\cos\varphi}{\left(\sin\varphi\right)^2} \\ a_{33} = \left(x_0 + dx\right)^2 + \dfrac{\left(y_0 + dy \pm h\cos\varphi\right)^2}{\left(\sin\varphi\right)^2} - h^2\left(\tan\alpha\right)^2 \end{cases} \tag{7}$$

Substitute Eq.(7) into Eq.(6) and make an arrangement, the equation of base axonometric projection will be taken the following form

$$\left[x-\left(x_0+dx\right)\right]^2+\frac{\left[y-\left(y0+dy\pm h\cos\varphi\right)\right]^2}{\left(\sin\varphi\right)^2}=h^2\left(\tan\alpha\right)^2 \quad (8)$$

Obviously, it's the standard form of equation for the ellipse. Substitute Eq.(5) into Eq.(4)， we derive the equation of cone axis (Z' axis) of OXY

$$x=x_0+dx \quad (9)$$

Base centers O'_1, O'_2 of the double cone in O'X'Y'Z' can be transformed to new coordinates of OXY which will be denoted $Q_1(x_0+dx,y_0+dy+h\cos\varphi)$ and $Q_2(x_0+dx,y_0+dy-h\cos\varphi)$, cone vertex O' transformed to $Q(x_0+dx,y_0+dy)$, as shown in Fig.2.

Determination of the tangent lines (limiting elements) of cone axonometric projection. In drawing cone axonometric projection, connect cone vertex Q to four tangent points P_{t1},P_{t2},P_{t3} and P_{t4} of two ellipses (bases projections of double cones), it can be obtained four tangent lines QP_{t1}, QP_{t2}, QP_{t3} and QP_{t4}, as shown in Fig.2. To accomplish it, the direction of tangent lines must be determined. In terms of the conditions of tangent line direction of quadrics [18], assuming X:Y to be the tangent line direction, along with

$$\begin{cases} F_1(x,y)=a_{11}x+a_{13} \\ F_2(x,y)=a_{22}y+a_{23} \end{cases} \quad (10)$$

Substitute every one of Eq.(7) into Eq.(6) and Eq.(10) , it can be obtained

$$F(x_0+dx,y_0+dy)=h^2\left(\frac{1}{\left(\tan\varphi\right)^2}-\left(\tan\alpha\right)^2\right) \quad (11)$$

$$\begin{cases} F_1(x_0+dx,y_0+dy)=a_{11}(x_0+dx)+a_{13}=0 \\ F_2(x_0+dx,y_0+dy)=a_{22}(y_0+dy)+a_{23}=\mp\dfrac{h\cos\varphi}{\left(\sin\varphi\right)^2} \end{cases} \quad (12)$$

Defining

$$\Phi(X,Y)=a_{11}X^2+a_{22}Y^2=X^2+\frac{Y^2}{\left(\sin\varphi\right)^2} \quad (13)$$

Following the conditions of tangency of conics [18]， it can be obtained

$$[XF_1(x_0+dx,y_0+dy)+YF_2(x_0+dx,y_0+dy)]^2-\Phi(X,Y)F(x_0+dx,y_0+dy)=0 \quad (14)$$

Substitute Eq.(11), (12) and (13) into Eq.(14), two tangent line directions are derived.

$$\frac{X}{Y} = \pm \frac{\tan\alpha}{\sqrt{(\cos\varphi)^2 - (\tan\alpha\cos\varphi)^2}} \tag{15}$$

Hence, the parametric equations of two tangent lines starting from Q are

$$\begin{cases} x = x_0 + dx + Xt \\ y = y_0 + dy + Yt \end{cases} \tag{16}$$

Where, t is the parameter. Denoting P_t（x_t,y_t）the tangent point, there is

$$\begin{cases} F(x_t, y_t) = 0 \\ XF_1(x_t, y_t) + YF_2(x_t, y_t) = 0 \end{cases} \tag{17}$$

where

$$\begin{cases} F_1(x_t, y_t) = a_{11}x_t + a_{13} = x_t - (x_0 + dx) \\ F_2(x_t, y_t) = a_{22}y_t + a_{23} = \dfrac{y_t - (y_0 + dy \pm h\cos\varphi)}{(\sin\varphi)^2} \end{cases} \tag{18}$$

Insert Eq.(18) into Eq.(17), y_t comes as

$$y_t = y_0 + dy \pm h\cos\varphi \mp \frac{\tan\alpha(\sin\varphi)^2\left[x_t - (x_0 + dx)\right]}{\sqrt{(\cos\varphi)^2 - (\tan\alpha\sin\varphi)^2}} \tag{19}$$

Substitute Eq.(19) into the first of Eq.(17), the coordinates of tangent points are obtained.

$$\begin{cases} x_t = x_0 + dx \pm h\tan\alpha\sqrt{1 - (\tan\alpha\tan\varphi)^2} \\ y_t = y_0 + dy \pm h\left(\cos\varphi - \dfrac{(\tan\alpha\sin\varphi)^2}{\cos\varphi}\right) \end{cases} \tag{20}$$

4. Cylinder axonometric transformation

Cylinder axonometric transformation matrix and base equations. Revolve the cylinder around X’ by β, translate it along X’ and Y’ axes by *dx’* and *dy’* successively to the destination, as shown in Fig.3. Then, transforming it to OXY coordinate system, the cylinder axonometric projection is drawn. Deriving the transformation matrix T_{cyl}

$$T_{cyl} = \begin{bmatrix} \sin\theta & -\cos\theta\sin\varphi & 0 & 0 \\ \cos\theta\cos\beta & \cos\beta\sin\theta\sin\varphi + \sin\beta\cos\varphi & 0 & 0 \\ -\sin\beta\cos\theta & \cos\beta\cos\varphi - \sin\varphi\sin\theta\sin\beta & 0 & 0 \\ dx'\sin\theta + x_0 + dx & dz'\cos\varphi - dx'\cos\theta\sin\varphi + y_0 + dy & 0 & 1 \end{bmatrix} \tag{21}$$

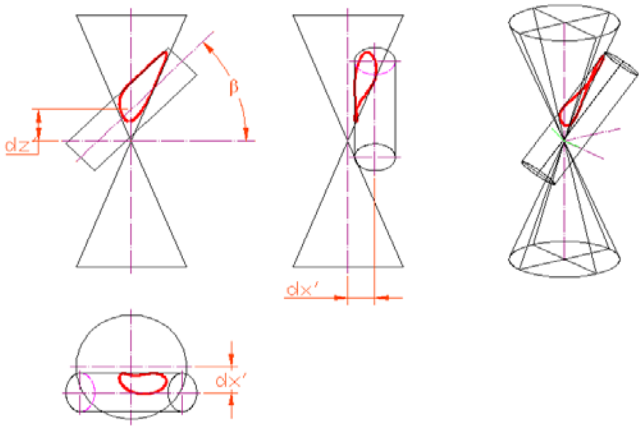

Fig.3 Cylinder position transformation

Replace all the elements of the left two columns of above matrix by *A*-*H* in sequence, Eq.(21) becomes

$$T_{cyl} = \begin{bmatrix} A & E & 0 & 0 \\ B & F & 0 & 0 \\ C & G & 0 & 0 \\ D & H & 0 & 1 \end{bmatrix} \tag{22}$$

So each point P'(*x'*,*y'*,*z'*) of the cylinder in 3D space of O'X'Y'Z' coordinate system will be transformed as P(x,y) of the axonometric projection of 2D OXY coordinate system, the new coordinates are

$$\begin{cases} x = Ax' + By' + Cz' + D \\ y = Ex' + Fy' + Gz' + H \end{cases} \tag{23}$$

In O'X'Y'Z', the equations of the cylinder circular base is

$$\begin{cases} x'^2 + z'^2 = r^2 \\ y' = \pm l \end{cases} \tag{24}$$

Where *l* is half of the cylinder height, Y' is the cylinder axis, $C'_{1,2}$ denote base centers with their coordinates (0,±*l*,0).

Substituting Eq.(24) into Eq. (23) yields the resolution *x'*,*z'* , and substitute them into Eq. (24), it can be obtained the equation F'(*x*,*y*) of the base axonometric projection. From its coefficient matrix, it can be also obtained two linear equations denoted $F'_1(x,y)$ and $F'_2(x,y)$

$$\begin{cases} F'(x,y) = b_{11}x^2 + b_{22}y^2 + 2b_{12}xy + 2b_{13}x + 2b_{23}y + b_{33} = 0 \\ F_1'(x,y) = b_{11}x + b_{12}y + b_{13} \\ F_2'(x,y) = b_{12}x + b_{22}y + b_{23} \end{cases} \tag{25}$$

Where

$$\begin{cases} b_{11} = G^2 + E^2 \\ b_{22} = A^2 + C^2 \\ b_{12} = -(CG + AE) = BF \\ b_{13} = -\left[b_{11}(D \pm Bl) + b_{12}(H \pm Fl)\right] \\ b_{23} = -\left[b_{12}(D \pm Bl) + b_{22}(H \pm Fl)\right] \\ b_{33} = b_{11}(D \pm Bl)^2 + b_{22}(H \pm Fl)^2 + 2b_{12}(D \pm Bl)(H \pm Fl) - r^2(AG - CE)^2 \end{cases} \quad (26)$$

Calculation of axonometric projection for cylinder base.

1) Determining the centers of the base of axonometric projection (ellipse)

Assume the axonometric projection of base center C (x_c, y_c), from the general theory of Quadrics, the center will satisfy the following equations [5]

$$\begin{cases} F_1'(x_c, y_c) = b_{11}x_c + b_{12}y_c + b_{13} = 0 \\ F_2'(x_c, y_c) = b_{12}x_c + b_{22}y_c + b_{23} = 0 \end{cases} \quad (27)$$

Solve the equations (27), it can be obtained the base centers written as $C_{1,2}$ ($x_{c1,2}$,$y_{c1,2}$) , as shown in Fig.4, making an arrangement results in

$$\begin{cases} x_{c1,2} = \pm l\cos\theta\cos\beta + dx'\sin\theta + dx + x_0 \\ y_{c1,2} = \pm(\sin\theta\cos\beta\sin\varphi + \sin\beta\cos\varphi) + dz'\cos\varphi - dx'\cos\theta\sin\varphi + dy + y_0 \end{cases} \quad (28)$$

Also substituting the base centers coordinates (0,$\pm l$,0) into the equations (23) , to obtain Eq.(28).

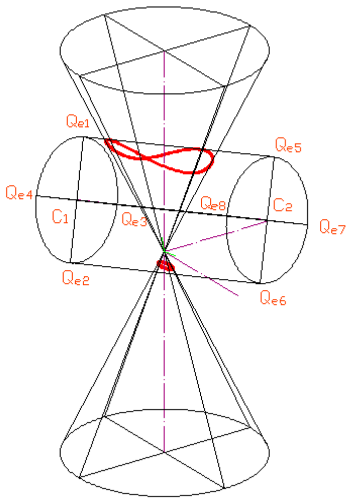

Fig.4 center points and quadrants of axonometric projection of cylinder base curves

2) Determination of ellipse principal directions (directions conjugated diameters)

In terms of the definition of conjugated principal directions of conics, assume $I_1=b_{11}+b_{22}$, $I_2=b_{11}b_{22}-b_{12}^2$, then two eigenvalues of the ellipse equation is denoted [18]

$$\lambda_{1,2}=\frac{I_1\pm\sqrt{I_1^{\,2}-4I_2}}{2} \tag{29}$$

From the equations (22) and Eq.(26), it can be obtained

$$b_{22}-b_{11}=F^2-B^2 \tag{30}$$

Assume $X_1{:}Y_1$ and $X_2{:}Y_2$ the two principal directions of the ellipse, then

$$\begin{cases}\dfrac{X_1}{Y_1}=\dfrac{b_{12}}{\lambda_1-b_{11}}=\dfrac{\lambda_1-b_{22}}{b_{12}}=\dfrac{B}{F}\\[2ex]\dfrac{X_2}{Y_2}=\dfrac{b_{12}}{\lambda_2-b_{11}}=\dfrac{\lambda_2-b_{22}}{b_{12}}=-\dfrac{F}{B}\end{cases} \tag{31}$$

3) Determination of the end points of principal diameters of ellipse

Assume $Q_{ei}(x_{ei}, y_{ei})$ the end points of principal diameters of ellipse, parametric equation of intersections of the ellipse and the arbitrary line passing through the ellipse center is

$$\Phi'(X,Y)t^2+2[XF_1'(x_c,y_c)+YF_2'(x_c,y_c)]t+F'(x_c,y_c)=0 \tag{32}$$

Where

$$\Phi'\left(X,Y\right)=b_{11}X^2+2b_{12}XY+b_{22}Y^2 \tag{33}$$

Substitute Eq. (28) into the Eq.(25), it can be obtained $F'(x_c, y_c)$, here be careful $F'(x_c, y_c)\neq 0$, and $F'(x_{c1}, y_{c1})= F'(x_{c2}, y_{c2})$, substitute this equation and Eq.(27), (33) into Eq.(32), the parameters $t_{1,2}$ and $t_{3,4}$ are obtained by which the end points of principal diameters of the ellipse are derived.

$$\begin{cases}t_{1,2}=\pm\sqrt{\dfrac{-F'(x_c,y_c)}{\Phi'(X_1,Y_1)}}\\[2ex]t_{3,4}=\pm\sqrt{\dfrac{-F'(x_c,y_c)}{\Phi'(X_2,Y_2)}}\end{cases} \tag{34}$$

So, the end points coordinates are

$$\begin{cases} x_{qei} = x_{cj} + X_k t_{2k-1+m} \\ y_{qei} = y_{cj} + Y_k t_{2k-1+m} \end{cases} \tag{35}$$

There are two bases of a cylinder or two ellipses of base axonometric projection, so two centers of the ellipses $C_{1,2}$ ($x_{c1,2}$, $y_{c1,2}$) exist, and j=1,2, each ellipse includes two principal directions X_1:Y_1 and X_2:Y_2, k=1,2; the principal direction for each center corresponds to two parameters $t_{1,2}$ or $t_{3,4}$ and m=0,1. So the point of equation (35) corresponds to eight and points, each ellipse has four end points, i=1 to 8, as shown in Fig.4.

Assume the halves of the ellipse principal diameters $R_{1,2}$

$$R_{1,2} = \sqrt{(x_{qe1,3} - x_{c1})^2 + (y_{qe1,3} - y_{c1})^2} \tag{36}$$

With the ellipses' centers, end points and halves of principal diameter, which are derived from the corresponding equations (28), (35) and (36) respectively, the bases of the axonometric projection of the cylinder or two ellipses will be drawn.

4) Determination of the tangent lines and tangent points at which the tangent lines are tangent to the ellipses for the cylinder.

From Eq. (23), deriving the cylinder axis parametric equations, with *y'* being the parameter

$$\begin{cases} x = By' + D \\ y = Fy' + H \end{cases} \tag{37}$$

The axis direction is X_a:Y_a=B:F, it's also the direction of tangent lines $Q_{e1}Q_{e5}$ *and* $Q_{e2}Q_{e6}$, as shown in Fig.4. This direction is equal to one of the directions of the ellipses, the base axonometric projections X_1:Y_1, so at this direction, the end points of the ellipses Q_{ei}(i=1,2,5,6) are the four tangent points. It can be also gotten the same result using the general theory of the conics.

5. Generation of the axonometric projection of intersections for cylinder and cone and determination of cone height

There are the size parameters *r*, *l*, *α*, *h* and position parameters *β*, *dx'*, *dz'* of cylinder and cone, and cone is selected as parameterization surface with its parameter equations, here

$$\begin{cases} x' = s \cdot \tan\alpha \cdot \cos\gamma \\ y' = s \cdot \tan\alpha \cdot \sin\gamma \\ z' = s \end{cases} \quad (38)$$

In O'X'Y'Z' coordinate system, rotate the cylinder around X' by β, then translate it along X' and Z' by *dx'* and *dz'* to the final position, respectively, its equation becomes

$$(x' - dx')^2 + (z'\cos\beta - y'\sin\beta - dz'\cos\beta)^2 = r^2 \quad (39)$$

Substitute Eq. (38) to Eq. (39), the parametric equation of the intersection line is obtained [16, 17]

$$a(\gamma)s^2 + b(\gamma)s + c = 0 \quad (40)$$

Where γ is the parametric angle of cone, $\gamma \in [0 \sim 2\pi]$, *s* is the parameter of height along Z' axis, and

$$\begin{cases} a(\gamma) = (\tan\alpha)^2(\cos\gamma)^2 + (\cos\beta)^2 + (\tan\alpha)^2(\sin\beta)^2(\sin\gamma)^2 - \tan\alpha\sin(2\beta)\sin\gamma \\ b(\gamma) = dz'\tan\alpha\sin(2\beta)\sin\gamma - 2dx'\tan\alpha\cos\gamma - 2dz'(\cos\beta)^2 \\ c = dx'^2 + dz'^2(\cos\beta)^2 - r^2 \end{cases} \quad (41)$$

Multiply the coordinates of the points coming from Eq. (40) and Eq. (41) by axonometric transformation of cone T_{con}, it can be gotten the points of axonometric projection of cylinder-cone intersections.

The variable *h* mentioned in Eq.(3) is of interest with the limit value of *s*, denoted s_{max}, that is s_{max} , multiplied by a height factor less than 1, yields *h*

$$h = s_{\max} i \quad (42)$$

Generally, i=1.2 to 1.5.

Determination *l*, the half of the height of cylinder is based on the maximum of distance from a point of intersection to the base point of cylinder, written as d_{max},, multiply d_{max} with *j*, which denote a length factor, *l* determined

$$l = d_{\max} j \quad (43)$$

Generally we define: j=1.5 to 1.8

6. Examples

On the basis of the algorithm produced before, an application program for drawing a combination of a cylinder intersecting a cone is developed, which can be loaded in AutoCAD. By entering the command “cone_cylinter” in AutoCAD,

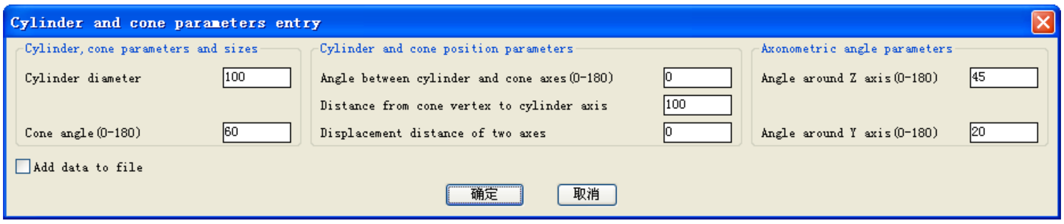

Fig 5. Dialog box for entering parameters

a dialog boxed appears, as showed in Fig.5, the parameters double r (cylinder diameter), double α (cone angle), β, dx', dz', θ and φ can be specified in it, here are two instances generating isometric and di-metric views by entering different θ and φ and other parameters. In the instances, r=50, α =30°, β =0°, dx' =0.

1) Drawing isometric view of a cone intersecting cylinder combination

For drawing isometric view, θ=45°, φ=35.27°, and dz' is specified -100, the isometric drawing is defined , as shown in Fig. 6.

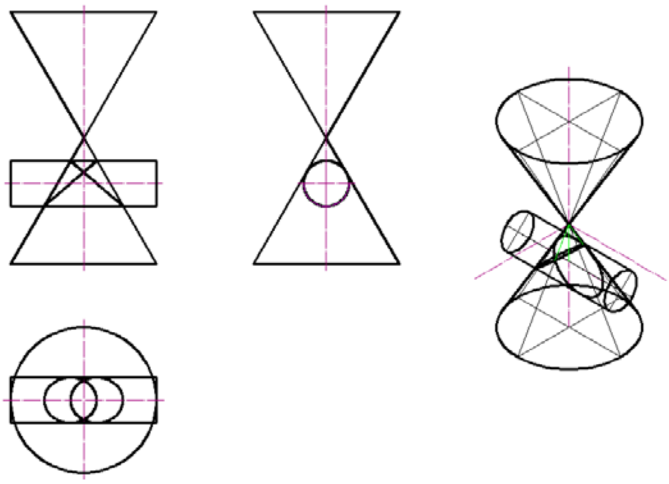

Fig.6 Isometric view

2) Drawing di-metric view

For drawing di-metric view, θ=20.70°, φ=19.61°, and dz' is specified 0, the di-metric drawing is defined , too, as shown in Fig. 7.

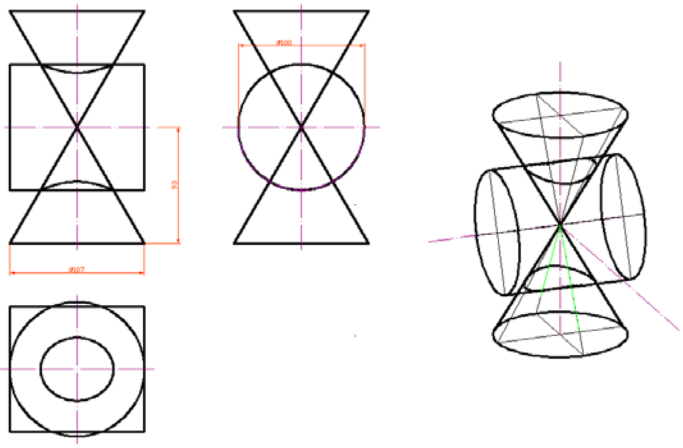

Fig.7 Di-metric view

If θ and φ are specified other values, then tri-metric view comes out.

7. Conclusion

In the paper, it was delivered the algorithm of axonometric projection of combination of cylinder intersecting a cone and related equations, which include operators of axonometric projection transformation and equation of ellipse, the result of axonometric projection of base of cone and cylinder. It also was developed the equations of tangent points and principal directions of tangent lines by which the axonometric limiting elements of cylinder and cone can accurately be drawn. In teams of the parametric equation of the Quadrics and axonometric transformation, the axonometric points of intersection are determined, too. For drawing the axonometric drawing of the combination of cylinder intersecting a cone with AutoCAD, it had been obtained all the dimensions and geometric parameters needed in terms of the algorithm and equations provided before. It can be drawn isometric, dimetric or trimetric drawings depending on different values' entry of angular parameters, such as β, θ and φ. Apart from the instances 6, all the figures of the paper are also drawn by running the application program developed in visual LISP of AutoCAD which is based on the algorithm of the paper. It can be used similar algorithm to derive corresponding equations to perform other types of pictorial like oblique or perspective projection drawings. It's important for us to develop AutoCAD self-defined commands and applications to construct different engineering drawings.

Acknowledgements

This work is Supported by the National Natural Science Foundation of China (NO.51375149),Key Technologies R & D Program of the Henan Province (NO.182102210303) and the landmark innovation project of Henan Institute of Science and Technology.

REFERENCES

[1]. *Gary R. Bertoline, Eric N. Wiebe, Craig L. Miller,* Fundamentals of Graphics Communication, McGraw-Hill, Boston, 1998.

[2]. *Avid R.Eide,Roland D.Jenison, Lane H. Mashaw, Larry L. Northup, C. Gordon Sanders,* Engineering Graphics Fundamentals, McGraw-Hill, New York,1985.

[3]. *Thomas E. French, Carl L. Svensen,* Mechanical Drawing, McGraw-Hill, ST. Louis, 1966.

[4]. *JinLiu, Zhong-keWu.* An adaptive approach for primitive shape extraction from point clouds, International Journal for Light and Electron Optics. 125 (2014) 2000-2008.

[5]. *Halalae, Ioan; Bojan, Patrik Silvester,* Experimental Generation of the Intersection Curve of Two Cylinders: An Algorithm Based on a New Paradigm, Analele Universitatii 'Eftimie Murgu' , 22 (2015) 217-222.

[6]. *Yuan Hao,* Study on 3D Reconstruction of Isometric Drawing,J, Jiangsu University Master Paper, 2003. (in Chinese)

[7]. *Lin Dajun,* Research on Converse Construction of Axonometric Projection Model, J. Journal of Graphics, 33 (2012) 1-6. (in Chinese)

[8]. *Xu Yueyue, Lin Dajun*, Optimizing Axonometric Projection of An Object on Stereoscopic Effect, J. JOURNAL OF GRAPHICS, 33 (2012) 1-4. (in Chinese)

[9]. *Hao Wuqiang, DONG Lijun,* 3D object reconstruction by point coordinates of 2D line drawings,J. CADDM, 23 (2013) 32-35.

[10]. *S. N. Mikhalev, I. Kh. Sabitov,* Isometric embeddings in R3 of an annulus with a locally euclidean metric which are multivalued of cylindrical type, Mathematical Notes, 98 (2015) 441–447

[11]. *C Trevisan.* Fast Algorithm for 4x4 Transformation Matrix of Generic Isometric Oblique Axonometry Projection, on https://www.researchgate.net/publication/236118178, 2012

[12]. *Lizaro Gimena, F austino N. Gimena and Pedro Gonzag.* A New Constructions in Axonometric System Fundamentals. Journal of Civil Engineering and Architecture, 6 (2012) 620-626.

[13]. *Ahmad Faiz Zubair, Mohd Salman Abu Mansor,* Automatic feature recognition of regular features for symmetrical and non- symmetrical cylinder part using volume decomposition method, Engineering with Computers. 34 (2018) 1-21

[14]. *Ja Aichen, ZHANG Jingqi, et al,* Analytical Equations of Normal Axonometric projection of Quadric Surfaces, J.Journal of Dalian University of Technology, 35 (1995) 410-412. (in Chinese)

[15]. *Feng Jinghua et al,* An Algorithm for the Contour Curve of Orthographic Axonometric projection of the Revolved Surface on a plane Curve, J. Journal of North China Institute of Technology , 16 (1995) 959-100. (in Chinese)

[16]. *Jamesr. Miller,* Geometric Approaches to Nonplanar Quadric Surface Intersection Curves, J.ACM Transactions on Graphics, 6 (1987) 274-307.

[17]. *Hu Zhigang, Zheng Qiubai,* Research on intersections of Cone and Cylinder, J. Journal of Graphics, 36 (2015) 671-677.(in Chinese)

[18]. Analytic Geometry Writing Group of Jiangsu Normal Collage, Analytic geometry, second ed., People ' s Education Press, Beijing, 1982