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# DESIGN PROCEDURE FOR THE INPUT/OUTPUT FILTER CIRCUITS USED FOR SINGLE PHASE AC CHOPPERS

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*Problema filtrării componentelor armonice produse de convertoarele statice este foarte importantă în rețelele moderne. Sinteza filtrelor trece jos, în cazul celulelor LC, plasate atât la intrare, cât şi la ieşire, are caracteristici specifice în prezența circuitelor chopper, şi nu permite o abordare clasică. In lucrare este prezentată o metodologie combinată, bazată pe anumite criterii impuse. Simplificarea ipotezelor de lucru, în concordanță cu aspectele tehnice, a permis deducerea unor ecuații simple pentru calculul componentelor. Pentru validarea metodei, după rezolvarea cu procesorul MathCad, s-a utilizat o simulare în PSpice pentru analiza unui circuit model. Generalitatea metodelor folosite indică rezultate promițătoare pentru diferite topologii de convertoare statice cu structuri mai complexe.*

*The harmonic filtering problem is more and more important for the modern power delivery networks. The low-pass filters synthesis, when the LC cells are present both at input and at output, has specific features, and a classic approach isn't valid. The paper presents a combined methodology, based on certain imposed criteria. The working hypotheses simplification, taking into account the technical aspects, enables to derive simple equations for the component evaluation. To prove the presented method, after the problem solving with the MathCAD interpreter, a PSpice simulation was used for a model circuit. The generality of the implied methods yields promising results for different static converters with more complex topologies.*

**Keywords**: low-pass-filter synthesis, static-converter, harmonic pollution.

## 1. Introduction

In the last decades new and powerful static converters structures were developed. These equipments are more and more used in different applications. The high frequency pollution is an important problem, not only for the power receiver but also from the supplying line point of view. In order to obtain a good filtering in both directions, output and input LC filters must be used. The literature

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contains relative many studies implying output filters (for example [1], [2], [3], [4]). As usual, only the output filter design is considered, following the consumer restrictions. The input problems must be solved using standard line filters. But this isn't an optimal situation. A better solution, proposed by the authors, gives a design procedure, taking into account the interaction between the output and the input filter. Because the presence of the switching nonlinear devices, the results from the classical filter theory aren't very useful. To this end, new design criteria are considered, giving the equations which enable the component evaluation.

In order to prove the validity of the derived equations, a numerical example is taken into account. The MathCAD interpreter is used to solve the synthesis problem. The quality of the obtained results is analyzed on a circuit model, by a PSpice simulation. Despite some approximation working hypothesis, the presented procedure is useful for more general and complex situations.

## 2. The physical approach

Because of the switching working mode, the chopper represents a high frequency perturbation source, not only for the output load circuit, but although for the supply input circuit (the 50Hz line). For a realistic analysis of the chopper behavior, this is considered as a voltage perturbation source at the output and as a current perturbation source at the input. This is justified because the chopper acts as a controlled voltage source at the output and as a current receiver at the input. The output current is determined by the load parameters and the input voltage is determined by the 50Hz line.

For analysis purposes, there are taken under consideration the serial switching device and the one necessary for freewheel working mode. The other parts present in a real chopper structure are neglected. This approach isn't restrictive for the general point of view because the main flux of energy is controlled by the serial device, which represents the main perturbation source of the circuit.

For the input and the output filters, there are considered first order low pass circuits. This represents the real situation because the chopper device uses a relative high switching frequency $f_s$ (tens of kHz), compared with the 50Hz line frequency. This allows a sufficiently good attenuation of the perturbation signal components, even using first order low pass filters. Thus, both for the input and output circuit there are necessary four reactive components, included in two LC circuits ($L_1$, $C_1$ and $L_2$, $C_2$).

In order to conceive a design procedure, determining the four values of the reactance, there are necessary four equations, having a unique solution. It is clear that two equations may be derived imposing two attenuation thresholds for the transfer functions of the input and the output circuits. The other two equations are derived from certain design criteria, taking under consideration some impedance aspects.

An important aspect is the determination of the first order filter topology. Placing the filters circuits before and after the serial switching device, the transmission additional impedance of the whole circuit must be maintained at low values. The additional reactance must generate a minimal delay for the 50 Hz signal and must ensure a good operation of the switching device (operating at $f_s$ frequency). The reactance components must have minimal values, ensuring simultaneously the imposed attenuations for the high frequency components. It is important to match the filter input or output impedances with the corresponding adjacent circuit requirements, so the main circuit behavior must be not altered by the presence of the filter elements. The behavior of the reactance components must be very different at 50Hz compared with the switching frequency. Thus the output filter impedance, viewed from the load, must be enough high at 50Hz, compared to the load impedance. On the other hand, the input filter impedance, viewed from the chopper, must be enough low at the switching frequency for a good operation of the chopper. It results that both the input and the output circuits must have reversed Γ (gamma) structures, containing a parallel capacitor at the right side.

In the following there are derived certain criteria, giving the equations necessary for the estimation of the filter elements. It must be mentioned that the classical filter theory in this case isn't useful because the presence of the nonlinear switching device.

## 3. The mathematical background and design criteria

The presented reasoning is based on the Fourier series signal representation and on the Laplace transform for the transfer function of an electrical circuit. If the input filter output impedance is enough low, the voltage signal, after the serial switching device, is a chopped sinusoidal wave $v_r(t)$. This results by multiplication of two Fourier series, corresponding to the signal of the supply source and to a duty factor pulse wave with unit amplitude and time parameter D [1]:

$$v_r(t) = S(\omega_s t) v_s(t) \tag{1}$$

The cosinus Fourier series of the pulse signal may be represented by:

$$S(\omega_s t) = D + \sum_{k=1}^{\infty} \frac{2\sin(kD\pi)}{k\pi} \cos(k\omega_s t) \tag{2}$$

where: $\omega_s$ – the switching angular frequency;

$D$ – the pulse duty factor;
$k$ – the number of the harmonic component $\omega_s$.
After the substitution of $S(\omega_s t)$ we obtain the following relation:

$$v_r(t) = \sqrt{2} D V_s \cos(\omega t) + \sum_{k=1}^{\infty} \sqrt{2} V_{rk} \cos[(k\omega_s \pm \omega)t] \quad (3)$$

where: $v_s(t) = \sqrt{2} V_s \cos(\omega t)$ - the instantaneous value of the supply voltage;
$V_{rk} = \frac{V_s \sin(kD\pi)}{k\pi}$ - the rms value of the k harmonic;
$V_s$ – the rms value of the supply voltage;
$\omega$ - angular frequency of the supply voltage.

In the following, in order to derive the four equations necessary to determine the four reactive components, there are used four criteria.

It must be mentioned that the high frequency perturbation is oriented to the consumer for the output filter and to the supply line for the input filter.

**a. The output attenuation criterion**

For our purpose, it is used the Laplace transform of the transfer function of the $R_2L_2C_2$ filter circuit (Fig.1):

$$\frac{V_0(s)}{V_r(s)} = \frac{R_2}{sL_2 + \left(1 + s^2 L_2 C_2\right) R_2} \quad (4)$$

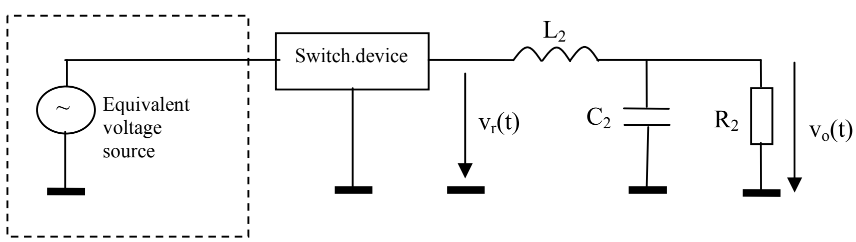


Fig.1. The output circuit model.

Using the $s = j\omega$ substitution and the module evaluation, the fundamental component, $V_{of}$ and the harmonic component, $V_{ok}$ of the output voltages may be computed. Using the hypothesis:

$$\omega L_2 << R_2 << \frac{1}{\omega C_2} \quad (5)$$

the fundamental component, $V_{of}$ is computed by:

$$V_{of} = \frac{R_2 D V_s}{\sqrt{\omega^2 L_2^2 + \left(1-\omega^2 L_2 C_2\right)^2 R_2^2}} \approx D V_s \tag{6}$$

By using the hypothesis:

$$\frac{1}{\omega_s C_2} << R_2 << \omega_s L_2 \quad \text{and} \qquad k\omega_s \pm \omega \approx k\omega_s \tag{7}$$

it is determined the component, $V_{ok}$ corresponding to the $k\omega_s$ harmonic:

$$V_{ok} = \frac{\sqrt{2} R_2 V_{rk}}{\sqrt{(k\omega_s)^2 L_2^2 + \left(1-k^2\omega_s^2 L_2 C_2\right)^2 R_2^2}} \approx \frac{\sqrt{2} V_{rk}}{(k\omega_s)^2 L_2 C_2} \tag{8}$$

The total harmonic distortion of the output voltage is defined by the following relation (having percent values):

$$THD_V = \frac{100}{V_{of}} \sqrt{\sum_{k=1}^{\infty} V_{ok}^2} \tag{9}$$

After the substitutions, we obtain:

$$THD_V \approx \frac{100\sqrt{2}}{\pi\omega_s^2 L_2 C_2} TH_1 \quad \text{where:} \qquad TH_1 = \frac{1}{D}\sqrt{\sum_{k=1}^{\infty} \frac{\sin^2(kD\pi)}{k^6}} \tag{10}$$

If the variables are rearranged, with $THD_V$ having decimal representation (with a maximal unit value), the above expression may be written as the following:

$$THD_V = \frac{\sqrt{2}}{\pi\omega_s^2 L_2 C_2} \sqrt{\sum_{k=1}^{\infty} \left(\frac{\sin kD\pi}{k^3 D}\right)^2} \tag{11}$$

In order to obtain a simpler representation, the series is approximated by a finite number of terms. This approach is justified by the $\frac{\sin(x)}{x}$ function evolution, which is a decreasing one. The square root factor may be considered as a function, with values having the vectorial representation $THS_i$. Figure 2 describes its plotted variation, for different $D_i$ values. In the following, the square root factor contribution will be substituted by its maximal value equaling 3.3, observed on the numerical simulation. This maximal value will corresponds to a near null value of the duty ratio D, but this is never accessible because the finite bandwidth of the electronic devices.

$$THS_i = \sqrt{\sum_{k=1}^{N} \left( \frac{\sin(kD_i \pi)}{k^3 D_i} \right)^2} \tag{12}$$

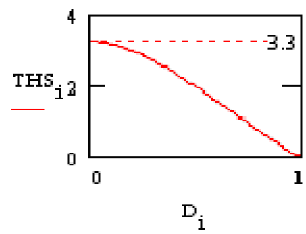


Fig.2. The $THS_i$ variation, computed by the MathCAD interpreter.

For the $THD_V$ value may be considered a covering greater value, denoted as $THDM_V$:

$$THDM_V = \frac{3.3 \cdot \sqrt{2}}{\pi \omega_s^2 L_2 C_2} \tag{13}$$

Imposing the total harmonic distortion $THDM_V$, using the above expression, the first equation for the $L_2$ and $C_2$ variables can be determined:

$$\text{(I)} \qquad L_2 C_2 = \frac{3.3 \cdot \sqrt{2}}{\pi \omega_s^2} \frac{1}{THDM_V} \tag{14}$$

**b. The input attenuation criterion**

For the analysis of the input low pass filter, it is used a simplified scheme (Fig.3). In this case the perturbation current signal comes from the chopper input and goes to the power supply line (from right side to the left side).

In the following, it is taken under consideration only the attenuation for the current component having the switching angular frequency $\omega_s$. This is due to the current divider $L_1C_1R_1$, where $R_1$ is the equivalent line resistance. Because $R_1 \ll X_{L1}$, in the following the $R_1$ will be neglected. In order to obtain an attenuation of the high frequency perturbation current, the condition $X_{C1} \ll X_{L1}$ must be also fulfilled.

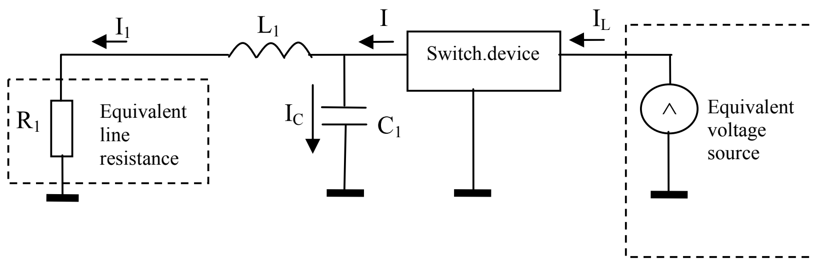


Fig.3. The input circuit model.

Neglecting the $\omega_s$ harmonics, the accuracy isn't serious affected because the signal frequency spectrum has a $\left(\frac{\sin(x)}{x}\right)$ type variation.

The voltage across the current divider (equaling the capacitor voltage) is given by:

$$V_C = I_1 \cdot j\omega_s L_1 = (I - I_1)\frac{1}{j\omega_s C_1} \tag{15}$$

It results the current attenuation factor K' of the $I_1$ current through $L_1$ against the total current $I$:

$$K' = \frac{I_1}{I} = \frac{1}{1 - \omega_s^2 L_1 C_1} \cong \frac{1}{\omega_s^2 L_1 C_1} \tag{16}$$

In our approach the current $I$ originate from the output circuit and equal the current through the $L_2$ inductance, when the serial switching device is under conduction state. Because at 50Hz the condition $X_{C2} >> R_2$ is fulfilled the fundamental component $I_f$ of this current may be approximated by the relation:

$$I_f \cong \frac{V_{rf}}{R_2} \tag{17}$$

where: $V_{rf} = \sqrt{2} \cdot D \cdot V_s$ represents the fundamental component of the voltage at the chopper output and $V_s$ is the line rms voltage.

The high frequency components of the $L_2$ inductance current are directly dependent on the $L_2$ value. These represent a ripple (with low level, for normal operation) superimposed on the 50Hz fundamental component. But, at the chopper input, when the serial switching device is in the conduction state, the current variation range is from zero to a value near the instantaneous value of the $L_2$ current fundamental component. It results that the current $I$ is mainly determined by the output resistance $R_2$.

If we denote by $I_k$ the $k\omega_s$ angular frequency components of the current $I$ and with $V_{rk}$ the rms value of the $v_r(t)$ components, then we have:

$$I_k = \frac{V_{rk}}{R_2} \tag{18}$$

where: $V_{rk} = \sqrt{2} V_s \dfrac{\sin kD\pi}{k\pi}$

Thus, it can be defined the $K''$ ratio between the $k\omega_s$ perturbation current component and the fundamental component:

$$K'' = \frac{I_k}{I_f} = \frac{\sqrt{2}V_s}{R_2} \frac{\sin kD\pi}{k\pi} \frac{R_2}{\sqrt{2}DV_s} \tag{19}$$

It results:

$$K'' = \frac{\sin kD\pi}{kD\pi} \tag{20}$$

But this is the "attenuated sinus" function, which is lower than unity:

$$\frac{\sin kD\pi}{kD\pi} < 1 \tag{21}$$

It results that $K'' < 1$. Adding the effect of the two attenuation factors K' and K", the ratio between the $\omega_s$ component of the perturbation current $I_1$, injected in the 50Hz line and the fundamental component $I_f$ of the current $I$, given by the chopper, it can be defined a factor $K_1$ by the relation:

$$\frac{I_1}{I_f} = K' \cdot K'' = K_1 \quad \text{and on the other hand:} \quad \frac{I_1}{I_f} = \frac{1}{\omega_s^2 L_1 C_1} \tag{22}$$

By this, it can be obtained a second equation implying the reactive elements, when the ratio $K_1 = \dfrac{I_1}{I_f}$ is imposed:

$$\text{(II)} \qquad L_1 C_1 = \frac{1}{\omega_s^2 \cdot K_1} \tag{23}$$

For the determination of all values for $L_1$, $C_1$ and $L_2$, $C_2$ we need two more equations. The above equations are derived from the attenuation requirements. The others are determined using different criteria. For our purposes we chose, on one hand the power factor maximization (for a resistive load, in our case) and on the other hand the minimization of the impedance at the chopper input. The first is a natural requirement for a good operation of the supplying network. The second ensures a good operation of the chopper. If the input impedance isn't enough low, having a reactive behavior, the shape of the voltage pulses at the chopper output will be altered, affecting the linearity proprieties and the dynamical characteristics of the entire circuit. Moreover the output voltage will be attenuated. Thus, it is necessary to have low capacitive impedance at the chopper input, imposing the satisfaction of the condition: $X_{C1} << X_{L2}$.

**c. The input impedance criterion**

This condition is imposed by a second factor denoted $K_2$. Thus, in our case, the condition is formulated as follows:

$$X_{L2} = K_2 X_{C1} \tag{24}$$

where for our purposes, it is chosen: $K_2$ = 100. It results the following relation between the $L_2$ and $C_2$:

$$\omega_s L_2 = K_2 \frac{1}{\omega_s C_1} \tag{25}$$

If we separate in the left side the unknown variables, the third equation is obtained:

$$\text{(III)} \qquad L_2 C_1 = \frac{K_2}{\omega_s^2} \tag{26}$$

**d. The power factor criterion**

The power factor is imposed for the fundamental component (at 50Hz frequency). For this, the serial switching device acts (integrating the pulse modulated wave) as a current controlling element. On the other hand, when it is in the conduction state the output voltage is determined by the input voltage. As we have seen, the maximal voltage distortions occur when the pulse duty ratio D goes to zero. But in this case the medium equivalent resistance of the switching device goes to infinity. So, the absorbed whole input power will be very low, having a negligible perturbation effect for the supplying line. In conclusion, the reactive behavior of the circuit must be limited (imposing a high power factor) when the received active power will be significant. By these reasons, for our purposes, will be considered a medium duty ratio D (D = 0.5), when the received power has a normal value. For this case, the output impedances will be seen at the input as a double value. (when the medium current controlled by the chopper is reduced at half).

The reactive elements implied in the power factor calculus are: $L_1$, $L_2$ and $C_2$. Now, the inductances have a serial configuration and the $C_1$ capacitor may be neglected because, at the fundamental frequency, its impedance is very high relative to the inductance impedance.

Thus, for the low frequency (50Hz) the equivalent impedance, denoted as $Z_e$ is given by the following relation, in complex representation, with the denominator transformed as a real value:

$$Z_e = \frac{R_0 X_C^2 + j\left[(R_2)^2(X_L - X_C) + X_L(X_C)^2\right]}{(R_0)^2 + (X_C)^2} \qquad (27)$$

When the power factor is maximized, the imaginary part will be nullified, obtaining:

$$X_L X_C^2 = R_2^2 (X_C - X_L) \qquad (28)$$

It must be specified that the impedance components are evaluated as viewed from the input. So, considering a ½ duty ratio the equivalent values for the $L_2$, $C_2$ and $R_2$ impedances are multiplied by two.

After some symbolical processing, it results the forth equation, necessary for the reactive element determination:

$$\text{(IV)} \quad \frac{L_1 + 2L_2}{C_2} = 2R_2^2 \qquad (29)$$

Cumulating the results given by the (a), (b), (c) and (d) paragraphs we obtain four equations with four unknowns corresponding to the reactive elements: $L_1$, $L_2$, $C_1$ and $C_2$:

$$\begin{cases} L_2 C_2 = \dfrac{3.3 \cdot \sqrt{2}}{\pi \omega_s^2} \dfrac{1}{THDM_V} & (I) \\ L_1 C_1 = \dfrac{1}{\omega_s^2 \cdot K_1} & (II) \\ L_2 C_1 = \dfrac{K_2}{\omega_s^2} & (III) \\ \dfrac{L_1 + 2L_2}{C_2} = 2R_2^2 & (IV) \end{cases}$$

For this nonlinear system, a real solution will be computed, using the following computing sequence:

1. The ratio $L_1/L_2$ from eq.(II) and (III);
2. It results the dependence $L_1(L_2)$;
3. The dependence $C_2(L_2)$ from eq.(I);
4. From eq.(IV), a 2-order equation with $L_2$ as unknown is obtained;
5. Thus, we have $L_1$ above dependence (2), $C_1$ from (II) and $C_2$ from (I).

## 4. Numerical example

In the following, in order to have a concrete image about the presented method, a numerical example is taken into account. The given parameters have the following significance:

- switching angular frequency $\omega_s = 10^5 \text{ s}^{-1}$;
- load resistance $R_2 = 50\Omega$;
- total harmonic distortion for output voltage $THD_V = 0.01$;
- the factor $K_1 = 0.01$ (from the input attenuation criterion);
- the factor $K_2 = 100$ (from the input impedance criterion);

The component values are computed using the MathCAD program, obtaining the values for the filters' reactive components. In order to verify the computed results, a simple model circuit is taken under consideration (Fig.4). This belongs to the single phase AC chopper category ([5], [6], [7]). The active devices are modeled by bidirectional ideal switching elements. This approach ensures the freewheeling mode (using the W2 switch), but neglects the dead time problems. This is no loss of generality because the energy transferred during the corresponding transient period is very low compared with the entire working cycle period and the switches are controlled in a synchronous manner.

The PSpice schematic diagram contain the 50Hz voltage supplying source (V1), the load resistance (R2), the input and the output low-pass filters (L1C1 and L2C2). For the control of the active switching elements additional component are inserted (high frequency pulse voltage source, DC voltage source, current limiting resistance, controlled switch W3).

The results obtained after the numerical simulations are plotted in Fig.5,6,7,8 for different input data concerning the parameters THDM, $K_1$ and K2.

The numerical simulation results show the following facts:

- the THDM value determines in principal the perturbation attenuation for the output voltage and is not important for the input current characteristics;
- the $K_1$ and $K_2$ factors determine in principal the perturbation attenuation for the input current and are not important for the output voltage characteristics.

The numerical experiments are based on the following input data and the corresponding values for the reactive elements:

*Best case* – using the scheme data (Fig.5,6):
Input data: THDM = 0.01 ; K1 = 0.01 ; K2 = 100
resulting: L1 = L2 = 5mH ; C1 = 2.01μF ; C2 = 2.98 μF.

*High perturbation on the output voltage* (Fig.7):
Input data: THDM = 0.1 ; K1 = 0.01 ; K2 = 100
resulting: L1 = L2 = 2mH ; C1 = 6.355μF ; C2 = 0.9441 μF.

*High perturbation on the input current* (Fig.8):
Input data: THDM = 0.01 ; K1 = 0.1 ; K2 = 10
resulting: L1 = L2 = 5mH ; C1 = 0.201μF ; C2 = 2.985 μF.

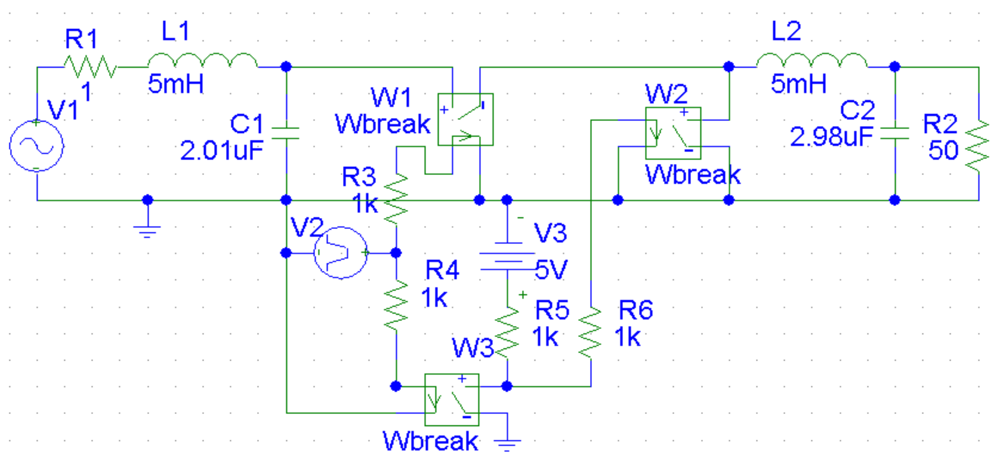


Fig.4. The PC Spice schematic diagram

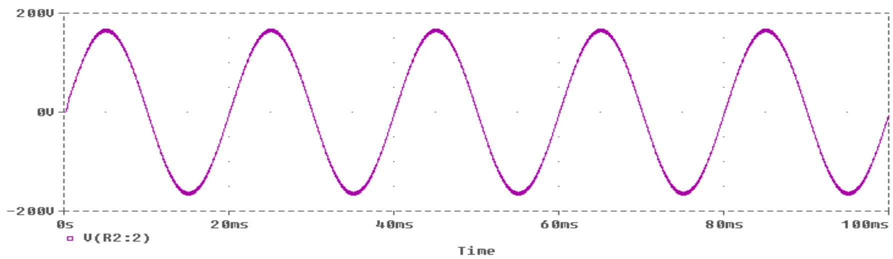


Fig.5. The variation of the output voltage (across R2).

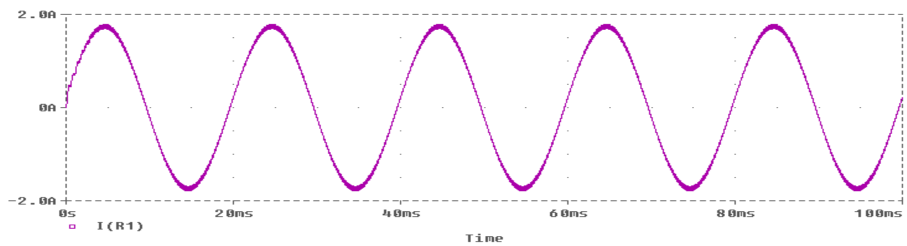


Fig.6. The variation of the input current (through R1).

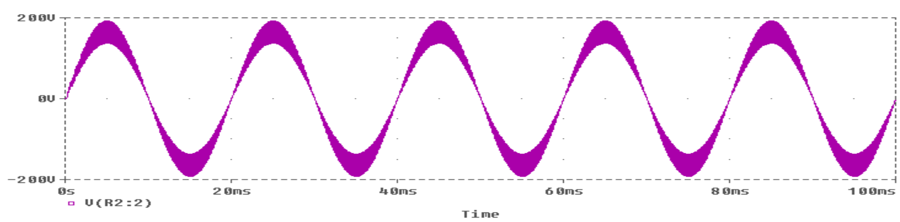


Fig.7. High perturbation on the output voltage.

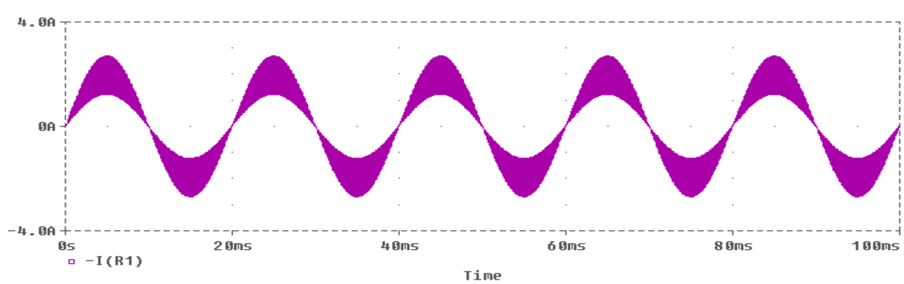


Fig.8. High perturbation on the input current.

## 5. Conclusions

The above presentation tries to find a unique solution for the AC chopper filtering, both for the input and the output circuit. The synthesis problem isn't a classical one because of the nonlinear operating of the chopper. Besides the imposed $THD_V$ level, our approach uses two additional constants $K_1$, $K_2$. Their dependence may be not so clear anytime.

Although a very covering values seems to be benefic, these lead to large capacitors and decreases the global stability characteristics. Therefore, once $K_1$ is determined, corresponding to a rated current value, $K_2$ must be enough large to maintain the efficiency of the filtering operation.

The presented methodology is based on certain idealizing hypothesis, and uses some circumstantial procedures from a technical point of view the procedure represent a good choice, with large applicability.

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