HYPERBOLIC FOUR VARIABLE REFINED SHEAR DEFORMATION THEORY FOR MECHANICAL BUCKLING ANALYSIS OF FUNCTIONALLY GRADED PLATES

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Buckling behavior of a thick rectangular plate made of functionally graded materials is investigated in this article. The material properties of the plate are assumed to vary continuously through the thickness of the plate according to a power-law distribution. The plate is assumed to be under three types of mechanical loadings, namely; uniaxial compression, biaxial compression, and biaxial compression and tension. The governing stability equations are derived based on the new four variable refined shear deformation theory. Unlike any other theory, the number of unknown functions involved is only four, as against five in case of other shear deformation theories. The theory takes into account the transverse shear effects and parabolic distribution of the transverse shear strains through the thickness of the plate, hence it is unnecessary to use shear correction factors. The resulted stability equations are decoupled and solved analytically for the functionally graded rectangular plates being simply supported and subjected to different types of mechanical loadings. A comparison of the present results with those available in the literature is carried out to establish the accuracy of the presented analytical method. The effects of the volume fraction exponent of the functionally graded material, plate thickness, aspect ratio and mechanical loading conditions on the critical buckling of aluminum/alumina functionally graded rectangular plates are investigated and discussed in detail.

Keywords: Mechanical buckling, Functionally graded plate, Thick rectangular plate, New four variable refined shear deformation theory, Analytical solution.

1. Introduction

Recent advances in material processing technology have led to a new class of materials called functionally graded materials (FGMs). FGMs are composites whose composition and microstructure vary continuously in some spatial

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directions. The advantage of FGMs is that no distinct internal boundaries exist and failures from interfacial stress concentrations developed in conventional structure components can be avoided. Featuring gradual transitions in microstructure and composition, they are designed to meet functional performance requirements varying with location within a structure component and to optimize the overall performance of the component. As a new concept of material design FGMs have found various available or potential applications in industries [1, 2]. Many works on FGM structures have been studied in literature. For example, Vel and Batra [3] have proposed a three-dimensional solution for free vibration of FG rectangular plates. Reddy [4] has analyzed the static behavior of FG rectangular plates based on his third-order shear deformation plate theory. Reddy and Cheng [5] have presented a three-dimensional model for an FG plate subjected to mechanical and thermal loads, both applied at the top of the plate. Park and Kim [6] studied thermal postbuckling and vibration of simply supported FGM plates with temperature-dependent materials properties by using finite element method. Bouazza et al. [7] used the first-order shear deformation theory to derive closed-form relations for buckling temperature difference of simply supported moderately thick rectangular power-law (linear, quadratic, cubic, and inverse quadratic) functionally graded plates. Bouazza et al. [8] presented the derivation of equations for mechanical buckling of rectangular thin functionally graded plates under uniaxial and biaxial compression using classical plate theory. Mohammadi et al. [9] investigated the buckling behavior of functionally graded material plate under different loading conditions based on the classical plate theory (Levy solution); the governing equations are obtained for functionally graded rectangular plates using the principle of minimum total potential energy.

Zenkour [10] derived the exact solution for FGM plates using generalized sinusoidal shear deformation theory and presented numerical results on displacement and stress response of FGM plates under uniform loading. Ying et al. [11] used a semi-analytical method to study thermal deformations of FG thick plates and the analysis is directly based on the 3D theory of elasticity. Yang and Shen [12] studied the postbuckling behavior of FGM thin plates under fully clamped boundary conditions. This work was then extended to the case of shear deformable FGM plates with various boundary conditions and various possible initial geometric imperfections by Yang et al. [13]. Woo et al. [14] studied the postbuckling behavior of FGM plates and shallow shells under edge compressive loads and a temperature field based on the higher order shear deformation theory. Javaheri and Eslami [15–18] presented the thermal and mechanical buckling of rectangular FGM plates based on the first- and higher-order plate theories. Three-dimensional deformations of a simply supported FG rectangular plate subjected to mechanical and thermal loads on its top and/or bottom surfaces have been analyzed by Vel and Batra [19].

Recently, a two variable refined plate theory (RPT) was first developed for isotropic plates by Shimpi [20], and was extended to orthotropic plates by Shimpi and Patel [21,22]. Kim et al [23], and Thai and Kim [24] have studied laminated composite plates using this theory. Mechab et al. [25] have developed this theory for the FGM plates. Benachour et al. [26] presented analytical solution for free vibrations of FG plates using this theory. Thai and Choi [27] developed the efficient and simple refined theory for buckling analysis of functionally graded plates. El Meiche et al. [28] proposed a new hyperbolic shear deformation theory for buckling and vibration analysis of functionally graded sandwich plates. Piscopo [29] also investigated refined buckling analysis of rectangular plates under uniaxial and biaxial compression. Hassaine Daouadji et al. [31] used a higher order theory which involves only four degrees of freedom for bending analysis of functionally graded plates. Bouhadra et al [32] studied the thermal buckling response of functionally graded plates with clamped boundary conditions using refined plate theory. Bellifa et al [33] used a new first-order shear deformation theory for bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position.

To the best of authors' knowledge, there are no research works for mechanical buckling analysis of functionally graded rectangular plates based on new four variable refined shear deformation theory. In this work, the application of a hyperbolic four-variable refined theory is extended for the FGM plates. For this purpose, the constitutive relations of buckling and vibration analysis of functionally graded sandwich plates are developed using four variable refined plate theory [28]. The novelty of this paper is the use of new four variable refined plate theory for mechanical buckling analysis of plates made of functionally graded materials. Unlike any other theory, the number of unknown functions involved is only four, as against five in case of other shear deformation theories. The theory presented is variationally consistent and does not require a shear correction factor. Introducing an analytical approach, the governing stability equations of functionally graded plates are decoupled and solved for a FGM rectangular plate with simply supported under different mechanical loads. The obtained results are compared with existing data in the literature. Moreover, mechanical loading conditions and geometric parameters of plate influence on the critical buckling of the FGM rectangular plate is comprehensively investigated.

2. Problem formulation

2.1 Material properties

Consider a rectangular plate of total thickness h. The FGM plate is made of aluminium and alumina, the material properties of the FGM such as material properties vary continuously across the thickness according to the following equations, which are the same as the equations proposed by Reddy [4]:

$$E(z) = E_m + E_{cm}\left(z/h + 1/2\right)^k \qquad E_{cm} = E_c - E_m \tag{1}$$

$$\nu(z) = \nu_0$$

where E_m denote the elastic moduli of metal; E_c denote the elastic moduli of ceramic . z is the thickness coordinate variable; and $-h/2 \le z \le h/2$, where k is the power law index that takes values greater than or equals to zero

For simplicity, Poisson's ratio of the plate is assumed to be constant in this study for that the effect of Poisson's ratio on deformation is much less than that of Young's modulus [33].

2.2. Present new hyperbolic shear deformation theory

The displacement field, which accounts for parabolic variation of transverse shear stress through the thickness, and satisfies the zero traction boundary conditions on the top and bottom faces of the plate using hyperbolic four variable refined shear deformation theory, is assumed as follows :

$$U(x,y,z) = u_0(x,y) - z\frac{\partial w_b}{\partial x} - \left(\frac{(h/\pi)\sinh\left(\frac{\pi}{h}z\right) - z}{\left[\cosh(\pi/2) - 1\right]}\right)\frac{\partial w_s}{\partial x}$$

$$V(x,y,z) = v_0(x,y) - z\frac{\partial w_b}{\partial y} - \left(\frac{(h/\pi)\sinh\left(\frac{\pi}{h}z\right) - z}{\left[\cosh(\pi/2) - 1\right]}\right)\frac{\partial w_s}{\partial y}\ , \tag{2}$$

$$W(x,y,z) = w_b(x,y) + w_s(x,y)$$

where, u and v denote the displacements along the x and y coordinate directions of a point on the midplane of the plate ; the transverse displacement W includes two components of bending w_b and shear w_s . Both these components are functions of coordinates x and y.

It should be noted that unlike the first-order shear deformation theory, this theory does not require shear correction factors. The kinematic relations can be obtained as follows:

$$\varepsilon_x = \frac{\partial u}{\partial x} - z\frac{\partial^2 w_b}{\partial x^2} + f(z)\frac{\partial^2 w_s}{\partial x^2};\ \varepsilon_y = \frac{\partial v}{\partial y} - z\frac{\partial^2 w_b}{\partial y^2} + f(z)\frac{\partial^2 w_s}{\partial y^2};\varepsilon_z = 0$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z\frac{\partial^2 w_b}{\partial x \partial y} + f(z)\frac{\partial^2 w_s}{\partial x \partial y};\ \gamma_{yz} = g(z)\frac{\partial w_s}{\partial y},\ \gamma_{xz} = g(z)\frac{\partial w_s}{\partial x} \tag{3}$$

where

$$g(z) = 1 - f'(z),\ f'(z) = \frac{df(z)}{dz};\ \ f(z) = \frac{(h/\pi)\sinh\left(\frac{\pi}{h}z\right) - z}{\left[\cosh(\pi/2) - 1\right]}. \tag{4}$$

The linear constitutive relations of a FGM plate can be written as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}. \tag{5}$$

The strain energy of the plate can be written as

$$U = \frac{1}{2}\int_V \sigma_{ij}\varepsilon_{ij}\,dV = \frac{1}{2}\int_V \left(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_{xy}\gamma_{xy} + \sigma_{yz}\gamma_{yz} + \sigma_{xz}\gamma_{xz}\right)dV. \tag{6}$$

The principle of virtual work for the present problem may be expressed as follows:

$$\iint \left[N_x\delta\varepsilon_x^0 + N_y\delta\varepsilon_y^0 + N_{xy}\delta\gamma_{xy}^0 + M_x^b\delta k_x^b + M_y^b\delta k_y^b + M_{xy}^b k_{xy}^b + M_x^s k_x^s \right.$$
$$\left. + M_y^s k_y^s + M_{xy}^s k_{xy}^s + Q_{yz}\gamma_{yz}^s + Q_{xz}\gamma_{xz}^s\right]dxdy = 0, \tag{7}$$

where (N_x, N_y, N_{xy}) denote the total in-plane force resultants, (M_x^b, M_y^b, M_{xy}^b), (M_x^s, M_y^s, M_{xy}^s) denote the total moment resultants of bending total moment resultants of shear, respectively, and (Q_{xz}, Q_{yz}) are transverse shear stress resultants, show a Fig. 1 with the system of axes and force resultants and they are defined as:

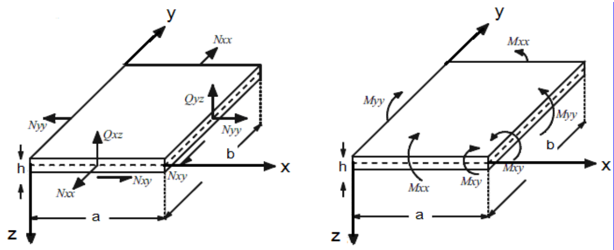

Fig. 1 Internal forces and moments of the FGM plate.

$$
\begin{aligned}
&(N_x, N_y, N_{xy}) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \sigma_{xy}) dz \\
&(M_x^b, M_y^b, M_{xy}^b) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \sigma_{xy}) z dz \\
&(M_x^s, M_y^s, M_{xy}^s) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \sigma_{xy}) f(z) dz \\
&(Q_{xz}, Q_{yz}) = \int_{-h/2}^{h/2} (\sigma_{xz}, \sigma_{yz}) g(z) dz
\end{aligned} \tag{8}
$$

Substituting Eq. (5) into Eq. (8) and integrating through the thickness of the plate, the stress resultants are given as:

$$
\begin{Bmatrix} \{N\} \\ \{M^b\} \\ \{M^s\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] & [B^s] \\ [B] & [D] & [D^s] \\ [B^s] & [D^s] & [H^s] \end{bmatrix} \begin{Bmatrix} \{\varepsilon^0\} \\ \{k^b\} \\ \{k^s\} \end{Bmatrix}, \tag{9}
$$

$$
\begin{Bmatrix} Q_{yz} \\ Q_{xz} \end{Bmatrix} = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} \tag{10}
$$

where A_{ij}, B_{ij}, etc. are the plate stiffness of extensional stiffness matrix , coupling stiffness matrix, defined by

$$
\begin{aligned}
&(A_{ij}, B_{ij}, D_{ij}, B_{ij}^s, D_{ij}^s, H_{ij}^s) = \int_{-h/2}^{h/2} Q_{ij} (1, z, z^2, f(z), zf(z), (f(z))^2) dz \quad (i, j = 1,2,6) \\
&A_{ij}^s = \int_{-h/2}^{h/2} Q_{ij} (g(z))^2 \, dz \qquad (i, j = 4,5)
\end{aligned} \tag{11}
$$

The stability equations of the plate may be derived by the adjacent equilibrium criterion. Assume that the equilibrium state of the FGM plate under mechanical loads is defined in terms of the displacement components $(u_0^0, v_0^0, w_b^0, w_s^0)$. The displacement components of a neighboring stable state differ by $(u_0^1, v_0^1, w_b^1, w_s^1)$ with respect to the equilibrium position. Thus, the total displacements of a neighboring state are:

$$u_0 = u_0^0 + u_0^1,\ v_0 = v_0^0 + v_0^1,\ w_b = w_b^0 + w_b^1,\ w_s = w_s^0 + w_s^1, \tag{12}$$

where the superscript 1 refers to the state of stability and the superscript 0 refers to the state of equilibrium conditions.

Substituting Equations (3) and (12) into Equation (7) and integrating by parts and then equating the coefficients of δu_0^1, δv_0^1, δw_b^1 and δw_s^1 to zero, separately, the governing stability equations are obtained for the new four variable refined shear deformation theory as

$$\begin{aligned}
&\frac{\partial N_x^1}{\partial x} + \frac{\partial N_{xy}^1}{\partial y} = 0 \\
&\frac{\partial N_{xy}^1}{\partial x} + \frac{\partial N_y^1}{\partial y} = 0 \\
&\frac{\partial^2 M_x^{b1}}{\partial x^2} + 2\frac{\partial^2 M_{xy}^{b1}}{\partial x \partial y} + \frac{\partial^2 M_y^{b1}}{\partial y^2} + \overline{N} = 0 \\
&\frac{\partial^2 M_x^{s1}}{\partial x^2} + 2\frac{\partial^2 M_{xy}^{s1}}{\partial x \partial y} + \frac{\partial^2 M_y^{s1}}{\partial y^2} + \frac{\partial Q_{xz}^{s1}}{\partial x} + \frac{\partial Q_{yz}^{s1}}{\partial y} + \overline{N} = 0
\end{aligned} \tag{13}$$

with

$$\overline{N} = N_x^0 \frac{\partial^2 (w_b^1 + w_s^1)}{\partial x^2} + N_y^0 \frac{\partial^2 (w_b^1 + w_s^1)}{\partial y^2} + 2N_{xy}^0 \frac{\partial^2 (w_b^1 + w_s^1)}{\partial x\, \partial y}. \tag{14}$$

The following approximate solution is seen to satisfy both the differential equation and the boundary conditions

$$\begin{Bmatrix} u_0^1 \\ v_0^1 \\ w_b^1 \\ w_s^1 \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}^1 \cos \lambda x \sin \mu y \\ V_{mn}^1 \sin \lambda x \cos \mu y \\ W_{bmn}^1 \sin \lambda x \sin \mu y \\ W_{smn}^1 \sin \lambda x \sin \mu y \end{Bmatrix}, \tag{15}$$

where

$U_{mn}^1, V_{mn}^1, W_{bmn}^1, W_{smn}^1$ are arbitrary parameters to be determined and $\lambda = m\pi/a$ and $\mu = n\pi/b$.

The pre-buckling forces can be obtained using the equilibrium conditions as [8, 9]

$$N_x^0 = \xi_1 N_0,\ N_y^0 = \xi_2 N_0,\ N_{xy}^0 = 0, \tag{16}$$

where N_0 is the force per unit length, ξ_1 and ξ_2 are the load parameter which indicate the loading conditions. Negative values for ξ_1 and ξ_2 indicate that plate is subjected to biaxial compressive loads while positive values are used for tensile loads. Also, zero value for ξ_1 or ξ_2 shows uniaxial loading in x or y directions, respectively.

3. Results and discussion

In this section, various numerical examples are presented and discussed for verifying the accuracy and efficiency of the present theory in predicting the critical buckling load of FGM plates subjected to in-plane loading. For the verification purpose, the results obtained by present theory are compared with those found in the literature using various theories. The following material Al/Al_2O_3 properties are used:

$$E_c = 380\,GPa, E_m = 70\,GPa, \nu = 0.3 \ ,$$

c and m represent the aluminum and alumina, respectively.

3.1. Comparisons

In order to validate the accuracy of the present method, a comparison has been carried out with previously published results by Hashemi et al [34], Bouazza and Adda [8] and Mohammadi et al [9] for rectangular plates for simply supported. Plates are subjected to monoaxial in-plane compressive applied loads in the x $(\xi_1 = -1, \xi_2 = 0)$ and equal biaxial in-plane compressive applied loads $(\xi_1 = -1, \xi_2 = -1)$.

The results of critical buckling load parameters $\tilde{N}_{cr} = N_{cr} a^2 / D$ of isotropic thin and moderately thick square rectangular plate are presented in Table 1 for different thickness to length ratios and aspect ratios. The obtained results are compared with those given by Hashemi et al [34] based on exact solution for linear buckling of rectangular Mindlin plates. It can be seen the present results are in excellent agreement with those given by Hashemi et al [34] for all loading types and geometric parameters.

Table 1

Comparison of critical buckling load parameters, $\tilde{N}_{cr} = N_{cr} a^2 / D$ of simply supported isotropic square plates $\left(\tilde{N}_1 = \xi_1 \tilde{N}_{cr},\ \tilde{N}_2 = \xi_2 \tilde{N}_{cr}\right)$

a/b	h/a	Critical buckling load			
		(ξ_1, ξ_2) =(-1,0)		(ξ_1, ξ_2) =(-1,-1)	
		Hashemi et al. [34]	Present study	Hashemi et al. [34]	Present study
0.4	0.001	13.280498	13.280496	11.448705	11.4487037
	0.01	13.27636	13.276201	11.445141	11.4450009
	0.1	12.875571	12.860397	11.099630	11.0865495
	0.2	11.796432	11.746961	10.169338	10.1266901
0.5	0.001	15.421205	15.421203	12.336964	12.3369620
	0.01	15.416032	15.415828	12.332826	12.3326625
	0.1	14.915722	14.896839	11.932578	11.9174714
	0.2	13.580179	13.519520	10.864143	10.8156159
2/3	0.001	20.592057	20.592054	14.256039	14.2560372
	0.01	20.584076	20.583761	14.250514	14.2502963
	0.1	19.816043	19.787249	13.718799	13.6988647
	0.2	17.803107	17.713481	12.325227	12.2631793
1	0.001	39.478204	39.478195	19.739102	19.7390976
	0.01	39.457021	39.456186	19.728510	19.7280931
	0.1	37.447690	37.373771	18.723845	18.6868855
	0.2	32.441432	32.230498	16.220716	16.1152490
1.5	0.001	96.381222	96.381158[a]	32.075932	32.0759206
	0.01	96.219798	96.213448 [a]	32.047973	32.0468723
	0.1	82.416286	81.974152 [a]	29.478533	29.3878842
	0.2	57.444097	56.714707 [a]	23.716511	23.5004635
2	0.001	157.910245	157.910111[a]	49.347353	49.3473269
	0.01	157.571876	157.55857 [a]	49.281211	49.2786081
	0.1	129.765726	128.92199 [a]	43.456572	43.2624640
	0.2	76.902078	75.829669 [a]	31.996718	31.6268055
2.5	0.001	255.022775[b]	255.02236[b]	71.553225	71.5531705
	0.01	253.983122[b]	253.94235 [b]	71.414247	71.4087824
	0.1	180.427919[e]	178.61856 [b]	59.799393	59.4387577
	0.2	87.667294[e]	87.563138[d]	40.057195	39.5232409

a Mode for plate is (m, n) = (2, 1).
b Mode for plate is (m, n) = (3, 1).
c Mode for plate is (m, n) = (4, 1).
d Mode for plate is (m, n) = (5, 1).

Table 2

Comparison of the critical buckling load (MN/m) for a FGM plate (b=1, h=0.01).

K	a/b	CRITICAL BUCKLING LOAD								
		(ξ_1, ξ_2) =(-1,0)			(ξ_1, ξ_2) =(0,-1)			(ξ_1, ξ_2) =(-1,-1)		
		REF[9]	REF[8]	PRESENT	REF[9]	REF[8]	PRESENT	REF[9]	REF[8]	PRESENT
0	0.5	2.14655	2.14655	2.14353	8.58619	8.58619	8.57412	1.71724	1.71724	1.71482
	1	1.37379	1.37379	1.37302	1.37379	1.37379	1.37302	0.68689	0.686896	0.68651
	1.5	1.49066[a]	1.49066[a]	1.48949[a]	0.71658	0.71658	0.71628	0.49609	0.49609	0.49589
1	0.5	1.06993	1.06993	1.06866	4.27971	4.27971	4.27464	0.85594	0.85594	0.85493
	1	0.68475	0.68475	0.68443	0.6847532	0.6847532	0.68443	0.34238	0.34238	0.34221
	1.5	0.74300	0.74300[a]	0.74252[a]	0.35717	0.35717	0.35705	0.24727	0.24727	0.24719
2	0.5	0.83488	0.83488	0.83382	3.33953	3.33953	3.3353	0.66791	0.66791	0.66706
	1	0.53432	0.53433	0.53405	0.53432	0.53433	0.53405	0.26716	0.26716	0.26703
	1.5	0.57978[a]	0.57978[a]	0.57937[a]	0.27871	0.27871	0.2786	0.19295	0.19295	0.19288

a Mode for plate is (m, n) = (2, 1).

The next comparison is performed for simply supported FGM plates subjected to various loading conditions. The plate is made from a mixture of aluminum (Al) and alumina (Al_2O_3). The critical buckling loads of simply supported plate for different values of aspect ratio *a/b*, and power law index *k* are shown in Table 1. As table shows, the present results have a good agreement with Refs. [8,9].

3.2. Buckling analysis of FGM plates

The variation of the nondimensional critical buckling load N_{cr} of square plate versus the variation of the modulus ratio E_m/E_c of FGM (i.e., different ceramic-metal mixtures) and dimensional parameter *a/h* have been plotted for various loading conditions in Fig. 1 through Fig. 6. In each figure, four arbitrary values of the power law index (k = 0; 1; 5; 10) are considered. As explained earlier, the variation of the composition of ceramics and metal is linear for k = 1. The value of k equal to zero represents a homogeneous (fully ceramic) plate.

Figs. 1, 3, and 5 show that nondimensional critical buckling load increases by increasing modulus ratio E_m/E_c of FGM and decreases by increasing power law index (k) from zero to 10. Figs. 2, 4, and 6 show that nondimensional critical buckling load increases with increasing dimension ratio a/h and also with decreasing power law index (*k*) from 10 to zero. It can be concluded from all the figures that, nondimensional critical buckling load for homogeneous plate (*k* = 0) is considerably greater than the values for nonhomogeneous functionally graded plates (*k* > 0) especially for thin plates.

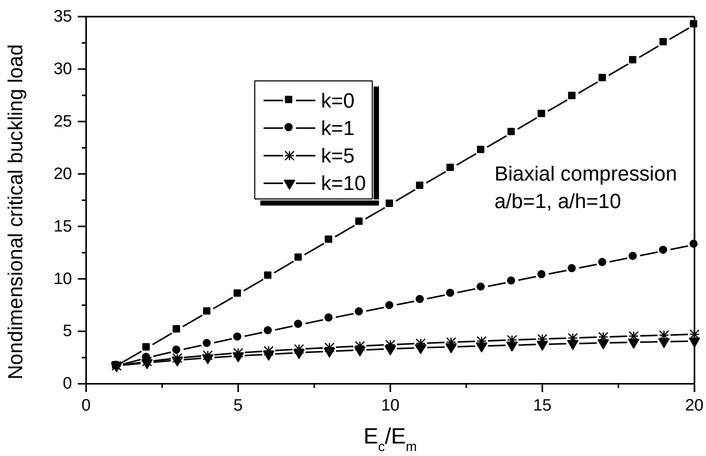

Fig.1. Critical buckling load of the FGM under biaxial compression versus E_c/E_m

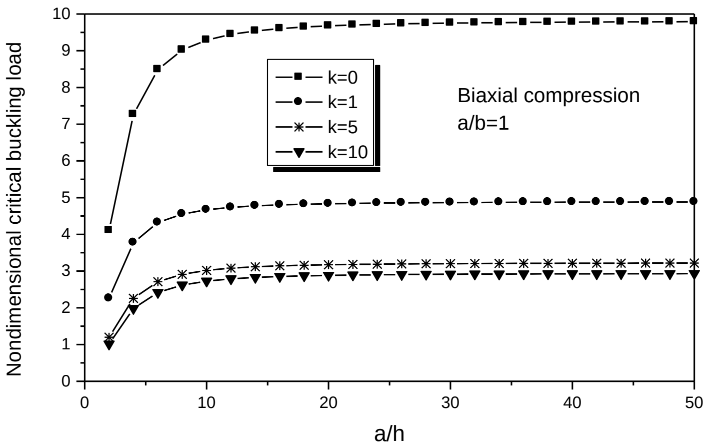

Fig.2. Critical buckling load of the FGM under biaxial compression versus *a/h*

A comparison of Figs. 1 and 2 with Figs. 3 and 4 shows that the nondimensional critical buckling load for the plate subjected to uniaxial compression $(\xi_1=-1,\ \xi_2=0)$, is greater than the corresponding values for the plate under biaxial compression $(\xi_1=-1,\ \xi_2=-1)$. The calculated values for $(\xi_1=-1,\ \xi_2=0)$ are twice those for $(\xi_1=-1,\ \xi_2=-1)$ for the square plate b/a= 1 but the difference decreases by increasing aspect ratio (a/h) and modulus ratio (E_m/E_c). Also, a comparison of Figs. 3 and 4 with Figs. 5 and 6 shows that the nondimensional critical buckling load for the plate subjected to compression along x-direction and tension along y-direction $(\xi_1=-1,\ \xi_2=1)$, is greater than the corresponding values for the plate under uniaxial compression $(\xi_1=-1,\ \xi_2=0)$. Obtained values for $(\xi_1=-$

1, ξ_2*=1)* are approximately twice those for *(*ξ_1*=-1,* ξ_2*=0)* for the square plate (*b/a*= 1) but the difference decreases by increasing aspect ratio (*a/h*) and modulus ratio (E_m/E_c). For the square plate under in-plane combined tension and compression ((*ξ_1=-1, ξ_2=1)*; *b/a = 1*), the plate buckles when *m = 1* and *n = 2*. In all other cases, buckling occurs for *m = n= 1*.

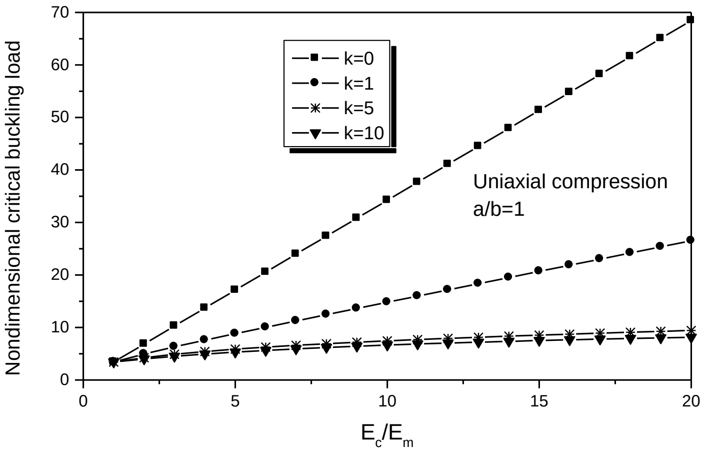

Fig.3. Critical buckling load of the FGM under uniaxial compression N_x versus E_c/E_m

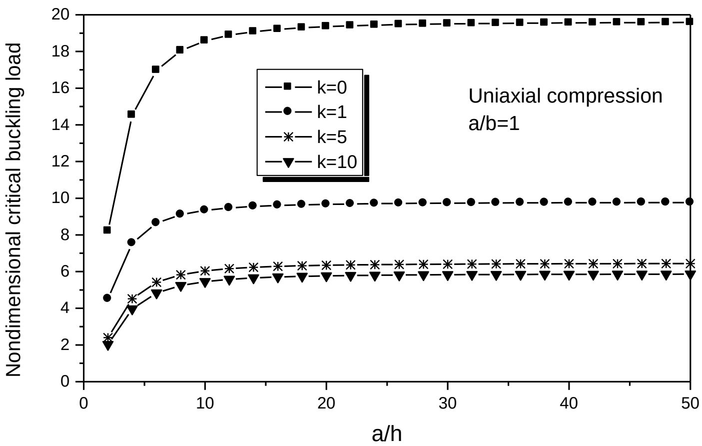

Fig.4. Critical buckling load of the FGM under uniaxial compression N_x versus *a/h*.

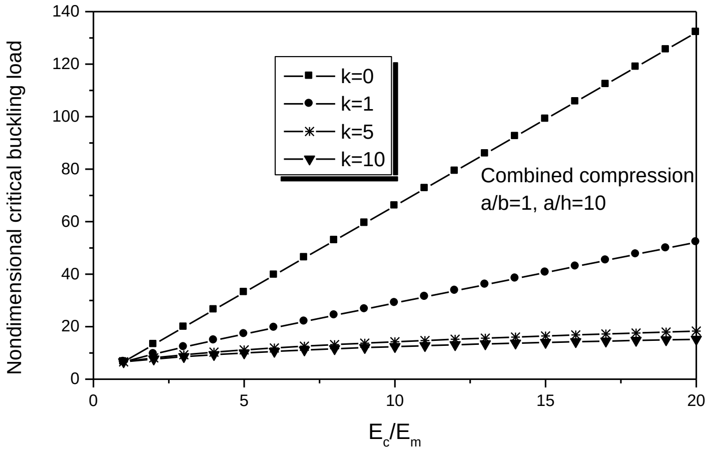

Fig.5. Critical buckling load of the FGM under combined compression N_x and tension N_y versus E_c/E_m.

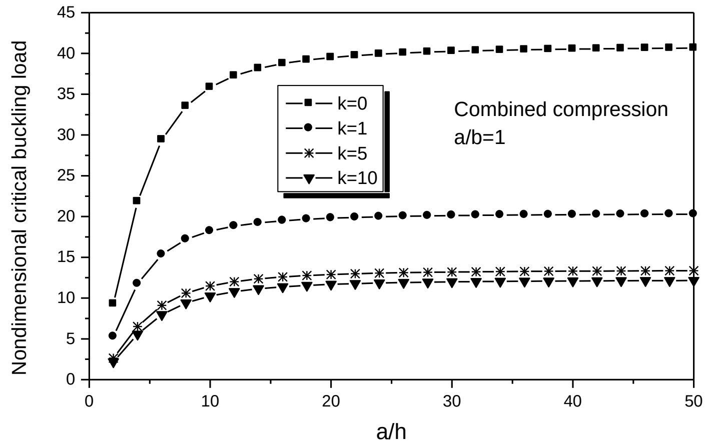

Fig.6. Critical buckling load of the FGM under combined compression P_x and tension P_y versus *a/h*.

The nondimensional critical buckling loads have been show in Figures 3–6 for simply supported plates subjected to uniaxial compression, and biaxial compression and tension, respectively. Fig. 3 shows the variation of nondimensional critical buckling load of square plate versus the modulus ratio E_c/E_m of FGM (i.e., different ceramic-metal mixtures) for different values of power law index. The thickness ratio a/h is assumed to be 10. It can be seen that the nondimensional critical buckling load increases as the ceramic-to-metal modulus ratio increases and decreases as the power law index increases.

Fig. 4 shows the nondimensional critical buckling load vs the thickness to span ratio a/h for different values of volume fraction exponent k (a/b= 1). It is seen that the nondimensional critical buckling load increases monotonically as the relative thickness a/h increases.

The modulus ratio E_c/E_m of FGM on nondimensional critical buckling load of simply supported plate under combined compression and tension is shown in Fig. 5. The thickness ratio a/h is assumed to be 10. It is shown that the nondimensional critical buckling load generally increases by the increase of the modulus ratio E_c/E_m.

Fig. 6 shows the variation trend of nondimensional critical buckling with respect to the thickness to span ratio a/h for different values of material gradient index k. The aspect ratio of the plate is set as a/b=1. It is observed that with increasing the thickness to span ratio a/h from 5 to 50, the nondimensional critical buckling also increases steadily, whatever the material gradient index k is.

4. Conclusions

In the present paper, mechanical buckling analysis of simply supported FGM plates has been analyzed using a new four-variable refined plate theory. Derivation was based on the new four variable refined shear deformation theory and with the assumption of power law composition for the material. Equilibrium and stability equations for rectangular simply supported functionally graded plates have been obtained. The buckling analysis of FGM plates under different types of mechanical loadings is presented. Closed-form solutions for the critical buckling of plates are presented. It is concluded that:

1. It is a displacement-based theory that includes the transverse shear effects.
2. Unlike any other theory, the number of unknown functions involved is only four, as against five in case of other shear deformation theories.
3. The theory takes account of transverse shear effects and parabolic distribution of the transverse shear strains through the thickness of the plate, hence it is unnecessary to use shear correction factors.
4. The critical buckling load for the functionally graded plates is reduced when the power law index *k* increases.
5. The critical buckling load for the functionally graded plates increases with increasing dimension ratio *b/a*.
6. The critical buckling load for the functionally graded plates decreases with increasing modulus ratio Ec/E_m.
7. The critical buckling load for the plates under uniaxial compression is greater than the plates under biaxial compression.

8. The critical buckling load for the plates under combined compression and tension is greater than for plates under uniaxial and biaxial compression. This conclusion confirms that the addition of a tensile load in the transverse direction is seen to have a stabilizing influence.

In conclusion, it can be said that the proposed theory is accurate and simple in solving the buckling behaviors of thick functionally graded plates.

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