FLOWS ACCOMPANIED BY NORMAL SHOCK CONSIDERING THE VARIATION WITH TEMPERATURE OF THERMODYNAMIC FUNCTIONS

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Efectul variației funcțiilor termodinamice cu temperatura este important şi nu poate fi neglijat în calculele cu acuratețea cerută, spre exemplu în problemele aerospațiale. Deoarece căldurile specifice ale gazelor uzuale cresc cu temperatura, temperatura reală după undele de şoc poate fi cu sute de grade Kelvin mai scăzută la numere Mach mari. Diferențe la fel de mari se constată şi în cazul temperaturilor de stagnare. Creşterile importante de entropie duc la pierderi semnificative de presiune. Metoda de calcul propusă permite păstrarea relațiilor analitice din cazul căldurilor specifice constante, prin introducerea unor numere Mach echivalente. În acelaşi timp, calculele sunt simplificate, fiind de rezolvat doar o ecuație într-o singură necunoscută. Sunt prezentate aplicații şi comparații pentru aer în intervalul de numere Mach (1.3; 8).

The effect of variation of the thermodynamic functions with temperature for the intensity of normal shock waves is important and cannot be neglected for an acurate calculation, as required, for example, in aerospace problems. Because the specific heat of usual gases increases with temperature, the real temperature after the shock wave can be smaller with hundreds of Kelvin degrees at higher Mach numbers. The same large differences are in the stagnation temperatures. Big differences exist in the entropy variation leading to much larger pressure losses. The proposed method of calculation was able to preserve the analytical relations from the constant caloric capacities, in terms of new introduced equivalent Mach numbers. At the same time, the calculation is simplifieed, only one unknown equation having to be solved. Applications and comparisons are presented for air in the Mach number interval (1.3; 8). The gas dissociation and/or ionisation at very high temperature were, for the moment neglected.

Keywords: equivalent Mach number; dimensionless quantities, mean stagnation temperature; entropy average caloric capacity.

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1. Introduction

One studies the 1D flows of an ideal gas initially at supersonic speeds. The initial stationary state 1 is a gas mixture, having a constant velocity u_1, at temperature T_1 and pressure p_1. The final state 2 is a gas of the same composition at subsonic regime with physical parameters modified due to a normal shock wave and depending on the initial state. At very high initial Mach numbers a gas dissociation, ionization etc. are possible; however this case is not studied here. The velocity, temperature and pressure after the shock wave are u_2, T_2 and p_2, respectively.

The overall transformation is considered at constant total mass enthalpy. The viscous effects are concentrated in the shock wave structure.

2. The governing equations

The transformation of the initial gas mixture from state 1 to the final mixture (state 2) (Fig.2.1) is subjected to the laws of mass, momentum and energy conservation, written as follows [1;2;8]:

$$\rho_1 u_1 = \rho_2 u_2 (mass); \tag{2.1}$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \ (momentum); \tag{2.2}$$

$$h_1(T_1) + \frac{1}{2}u_1^2 = h_2(T_2) + \frac{1}{2}u_2^2 \ (energy). \tag{2.3}$$

In the above equations, ρ_1, ρ_2 are gas mixture densities and h_1, h_2 mass enthalpies.

One replaces the enthalpy $h_2(T_2)$ as follows:

$$h_2(T_2) = h_2(T_1) + \left(h_2(T_2) - h_2(T_1)\right) = h_2(T_1) + (T_2 - T_1)\, C_{pm2}(T_1, T_2). \tag{2.4}$$

where $C_{pm2}(T_1,T_2)$ represents the mean specific caloric capacity at constant pressure, for the temperature interval $[T_1, T_2]$.

Because the gas mixture composition is unchanged within large temperature intervals, one has:

$$h_1(T_1) - h_2(T_1) = 0, \tag{2.5}$$

and the energy equation becomes:

$$c_{pm2}(T_1, T_2)T_1 + \frac{1}{2}u_1^2 = c_{pm2}(T_1, T_2)T_2 + \frac{1}{2}u_2^2. \tag{2.6}$$

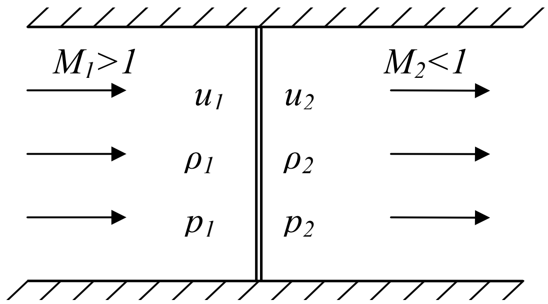

Fig.2.1. 1-D supersonic flow with normal shock wave

By introducing the ratios: ratio of specific volumes τ_r, the pressure ratio p_r and the temperature ratio T_r, defined by:

$$\tau_r = \frac{\rho_1}{\rho_2}; p_r = \frac{p_2}{p_1}; T_r = \frac{T_2}{T_1}, \tag{2.7}$$

from conservation laws one obtains:

$$p_r = 1 + \overline{m}(1 - \tau_r) \tag{2.8-a}$$

$$T_r = \frac{R_1}{R_2}\left[1 + \overline{m}(1 - \tau_r)\right]\tau_r \tag{2.8-b}$$

where R_1, R_2 are the gas mixtures constants.

The Clapeyron equation was also used; $\overline{m}$ is a dimensionless mass flow parameter given by:

$$\overline{m} = \frac{v_1^2}{R_1 T_1} = k_1 M_1^2;\ k_1 = \frac{C_{p1}(T_1)}{C_{p1}(T_1) - R_1}; \tag{2.9}$$

k_1 is the ratio of specific heats in state 1 and M_1 the corresponding Mach number.

For a given flow rate parameter, $\overline{m}$, for τ_r one gets the quadratic equation:

$$\overline{m}\left(1 - \frac{R_2}{2C_{pm2}}\right)\tau_r^2 - \left(1 + \overline{m}\right)\tau_r + \frac{R_2}{R_1} + \frac{\overline{m}R_2}{2C_{pm2}} = 0 \tag{2.10}$$

If the gas composition is unchanged, one may write:

$$R_1 = R_2 = R. \tag{2.11}$$

Depending on the sign of the discriminant, Δ, of the equation (2.10) given by:

$$\Delta = \left(1 + \overline{m}\right)^2 - 4\overline{m}\left(1 - \frac{R_2}{2C_{pm2}}\right)\left[\frac{R_2}{R_1} + \frac{\overline{m}R_2}{2C_{pm2}}\right] \tag{2.12}$$

one obtains, for a chosen $\overline{m}$: a) 2 solutions; b) 1 solution and c) no solution. These possibilities are studied in the following.

3. The case of unchanged gas composition

In this case, one has the same mixture in the two states 1 and 2. The previous relations are simplified, by writing:

$$R_1 = R_2 = R\,; \tag{3.1-a}$$

$$C_{pm2}\left(T_1, T_2\right) = C_{pm};\; k_m = \frac{C_{pm}}{C_{pm} - R}. \tag{3.1-b}$$

The equation (2.10) becomes:

$$\tau_r^2 - \frac{2k_m\left(1+\overline{m}\right)}{\overline{m}\left(k_m+1\right)}\tau_r + \frac{2k_m + \overline{m}\left(k_m-1\right)}{\overline{m}\left(k_m+1\right)} = 0 \tag{3.2-a}$$

or:

$$\left(\tau_r - 1\right)\left(\tau_r - \frac{2k_m + \overline{m}\,\left(k_m-1\right)}{\overline{m}\,\left(k_m+1\right)}\right) = 0 \tag{3.2-b}$$

Therefore, in case the gas composition is unchanged, there are always two solutions:

a) a trivial solution when the gas flow remains unmodified:

$$\tau_r = 1;\; p_r = 1;\; T_r = 1\,. \tag{3.3}$$

b) a non trivial solution:

$$\tau_r = \frac{2k_m + \overline{m}\left(k_m-1\right)}{\overline{m}\left(k_m+1\right)};\left(\overline{m} = k_1 M_1^2\right) \tag{3.4-a}$$

$$p_r = \frac{2\overline{m} - \left(k_m-1\right)}{k_m+1};\; T_r = p_r\tau_r\,. \tag{3.4-b}$$

A discussion via the mass rate parameter $\overline{m} = k_1 M_1^2$ is interesting. Thus:

1) for $\overline{m} > k_m > 1$ one obtains a shock wave:

$$\overline{m_s} > k_m > 1; \frac{k_1}{k_m} M_1^2 > 1 \tag{3.5-a}$$

$$\tau_{rs} < 1; p_{rs} > 1; T_{rs} > 1\,, \tag{3.5-b}$$

the index *s* indicating a state with shock wave.

An interesting fact can be pointed out: the condition for a shock wave (3.5-a) is not M_1>1 as for constant caloric capacities with temperature: an ***equivalent Mach number*** of the initial flow, M_{1e} is obtained from (3.5-a), namely:

$$M_{1e} = M_1 \sqrt{\frac{k_1}{k_m}};\ M_{1e} > 1;\ T_2 > T_1, \tag{3.6}$$

and the shock wave occurence condition is $M_{1e}>1$. As for the most gases the ratio of specific heats, k, decreases with temperature, one obtains:

$$T_2 > T_1\ ;\ k_1 > k_m;\ M_{1e} > M_1;\ k_m = k_m\left(T_1, T_2\right)\ . \tag{3.7}$$

Therefore, in principle, a shock wave could be produced even for $M_1 < 1$ if the ratio k_1/k_m is sufficiently large. In fact, k_1/k_m is pretty close to unity ($k_1/k_m>1$, $T_2>T_1$); on the other hand, the entropy source should be non-negative. In fact, only in case of reacting flows one finds a value M_{2e}=1, with M_2 < 1 (so called Chapman-Jouguet detonation wave).

2) a weak wave (Mach wave) can be obtained for $M_{1e}=1$, or:

$$\overline{m} = k_m;\ \frac{k_1}{k_m} M_1^2 = 1;\ \tau_r = p_r = T_r = 1. \tag{3.8-a}$$

Because $T_r=1$ $(T_1=T_2)$, one yields:

$$k_1 = k_m;\ M_1 = 1\,, \tag{3.8-b}$$

that is the Mach wave occurrence is still $M_1=1$;

3) an expansion wave is theoretically possible for:

$$0 < \overline{m} < k_m;\ p_r < 1;\ T_r < 1 \tag{3.9-a}$$

$$\frac{k_1}{k_m} M_1^2 < 1; k_1 < k_m \tag{3.9-b}$$

Because $T_r<1$, $(T_2<T_1)$, it results $k_m> k_1$. If the initial flow is supersonic, the conditions (3.9) give:

$$1 < M_1^2 < \frac{k_m}{k_1}; \left(k_m > k_1\right) \tag{3.9-c}$$

therefore a small interval of M_1 is possible to obtain an expansion.

If the initial flow is subsonic ($M_1<1$) the condition (3.9-b) seems easier to be satisfied, however a wave can travel only in the neighbourhood of the speed of sound .

Remark.1 The above discussed cases require however some additional conditions to be actually produced. For example, a shock wave requires a downstream obstacle or a downstream narrowing of the channel.

Similarly, to produce a Mach wave (both for compression or expansion) a small perturbation of the flow is necessary.

3.1. Expressions in terms of equivalent Mach numbers

The above relations for density, pressure and temperature ratios can be written in terms of the equivalent Mach number M_{1e} [4].

Thus one obtains:

- for the pressure ratio:

$$p_r = \left(1 + \chi_m\right) M_{1e}^2 - \chi_m \quad ; \quad M_{1e}^2 = \frac{k_1}{k_m} M_1^2 \quad ; \qquad (3.10\text{-a})$$

- for the density ratio:

$$\tau_r = \frac{1 - \chi_m}{M_{1e}^2} + \chi_m \ , \ T_r = p_r \tau_r \ , \qquad (3.10\text{-b})$$

where one has denoted:

$$\chi_m = \frac{k_m - 1}{k_m + 1} = \chi_m \left[T_1, T_r\right] = \chi_m \left[T_1, p_r \tau_r\right] . \qquad (3.10\text{-c})$$

<u>Determination of the Mach number M_2</u>

In order to determine the final state Mach number M_2, one writes:

$$M_2^2 = \frac{u_2^2}{k_2 R_2 T_2} = \left(\frac{\rho_1}{\rho_2}\right)^2 \frac{v_1^2}{k_2 R T_2} = \frac{\overline{m}}{k_2} \frac{\tau_r^2}{T_r} = \frac{\overline{m}}{k_2} \frac{\tau_r}{p_r} , \qquad (3.11)$$

k_2 corresponding to the temperature T_2.

By replacing the ratios τ_r, p_r from (3.4), finally, it yields:

$$M_2^2 = \frac{2k_m + \overline{m}\left(k_m - 1\right)}{k_2\left(2\overline{m} - \left(k_m - 1\right)\right)} \qquad (3.12\text{-a})$$

or:

$$\left(\frac{1}{M_{1e}^2} + \frac{k_m - 1}{2}\right)\left(\frac{1}{M_{2e}^2} + \frac{k_m - 1}{2}\right) = \left(\frac{k_m + 1}{2}\right)^2 , \qquad (3.12\text{-b})$$

M_{1e} and M_{2e} being ***the equivalent Mach numbers*** of the two states, in case of transformation without change of gas composition:

$$M_{1e}^2 = \frac{k_1}{k_m} M_1^2 ; \ M_{2e}^2 = \frac{k_2}{k_m} M_2^2 . \qquad (3.13)$$

One can see the analogy with the case of constant specific heats [1;2;8] where:

$$k_1 = k_2 = k_m\left(T_1, T_2\right) ; M_1 = M_{1e} ; \ M_2 = M_{2e} . \qquad (3.14)$$

4. Calculation of the caloric capacities, enthalpies and entropies

By introducing the temperature ratio, $\theta = T / \Delta T_{\dim}; \Delta T_{\dim} = 1000K$, a temperature interval [0,2;6], and by writing the dimensionless caloric capacity $\overline{C_p}(\theta) = F_c(\theta)$ as functions on intervals $I_1 = [0.2\,;1]; I_2 = [1;6]$ as follows:

$$\frac{C_p(\theta)}{R_u} \equiv \overline{C_p}(\theta) = F_c(\theta) = \begin{cases} F_{c1}(\theta), if\ \theta \in I_1 = [0.2\,;1]; \\ F_{c2}(\theta), if\ \theta \in I_2 = [1;6]; \end{cases} \tag{4.1}$$

where the caloric capacity functions $F_{ci}(\theta),\ i = \overline{1,2}$, are:

$$F_{ci}(\theta) = \sum_{j=1}^{7} \alpha_{ji} \theta^{j-3}, j = \overline{1;7}, i = \begin{cases} 1,\ if\ \theta \in I_1 = [0.2\,;1]; \\ 2, if\ \theta \in I_2 = [1;6]; \end{cases} \tag{4.1-a}$$

one can introduce the coefficients $\alpha_{ji}, j = \overline{1;7}, i = \overline{1;2}$ as pure numbers. The only reference to Kelvin degree appears in connection with the temperature interval $\Delta T_{\dim}$. In particular, for $\Delta T_{\dim}$ = 1000K, one has the advantage of maintaining the mantissa of the NASA [6] existing data. Only the exponents are modified, the difference in orders of magnitude being reduced up to 18 orders and a few other small inconveniences are thus avoided.

<u>The dimensionless enthalpy</u>, $\overline{H}(\theta)$, is written as follows :

$$\overline{H}(\theta) = \begin{cases} \overline{b_{h1}}(\theta_{ref}) + F_{h1}(\theta), if\ \theta \in I_1 = [0.2\,;1]; \\ \overline{b_{h2}}(\theta_{ref}) + F_{h2}(\theta), if\ \theta \in I_2 = [1;6]; \end{cases} \tag{4.2-a}$$

where $F_{hi}(\theta), i = \overline{1;2}$, are the enthalpy functions on intervals, given by the relations:

$$F_{hi}(\theta) = \sum_{j \neq 2} \frac{\alpha_{ji}}{j-2} \theta^{j-2} + \alpha_{2i} \ln\theta, j = \overline{1;7}, i = \begin{cases} 1,\ if\ \theta \in I_1 = [0.2\,;1]; \\ 2, if\ \theta \in I_2 = [1;6]; \end{cases} \tag{4.2-b}$$

The dimensionless coefficients $\bar{b}_{hi}(\theta_{ref}),\ i = \overline{1;2}$, are:

$$\overline{b_{h1}}(\theta_{ref}) = \overline{H}(\theta_{ref}) - F_{h1}(\theta_{ref});\ \overline{b_{h2}}(\theta_{ref}) = \overline{b_{h1}}(\theta_{ref}) + F_{h1}(1) - F_{h2}(1). \tag{4.2-c}$$

In this way one obtains for the dimensionless enthalpy, $\overline{H}(\theta)$, more compact if- expressions.

The average caloric capacity at constant pressure for the temperature interval $[\theta_0,\theta]$, $C_{pm}(\theta_0,\theta)$, and its dimensionless correspondent, $\overline{C_{pm}}(\theta_0,\theta)$ are “3- if”- functions:

$$(\theta-\theta_0)\overline{C}_{pm}(\theta_0,\theta)=\begin{cases} F_{h1}(\theta)-F_{h1}(\theta_0),\ if\ \theta_0,\theta\in I_1; \\ F_{h1}(1)-F_{h2}(1)+F_{h2}(\theta)-F_{h1}(\theta_0),\ if\ \theta_0\in I_1,\theta\in I_2; \\ F_{h2}(\theta)-F_{h2}(\theta_0), if\ \theta_0,\theta\in I_2; \end{cases} \tag{4.3}$$

$$C_{pm}(\theta_0,\theta)=R_u\overline{C}_{pm}(\theta_0,\theta). \tag{4.3-a}$$

An average ratio of specific heats, $k_m(\theta_0,\theta)$, has also been defined by:

$$k_m(\theta_0,\theta)=\frac{\overline{C}_{pm}(\theta_0,\theta)}{\overline{C}_{pm}(\theta_0,\theta)-1}. \tag{4.4}$$

The dimensionless entropy.

As regards the gas entropy, this is a function, $S(\theta,p)$, of both temperature and pressure for any gas. For the molar entropy variation, $S(\theta,p)$ and its dimensionless correspondent, $\overline{S}(\theta,p)$, one applies the relation[7]:

$$\overline{S}(\theta,p)=\overline{b_s}(\theta_{ref},p_{ref})+F_s(\theta)-\ln\left(\frac{p}{p_{ref}}\right), \tag{4.5}$$

where this time $\overline{b_s}(\theta_{ref},p_{ref})$ and $F_s(\theta)$ are interval functions. One writes:

$$F_s(\theta)=\begin{cases} F_{s1}(\theta), if\ \ \theta\in I_1=[0.2\ ;1]; \\ F_{s2}(\theta),\ if\ \ \theta\in I_2=[1;6]; \end{cases} \tag{4.6}$$

with:

$$F_{si}(\theta)=\sum_{j\neq 3}^{7}\frac{\alpha_{ji}}{j-3}\theta^{j-3}+\alpha_{3i}\ln\theta,\ j=\overline{1;7},i=\overline{1;2}; \tag{4.6-a}$$

$$\begin{aligned}&\overline{b_{s1}}(\theta_{ref},p_{ref})=\overline{S}(\theta_{ref},p_{ref})-F_{s1}(\theta_{ref});\ \overline{b_{s2}}(\theta_{ref},p_{ref})=\\ &=\overline{b_{s1}}(\theta_{ref},p_{refr})+F_{s1}(1)-F_{s2}(1).\end{aligned} \tag{4.6-b}$$

In connection with the entropy variation with temperature, one has defined a ***referred to entropy average caloric capacity at constant pressure***,

$C_{pms}(\theta_0,\theta)$, for the temperature interval $[\theta_0,\theta]$, and its dimensionless correspondent, $\overline{C}_{pms}(\theta_0,\theta)$:

$$\overline{C}_{pms}(\theta_0,\theta).\ln\frac{\theta}{\theta_0}=\begin{cases}F_{s1}(\theta)-F_{s1}(\theta_0),\ if\ \theta_0,\theta\in I_1;\\ F_{s1}(1)-F_{s2}(1)+F_{s2}(\theta)-F_{s1}(\theta_0),\ if\ \theta_0\in I_1,\theta\in I_2;\\ F_{s2}(\theta)-F_{s2}(\theta_0),if\ \theta_0,\theta\in I_2.\end{cases} \tag{4.6-c}$$

$$C_{pms}(\theta_0,\theta)=R_u\overline{C}_{pms}(\theta_0,\theta). \tag{4.6-d}$$

A referred to entropy average ratio of specific heats, $k_{ms}(\theta_0,\theta)$, was also defined by:

$$k_{ms}(\theta_0,\theta)=\frac{\overline{C}_{pms}(\theta_0,\theta)}{\overline{C}_{pms}(\theta_0,\theta)-1}. \tag{4.7}$$

The coefficients $\alpha_{ji}, j=\overline{1;7}, i=\overline{1;2}$, are given in [3]. In dimensional form, these coefficients are given in [6].

5. Applications

For calculations, one starts with two estimates of the temperature θ_2, in order to obtain a sign change in the temperature function (3.4-a or 3.10-b) and apply a chord method. The value $\tau_{r\ const}$ is always an overestimation. Thus one has only to solve one variable equations. For $M_1= 8$, the extension of formulae in chapter 4 to the temperature interval [6;20], as given in Ref. [3] was necessary.

In Table 5.1, a comparison for the main parameters in case of constant and variable specific heats is given. The fluid is air; and the equations (3.4) are applied.

As one can see, the effect of the specific heat variation with temperature is important for higher Mach numbers (M_1>2), depending on the initial gas temperature θ_1, as well. Two initial temperatures, 300 K (θ_1 = 0.3; k_1= 1.4) and 600 K (θ_1 = 0.6; k_1= 1.37575) were selected.

Table 5.1(Air)

Flow and thermodynamic parameters at normal shock

M_1	θ_1	M_{1e}	$p_{r\ const}$	$p_{r\ var}$	θ_{2const}	$\theta_{2\ var}$	τ_{rconst}	τ_{rvar}
1.3	0.3	1.300462	1.8050	1.80614	0.35726	0.35721	0.659763	0.65926
	0.6	1.302940	1.8050	1.80645	0.71452	0.70792	0.659763	0.65313
2	0.3	2.004028	4.5000	4.51283	0.50625	0.50454	0.375000	0.37267
	0.6	2.015098	4.5000	4.52222	1.0125	0.97664	0.375000	0.35994

3	0.3	3.021579	10.3333	10.4280	0.80370	0.78743	0.25926	0.25170
	0.6	3.043616	10.3333	10.4540	1.60741	1.45464	0.25926	0.23645
5	0.3	5.100631	29.000	29.6990	1.74000	1.565978	0.20000	0.17998
	0.6	5.125082	29.000	29.6516	3.48000	2.942780	0.20000	0.16695
8	0.3	8.265574	74.500	77.3895	4.01601	3.401180	0.179687	0.14739
	0.6	8.268567	74.500	76.8554	8.03203	6.382471	0.179687	0.13847

The differences in the final temperature behind the shock wave – that can reach hundreds of Kelvin degrees – is increase with the initial temperature θ_1 At Mach number $M_1 = 3$, for example, the difference in the final temperature T_2 is 152.77 K for $T_1 = 600\ K$ and only 16.27 K for $T_1 = 300K$. As regards the pressure jump, there is some increasing due to the specific heat variation with temperature, but less than 1.5% at $M_1 = 3$. The equivalent Mach numbers are slowly larger than the current Mach numbers, the increment being larger at larger initial temperatures.

In Table 5.2, the enthalpy-average capacity $\overline{C_{pm}}$ (eqs.4.3), the entropy-average capacity $\overline{C_{pms}}$ (eqs.4.6-c) and the corresponding ratios k_m (eq.4.4) and k_{ms} (eq.4.7) for the same initial Mach numbers and temperatures as in Table 5.1 are given. In addition, the Mach numbers behind the shock wave, M_2, are also given.

As one can see, it results $\overline{C_{pm}} > \overline{C_{pms}}$, and $k_m < k_{ms}$, that is the isentropic exponent is larger then the enthalpy-average capacities ratio.

Table 5.2 (Air)

Flow and thermodynamic parameters at normal shock

M_1	θ_1	$\overline{C_{pm}}$	k_m	M_{2e}	M_2	$\overline{C_{pms}}$	k_{ms}
1.3	0.3	3.506705	1.398929	0.785687	0.785387	3.506504	1.398961
	0.6	3.706018	1.369546	0.783454	0.774886	3.704776	1.369716
2	0.3	3.536114	1.394303	0.575895	0.574723	3.532327	1.394893
	0.6	3.815236	1.355209	0.568510	0.559342	3.803014	1.356759
3	0.3	3.631567	1.380001	0.469440	0.466075	3.606006	1.383728
	0.6	3.970876	1.336601	0.457743	0.447259	3.930077	1.341287
5	0.3	3.896604	1.345231	0.397073	0.389229	3.785813	1.358961
	0.6	4.231895	1.309415	0.384565	0.371916	4.122542	1.320251
8	0.3	4.211185	1.311411	0.360719	0.349121	3.984740	1.335037
	0.6	4.478026	1.287519	0.351019	0.336623	4.288511	1.304088

5.1 The state of stagnation. Pressure losses through normal shock waves

The state of stagnation is a state corresponding to null velocity. From this condition and from the equation of energy (2.3) one gets the stagnation temperature T^* or θ^*:

$$h_1(T_1)+\frac{1}{2}u_1^2=h_1(T^*);\ h_1(T^*)=h_1(T_1)+\int_{T_1}^{T^*}c_{p1}(T)\,dT. \tag{5.1}$$

By introducing the average specific heats, one obtains for the two states:

$$c_{pmi}(T_i,T^*)T_i+\frac{1}{2}u_i^2=c_{pmi}(T_i,T^*)T^*;\ i=\overline{1;2},$$

$$\frac{\theta^*}{\theta_1}=1+\frac{k_{m1}^*-1}{2}M_{1e}^{*\,2};\ M_{1e}^{*\,2}=\frac{k_1}{k_{m1}^*}M_1^2, \tag{5.1-a}$$

$$T_1^*=T_2^*=T^*;\ c_{p1}(T)=c_{p2}(T),$$

the two stagnation temperatures being equal as the gas composition is unchanged.

As regards the pressure at stagnation state, by definition, one takes isentropic gas evolutions between the temperatures $T_i, i=\overline{1;2}$, and T^*, the isentropic exponent being $k_{msi}(T_i,T^*); i=\overline{1;2}$; (see (4.7)).

By denoting:

$$k_{msi}(T_i,T^*)=k_{msi}^*;\ \overline{C}_{p\,m\,s\,i}(T_i,T^*)=\overline{C_{pmsi}^*},\ i=\overline{1;2}, \tag{5.2}$$

one can write the ratio of stagnation pressures under the form:

$$p_i^*/p_i=(\theta^*/\theta_i)^{\overline{C_{pmsi}^*}},\ i=\overline{1;2}. \tag{5.3}$$

The ratio of stagnation pressure is then:

$$p_2^*/p_1^*=p_r\,(\theta^*/\theta_2)^{\overline{C_{pms2}^*}}\,(\theta_1/\theta^*)^{\overline{C_{pms1}^*}}, \tag{5.3-a}$$

the entropy variation being given by the equation:

$$\Delta\overline{S}=\ln(T_r^{\overline{C_{pms}}}/p_r);\ T_r=\theta_2/\theta_1, \tag{5.4}$$

Thus three entropy average heat capacities: $\overline{C}_{p\,m\,s\,i}(T_i,T^*), i=\overline{1;2}$, and $c_{pms}(T_1,T_2)$ are involved.

We also introduce an ***average stagnation temperature***, T_a^*, defined bellow:

$$c_{pm}\left(T_1,T_2\right)T_i+\frac{1}{2}u_i^2=c_{pm}\left(T_1,T_2\right)T_a^*,\ i=\overline{1;2},$$
$$\frac{\theta_a^*}{\theta_1}=1+\frac{k_m-1}{2}M_{1e}^2; \tag{5.5}$$

by using the average specific heat $c_{pm}\left(T_1,T_2\right)$. In terms of the average stagnation temperature, ***average stagnation pressures***, $p_{ia}^*, i=\overline{1;2}$, can also be defined, and their ratio can be simpler expressed, as follows:

$$p_{2a}^*/p_{1a}^*=\sigma;\ \sigma=e^{-\Delta\overline{S}}. \tag{5.6}$$

where σ is a ***pressure loss coefficient*** depending on the entropy variation through the shock wave only.

In Table 5.3 the ratios of the stagnation temperatures θ^*/θ_1 and θ_a^*/θ_1, the variation of the dimensionless entropy, $\Delta\overline{S}$, and the ratios of the stagnation pressures after and before the shock wave jump, p_2^*/p_1^*, (representing the pressure losses through the normal shock wave) are compared for constant and variable caloric capacities. One can see that the two stagnation temperatures, θ^* and θ_a^* are very closed; the differences do not exceed 1% even for high Mach numbers (M_1 = 8). In exchange, some formulae are much simpler in terms of the mean stagnation temperature.

Table5.3 (Air)

Flow and termodynamic parameters at normal shock. Comparisons for constant [5] and variable caloric capacities

M_1	θ_1	$(\theta^*/\theta_1)_c$	$(\theta^*/\theta_1)_{var}$	$(\theta_a^*/\theta_1)_{var}$	$\Delta\overline{S}_{ct}$	$\Delta\overline{S}_{var}$	$(p_2^*/p_1^*)_{var}$	$(p_{2a}^*/p_{1a}^*)_{var}$
1.3	0.3	1.3380	1.336689	1.337337	0.02084	0.020897	0.979314	0.9793198
	0.6	1.3380	1.310915	1.310917	0.02084	0.021411	0.978766	0.9788166
2	0.3	1.8000	1.789822	1.791787	0.32729	0.329447	0.719166	0.7193214
	0.6	1.8000	1.717380	1.721187	0.32729	0.343819	0.708618	0.7090573
3	0.3	2.8000	2.729000	2.734697	1.11369	1.135309	0.320994	0.3213228
	0.6	2.8000	2.551240	2.559068	1.11369	1.209683	0.297640	0.2982918
5	0.3	6.0000	5.468163	5.490853	2.78521	2.954786	0.051791	0.0520898
	0.6	6.0000	5.053860	5.063632	2.78521	3.204330	0.040444	0.0405861
8	0.3	13.800	11.61501	11.63777	4.76912	5.350805	4.7164D-3	4.7443D-3
	0.6	13.800	10.81894	10.83111	4.76912	5.799841	3.0178D-3	3.0280D-3

One obtains lower stagnation temperatures for variable caloric capacities, the diminution being more important for larger initial temperature of gas.

As regards the entropy, at lower initial temperatures the adverse effect of pressure increase and isentropic exponent increase leads to a slightly smaller entropy growth, as one can also see from the formula (5.4).

6. Conclusions

The effect of the variation of the specific heats with temperature on the intensity of normal shock wave is important, leading to large differences in the temperature values and pressure losses, for higher Mach numbers *(M_1 >2), depending on the initial gas temperature θ_1, as well*. Although the expressions of the caloric capacities, enthalpies and entropies as functions of temperature are rather complicated, *the main analytical formulas for pressure, density etc. ratios are preserved in terms of the equivalent Mach numbers* introduced by authors [3;4]. The calculations now necessary in terms of the new introduced parameters are simplified. One recommends the use of dimensionless parameters.

The extension of the presented study for the case when the gas composition changes is possible in terms of the defined equivalent parameters.

For $M_1 = 8$, the air properties were calculated by using NASA values for oxygen and nitrogen at temperatures higher the 6000K. The dissociation, ionization etc. at high temperatures were for the moment neglected.

An ***average stagnation temperature*** and an ***average stagnation pressure*** were also defined presenting some advantages in order to simplify the calculation.

R E F E R E N C E S

[1] *E. Carafoli, V .N. Constantinescu*, "Dinamica fluidelor compresibile" (Dynamics of compressible fluids), Editura Academiei Române, Bucureşti ,1984. (in Romanian)

[2] *C. Berbente, N. V. Constantinescu*, "Dinamica gazelor şi aerotermochimia"(Gasdynamics and aerothermochemistry), Institutul Politehnic Bucureşti, 1980; 1985. (in Romanian)

[3] *C. Berbente, D. Crunţeanu, D. Racoţi*, "On the interpretation of some thermodynamic expressions for ideal gases in terms of average values in temperature intervals", Termotehnica, Anul XIII, nr.1/2009, p.33-42, Ed.AGIR, Buc. 2009, ISSN 1222-4057.

[4] *C. Berbente, S. Berbente*, "An Extension of Hugoniot – Rankine Relations for Reacting and Nonreacting Flow Considering the Variation of Specific Heats with Temperature", The 32nd "CAIUS IACOB" Conference on Fluid Mechanics and its Technical Applications", Bucharest, 16-17 Oct. 2009.

[5] *** Handbook of Supersonic Aerodynamics, NAVORD REPORT 1488, **vol. 2**, The Johns Hopkins University Applied Physics Laboratory, Maryland, USA,1950.

[6] *Mc.Bride B.J., Zehe, M.J. and Gordon S.* "NASA Glenn Coefficients for Calculating Thermodynamic Properties of Individual Species", NASA/TP-2002-211556, September 2002.

[7] *D. Racoți and C. Berbente*,"The Analytical Calculations of the Diagram Enthalpy-Entropy",(in Romanian), Proceedings of the 31-st Congress of the American Romanian Academy of Arts and Sciences, Brașov (2007).
[8] *F.A, Williams*, "Combustion Theory", Adison Wesley Palo Alto, London, 1967.