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SYNCHRONIZATION AND CONTROL IN THE DYNAMICS OF DOUBLE LAYER CHARGE STRUCTURES. AUTONOMOUS STOCHASTIC RESONANCE

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În această lucrare se prezintă obervații experimentale şi rezultate numerice asupra rezonanței stocastice autonome în sisteme de sarcini electrice de tip strat dublu. Structura de acest tip este generată în spațiul inter-anodic al unei descărcări duble simetrice. Dinamica sistemului este studiată prin intermediul luminii emise din zona de interes a plasmei. Într-un domeniu restrâns al polarizării relative inter-anodice, structura tip strat dublu prezintă o dinamică de tranziție între o stare staționară şi comporare periodică. Suprapunerea unui zgomot Gaussian poate induce o asemenea tranziție fără aplicarea unui semnal periodic din exterior. Cu creşterea polarizării relative, curba raportului semnal/zgomot ca funcție de nivelul zgomotului prezintă un maxim, comportare caracteristică fenomenului de rezonanță stocastică autonomă. Modelarea sistemului este realizată pe baza unui model de oscilator van der Pol modificat, perturbat de zgomot Gaussian. Valorile numerice obținute sunt în bună concordanță cu rezultatele experimentale.

In this paper we present experimental observations and computational results on autonomous stochastic resonance in a double layer (DL) charge structure. The DL under investigation is generated in the inter-anode space of a twin electrical discharge. We investigate the dynamics of this structure as reflected in the light emission from the DL area of the plasma. In a restricted range of the inter-anode biasing, the DL shows a transition between steady state and periodical dynamics. The superposition of Gaussian noise can induce such a transition without any periodic signal being injected into the system. With increasing of the biasing, the signal to noise ratio versus the noise level is a curve with a maximum, characteristic of stochastic resonance. As computational model, we consider a modified van der Pol oscillator perturbed by Gaussian noise. This model is found to well reproduce the experimentally observed dynamics.

Keywords: autonomous stochastic resonance, van der Pol oscillator, double layer

1. Introduction

The transition from steady state to periodical dynamics under the influence of noise in the absence of an injected periodical perturbation is known as autonomous stochastic resonance (ASR). During the twenty five years since the

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first mention of stochastic resonance (SR) in connection with the apparent periodicity of ice ages on earth, the subject has received considerable attention both theoretically and experimentally. The phenomenon can take place in nonlinear systems only and consists in a noise induced cooperative process leading to resonance between a weak periodic modulation and the stochastic signal. As a result, the small, hardly detectable deterministic modulation becomes clearly observable after the addition of noise. The original and still most popular model considers a weak periodical signal and a noise (usually Gaussian) simultaneously injected into a nonlinear system characterized by a two well potential [1-4]. While the model is applicable for many observed situations, systems where noise can lead to periodical dynamics without the necessity of injecting a periodical signal from outside were reported [5, 6]. This phenomenon was initially described in the last decade of the twentieth century [7-9] and was named autonomous stochastic resonance.

In this paper we present the analysis of experimental observations on the stochastically induced harmonic oscillations in a DL charge structure generated in the inter-anode space of a twin electrical discharge with suitable biasing of one anode against the other. A computational analysis based on a modified van der Pol system working with parameters in the range corresponding to a supercritical Hopf bifurcation is presented. Addition of Gaussian noise under these circumstances can induce the transition from steady state to periodic dynamics.

2. Experimental set-up and results

A sketch of the experimental device is shown in Fig.1. Two independent electric discharges at low pressure (80 mTorr) in flowing Argon are running between the electrodes K1-A1 and K2-A2 respectively, placed in the same glass tube. A dc voltage source maintains a constant biasing U of one anode against the other. A perturbed regime can be generated if a small variable voltage is connected in series with the dc biasing. In the present study, the perturbation is generated by a Gaussian noise supply (denoted U_n on Fig.1), with an equivalent standard deviation U_s=10V$_{rms}$, coupled to the discharge through an attenuation network. The DL is the source of oscillations in the inter-anode plasma and its behavior can be efficiently controlled by the characteristics of the biasing [10-12].

In this study, we are interested in a small range of inter-anode biasing where a transition between steady state and periodical dynamics is taking place. From the dynamical point of view, in the transition region, a limit cycle can disappear if the control parameter (related to the biasing voltage) is adjusted below a critical value. At higher biasing, the DL behavior is more complex and correspondingly, its dynamics is different showing period doubling sequences and chaos. For this range, a system of two coupled van der Pol oscillators was found to satisfactorily model the experimental dynamics [10].

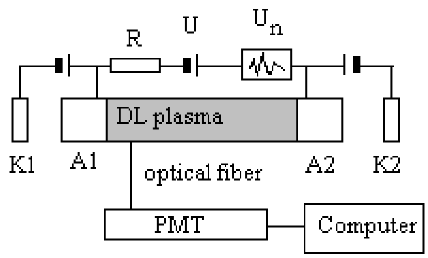

Fig. 1. Sketch of the experimental set-up.

We study the dynamics of the DL as reflected in the temporal behavior of the light emitted from the DL structure, collected by an optical fiber placed in the neighborhood of the DL region and recorded by a photomultiplier (PMT). Data acquisitions are analyzed using a personal computer. The main method of investigation is by spectral analysis of the PMT response. The stability of our experimental system allowed for a sixteen time repetition of the data acquisition process and the present analysis is based on the resulting ten sample averaged power spectra.

We focus on the transition from steady state to periodical oscillations which takes place for biasing in the range between 17 and 20V where a Hopf bifurcation is observed.

For values of the biasing below a certain threshold, the DL exists in a steady state. In this range, the intrinsic oscillation is only present as a transient behavior corresponding to the evolution of the system towards its steady state. The addition of Gaussian noise to the system in the steady state can induce the transition towards a harmonic oscillation through a Hopf bifurcation. For a particular noise level the regular dynamics shows an optimum degree of coherence that is interpreted as fingerprint of autonomous stochastic resonance.

The autonomous stochastic resonance behavior can be detected by various measures [1, 13]. The one we use in the present work is the dependence of the signal to noise ratio (SNR) on the noise level.

Spectra of the type presented in Fig. 2a where used to generate the diagram in Fig. 2b that shows the SNR as function of the noise level. The SNR (in dB) is computed from the power spectra as proportional to the logarithm to base ten of the ratio between the signal power and the noise power at the frequency of the harmonic component:

$$SNR(\mathrm{dB}) = 10 \log_{10} \frac{S}{N} . \tag{1}$$

Here we use the approximate method to estimate *S/N* illustrated on Fig. 2a. The signal and the noise level are measured from a common base line. The curves of the SNR versus the noise level, for different values of the dc voltage in the range of interest, are shown in Fig.2b. The experimental values are represented by the marks while the lines are simply drawn for eye guiding. The graphs clearly show two different responses of the system to the noisy perturbation. Below a threshold of about 18.5V, the curves show the well-known SR shape presenting a clear maximum. The amplitude of the generated oscillation is maximal for an optimum value of the noise. Above the mentioned threshold, the SNR is continuously decreasing with increasing of the noise level. This is the expected behavior because above 18.5V the dynamics of the system consists of regular oscillations and the added noise only degrades the regularity.

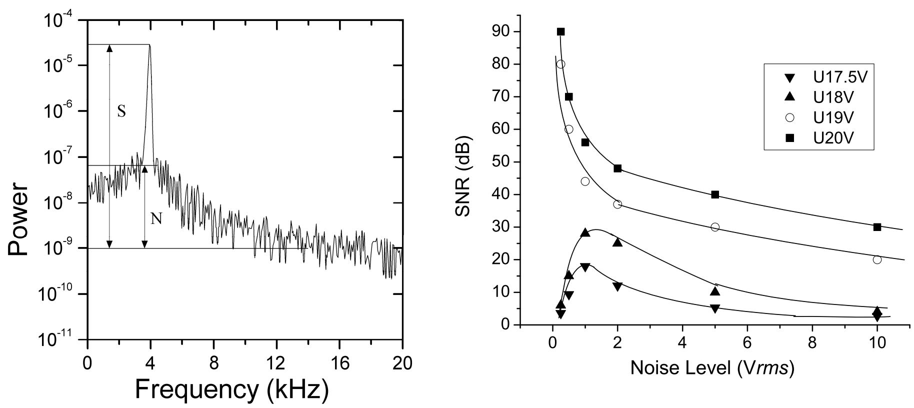

Fig. 2 (a) Power spectrum of the experimental data for the noise level corresponding to the maximum of the U=18V curve in Fig. 2b (sixteen spectra average); (b) Curves of SNR versus noise level for the specified experimental values

Stochastic resonance-like behavior in electrical discharges was previously reported, in weakly ionized radio frequency magnetoplasma [14] as manifested in the spontaneously generated nonlinear ionization drift waves; it was also observed in waves of stratification generated by a convective instability of the positive column in a simple configuration electrical discharge in neon [15].

3. Computational model

As computational model, we consider a slightly modified noise perturbed van der Pol oscillator adapted to take into account the effect of the biasing as control parameter. This model is supported by some works (e.g. [16]) that suggest that the dynamics of a DL can be described by a van der Pol oscillator. Accordingly, the following system is analyzed:

$$\dot{x}_1 = ax_2 - mx_1 + D\xi(t) \tag{2}$$

$$\dot{x}_2 = -bx_2(x_1^2 - 1) - cx_1 \tag{3}$$

where a, b, c and D are positive parameters. Here, $\xi(t)$ is a Gaussian noise with zero mean and delta correlation:

$$< \xi(t)\xi(t') >= \delta(t - t') \tag{4}$$

In the system (2-3), x_1 stands for the electric charge distribution across the DL. As the DL is generated by the biasing, the effect thereof will be mostly manifest in the charge distribution. This is the reason why, both the dc biasing term and the noise term are introduced in the charge/current equation (2) rather than in the current/voltage equation (3).

For the selected range of the parameters and in the absence of noise (D=0), the system (2-3) presents two distinctive dynamics, depending on the control parameter m. Below some threshold value the system presents periodic oscillation while above that value, it evolves by damped oscillation towards a stationary state.

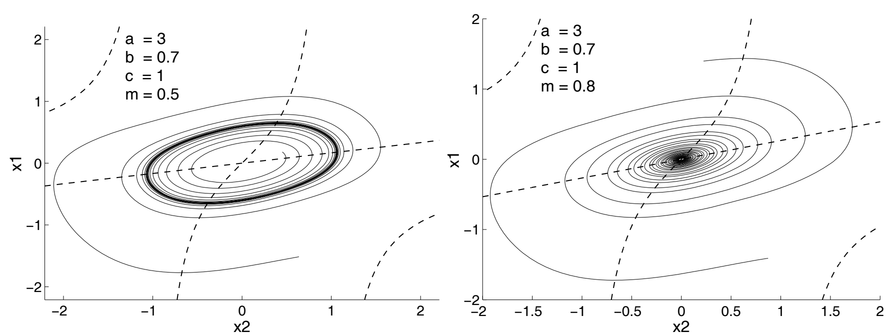

Fig.3 Phase portraits of the system (2-3) without noise (D=0) for the values of the parameters indicated on each graph: a) limit cycle oscillation; b) stationary state (spiral sink).

Fig. 3 shows phase portraits of the system (2-3) without noise for the values of the parameters indicated on each diagram: limit cycle oscillation (a) and stationary state (spiral sink) (b). The broken lines represent the nullclines of the system.

The dynamics of the model system is similar to the behavior of the experimental one with respect to their control parameters. However, in the experimental system the transition from steady state to periodic oscillation takes place with the increasing of the biasing (U), while in the model system, the same transition takes place for decreasing of the control parameter (m).

Similarity also exists in the dependence of the fundamental frequency of oscillation on the control parameter. As presented in a previous work [10], the fundamental frequency of the DL shows linear dependence on the biasing for a large range of the values, namely between the threshold $U_{th} \approx 18.5$V and about 30V.

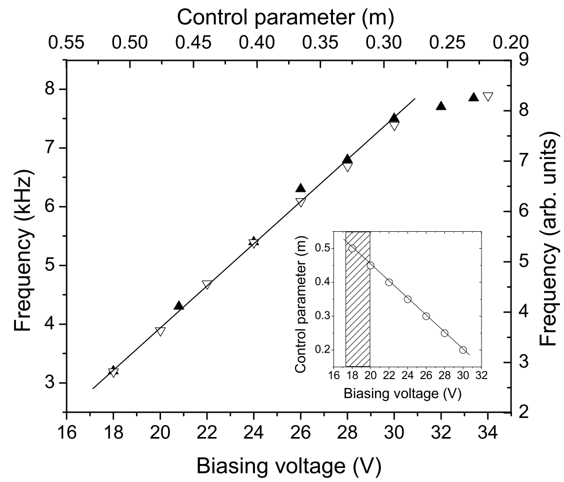

Fig. 4 Frequency versus control parameter curves: experimental, solid triangles and computed, open triangles. Inset is shown the $m=f(U)$ dependence; the shaded area represents the range of interest for this study.

The change of the oscillation frequency of the model system (2-3) with respect to the control parameter m in the absence of noise is also linear between the threshold value $m_{th} \approx 0.51$ and m=0.30. Below this value, the frequency quickly evolves towards saturation. Based on these facts, we consider a linear dependence of m on U namely $m = p(q-U)$. By fitting the linear parts of the two frequency versus the control parameter curves (experimental and computed), as shown in Fig.4, we find p=0.02 and q=44. Inset on Fig.4 the $m=f(U)$ dependence is shown.

It should be observed that the range of validity of the model extends over a considerably larger domain than that used in this study - shaded on this graph. We consider this wide range correspondence as solid argument to justify the extension of the model for the region of stationary state dynamics.

Fig. 5 shows the behavior of the model with respect to the injected noise, corresponding to the experimental curves in Fig. 2. For values of m in the stationary state range (m>0.51) the curves of the SNR versus the noise level (D) show a maximum characteristic of stochastic resonance (Fig. 5b). This is in agreement with the experimental stochastic resonance curves obtained for values of U below the oscillation threshold (Fig. 2b). For values of m below the threshold ($m \approx 0.51$), in the oscillatory regime, the curves of the SNR versus the noise level show the expected monotonous decreasing with increasing of the noise level.

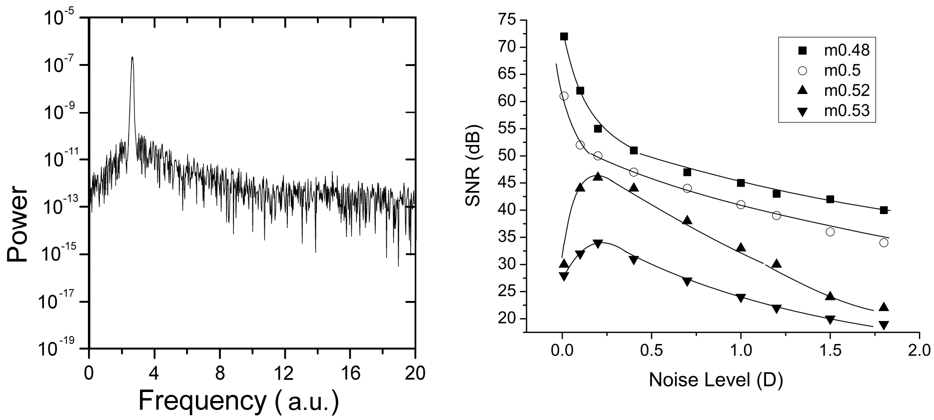

Fig. 5 Results of the model corresponding to the experimental curves in Fig.2; a) Power spectrum of the computed data (ten sample average) for the noise level corresponding to the maximum of the m=0.52 curve in Fig. 4b;(b) Curves of SNR versus the noise level computed for values of m in the range 0.48-0.53.

This is in agreement with the experimental behavior observed for U larger than the threshold.

4. Conclusions

We observed that the behavior of a DL dynamics with respect to injected noise is changing as the inter-anode biasing transcends a certain threshold value that separates stationary and oscillatory dynamics. Although the curves in Fig.2b are drawn through a reduced number of points, restricted by the possibilities of the available noise generator, we consider the experimental results fully conclusive. The numerical treatment of this behavior is based on a modified van der Pol system working in the range of the parameters corresponding to a supercritical Hopf bifurcation. Addition of Gaussian noise under these circumstances can cause the transition from steady state to periodic dynamics. Depending on the value of the control parameter, in the oscillatory regime, the curves of the SNR versus the noise level show the expected monotonous decreasing with increasing of the noise level corresponding to a degrading of the spectrum. In the stationary state, the curves show a well defined maximum, characteristic of autonomous stochastic resonance. The good agreement between the experimental data and the dynamical treatment demonstrates that a properly modified van der Pol oscillator represents a suitable model for the behavior of the DL charge structure in the considered circumstances.

R E F E R E N C E S

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