AN ITERATIVE NONLINEAR REGRESSION FOR POLARIZATION/DEPOLARIZATION CURRENT

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Pentru a defini legătura dintre răspuns – variabila dependentă y şi cauză predeterminată – variabila independentă x, se pot folosi funcţia lineară sau funcţia concavă respective convexă afectate de erori aleatoare. Folosind ajustarea prin metoda celor mai mici pătrate aplicată unor submulţimi de date, se propune o legătură bazată pe un polinom de grad minim m, adică regresie lineară sau bazată pe o curbă exponenţială, adică regresie nelineară.

S-au utilizat tehnici de testarea domeniului de timp dielectric, adică RVM – măsurarea voltajului returnat, respectiv PDC – măsurarea curentului de polarizare "by the power utilities" pentru stabilirea condiţiei transformatorilor de separare (izolare) ulei – hârtie.

PDC şi RVM Nova software furnizează un "toolbox" pentru a încărca setul de date, pentru a filtra datele şi pentru a estima parametrii pe fiecare submulţime.

To define the relationship between the response – a random or dependent variable y and a predetermined or independent variable x, a straight line or concave respectively convex functions affected by random errors must be used.

A locally weighted low-degree polynomial regression i.e. linear is fit to several subsets of data using linear least square fitting and a first-degree polynomial or the exponential curve.

Time-domain dielectric testing techniques, namely the Return Voltage Measurement (RVM) and Polarisation and Depolarisation Current (PDC) measurement are being used by the power utilities for assessment of the condition of transformers oil-paper insulation.

PDC RVM Nova software provides a toolbox to load the dataset, to filter the data and to estimate the parameters of each subset.

Keywords: transformer oil/paper insulation, polarization/depolarization current, nonlinear regression, "peeling-off" exponentials estimates.

Introduction

Transformer is one of the most important and costly apparatus in a power system. The reliable and efficient fault-free operation of the high-voltage transformer has a decisive role in the availability of electricity supply. The transformer oil/paper insulation gets degraded under a combination of thermal, electrical, chemical, mechanical, and environmental stresses during its operation.

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In recent years, there has been growing interest in the condition assessment of transformer insulation.

To estimate the status of transformer insulation it has been elaborated a modern diagnose method based on measuring and processing of polarisation/depolarisation currents which characterize the transformer.

The PDC-Analysis is a non-destructive method for determining the transformer's moisture content in the solid insulation material like paper and pressboard. A DC(direct current) voltage step of some 2000 V is applied between HV and LV windings of the transformer during a certain time *TP*, the so-called polarization duration. Thus, a charging current of the transformer capacitance, i.e. insulation system, the so-called polarization current, flows. It is a pulse-like current during the instant of voltage application which decreases during the polarization duration to a certain value given by the DC conductivity of the insulation system. After elapsing the polarization duration *TP*, the dielectric is short circuited via the ammeter. Thus, the discharging current jumps to a negative value, which goes gradually towards zero. Both kinds of currents called relaxation currents are stored in the apparatus. We analyze this currents by fitting them using a mathematical model.

The problem of fitting experimental data by a linear combination of exponential is very common in technical and biological researches.

The corresponding regression model reflecting the effect of random errors on this fitting is non-linear with respect to its parameters:

$$y_i = \alpha + \sum_{j=1}^{k} \beta_j \rho_j^i + e_i, i = 1,...,n \quad (1)$$

where k is the number of exponentials; n is the number of observations; here $\rho_j = \exp(-\lambda_j)$, α and β_j are constants and e_i are random errors with zero mean and variance σ^2.

The iterative methods of curve-fitting for data to sum of *n*-exponentials require initial estimates of the parameters involving linear zed by expanding in a Taylor series.

1. Model assumptions

The model (1) may be written more generally as:

$$y = \alpha + \beta g(x) + e \quad (2)$$

i.e. $y_i = \alpha + \beta\, g(x_j) + e_i, i = 1...n, j = 1...k$

where $g(x)$ represents either a concave or convex continuous function.

The null hypothesis may be:

$$H_0: g(x) = x \quad (3)$$

under various assumption concerning the probability distribution of the e's. The random errors e_i are independent and symmetrical, but not necessarly identically distributed around 0 with continuous distribution function:

$$F_i(z) = P\{e_i < z\}$$

$$F_i(0) = 1/2, \mathrm{i} = 1,..,n$$

For example, in normal regression analysis, the problem is to test for linearity against specific alternatives (i.e. concave or convex) if the errors are i.i.d. (independently and identically distributed) with unknown variance σ^2 or if they have a covariance structure known except for a scale factor.

However with a sufficient number of replicated points, an independent estimate of σ^2 or of the scale factor is available and then it is possible to test for linearity against the mentioned alternatives.

3. Test statistics

Let the pairs $\{y_j, x_i\},\ j = 1...k$ be ordered so that $x_1 < x_2 < ... < x_n,\ i = 1...n$ and define three disjoint intervals T_1, T_2, and T_3 on the x–axis such that containing the first n_1 observed the x's lie in T_1, the next n_2 ordered the x's lie in T_2 and the remaining $n_3 = n - n_1 \text{-} n_2$, ordered the x's lie in T_3.

Let us define the statistics:

$$h_{ijk} = h(e_i, e_j, e_k) = \begin{cases} 1, \text{ if } (y_j - y_i)/(x_j - x_i) > (y_k - y_i)/(x_k - x_i) \\ -1, \text{ if } (y_j - y_i)/(x_j - x_i) < (y_k - y_i)/(x_k - x_i) \\ 0, \text{otherwise} \end{cases} \quad (4)$$

The proposed statistics are:

$$H_n = (n_1 n_2 n_3)^{-1} \sum h_{ijk} \quad (5)$$

for all $n_1 n_2 n_3$ sets of pairs.

***The decision*:** the regression is concave if $H_n > 0$ respectively convex if $H_n < 0$.

The estimators r_i^* of ρ_i are the roots of the k^{th} degree polynomial:

$$r^{*k} - \frac{|A_1^*|}{|A^*|} r^{*(k-1)} - ... - \frac{|A_k^*|}{|A^*|} = 0 \quad (6)$$

with coefficients the difference of successive pairs of the experimental data $y_i,\ i = 1,...,n$:

$$Y'_h = Y_{2h} - Y_{2h-1},\ h = 1,2,\ldots,2k \tag{7}$$

where

$$Y_{2h} = \sum_{i=m+1+h-k}^{n+h-2k} y_i \ ; \quad Y_{2h-1} = \sum_{i=h}^{m+h-k-1} y_i\ , \quad n = 2m \tag{8}$$

respectively,

$$Y_{2h} = \sum_{i=m+1+h-k}^{n+h-2k} y_i \ ; \quad Y_{2h-1} = \sum_{i=h}^{m+h-k} y_i, \quad n = 2m+1 \tag{9}$$

Observation. For the small size of experimental data or the value of σ^2 too large, the lineary interpolated values, the pair is introduced between each successive points: (t_i^*, y_i^*), with $t_i^* = x_i^*$, where $y_i^* = \frac{y_i + y_{i+1}}{2}$ respectively $t_i^* = \frac{t_i + t_{i+1}}{2}$ or $x_i^* = \frac{x_i + x_{i+1}}{2}$.

4. Iterative method

Iterative method using initial estimates involves l.s.–process after linearization by expanding in a Taylor series in the nonlinear regression ([5]).

A method for obtaining initial estimates of the parameters in (2) is applied using a least square (l.s.) “peeling-off” technique ([3]).

By the l.s. method the estimates of a_k and b_k can be obtained by fitting a straight line to the last three data points. Then :

$$y_{residual} = y - a_k e^{-b_k t} \tag{10}$$

By taking the next three points for $y_{residual}$ against $t\ (= x)$ and fitting a straight line to them by l.s. method, the new $y_{residual}$ is obtained corresponding to the determined a_{k-1} and b_{k-1}, etc:

$$\text{New } y_{residual} = y_{residual} - a_{k-1} e^{-b_{k-1} t} \tag{11}$$

The remaining data points are used to calculate a_1 and b_1.

The ordinary l.s (OLS) estimator respectively the weighted l.s. (WLS) estimator of β – the vector of regression parameters are given in the literature: ([6], [7]).

$$\hat{\beta}_{OLS} = (X'X)^{-1} X'y \tag{12}$$

respectively

$$\hat{\beta}_{WLS} = (X'WX)^{-1} X'Wy \tag{13}$$

with W= block diag $\left(w_1 I_{n_1}, \ldots, w_1 I_{n_k}\right)$

$$w_i = u_i^{-1} = \left(\frac{1}{n_i}\sum_{j=1}^{n_i}\hat{e}_{ij}^2\right)^{-1}, \text{ the weights} \tag{14}$$

It is the *t x t* identity matrix.

For nonlinear least squares fitting to a number of unknown parameters, linear least squares fitting may be applied iteratively to a linearized form of the function until convergence is achieved. However, it is often also possible to linearize a nonlinear function at the outset and still use linear methods for determining fitted parameters without resorting to iterative procedures. This approach does commonly violate the implicit assumption that the distribution of errors is normal, but often still gives acceptable results using normal equations, a pseudoinverse, etc. Depending on the type of fit and initial parameters chosen, the nonlinear fit may have good or poor convergence properties. If uncertainties (in the most general case, error ellipses) are given for the points, points can be weighted differently in order to give the high-quality points more weighted.

Vertical least squares fitting proceeds by finding the sum of the *squares* of the *vertical* deviations R^2 on a set of *n* data points

$$R^2 \equiv \sum\left[y_i - f\left(x_i, \alpha_1, \alpha_2, \ldots, \alpha_n\right)\right]^2 \tag{15}$$

from a function f. Note that this procedure does *not* minimize the actual deviations from the line (which would be measured perpendicular to the given function). In addition, although the *unsquared* sum of distances might seem a more appropriate quantity to minimize, use of the absolute value results in discontinuous derivatives which cannot be treated analytically. The square deviations from each point are therefore summed, and the resulting residual is then minimized to find the best fit line. This procedure results in outlying points being given the disproportionately large weighting.

The condition for R^2 to be a minimum is that

$$\frac{\partial\left(R^2\right)}{\partial\alpha_i} = 0,\ i = 1, \ldots, n \tag{16}$$

5. Software description

The polarization/depolarization currents are affected by noise so it must be processed. After this, it must be transformed into a computed function of this type: $I_{pol}(t) = I_0 + \sum_{i=0}^{5} Ip_i e^{\frac{-t}{Tp_i}}$.

The processing of the currents it is made in two stages. First the signal is filtered, eliminating the noise partially or even totally and then to the next stage, the signal is parameterized.

The 4 methods of signal filtering are: Moving Average, Lowess, Loess and Savitzky-Golay. These methods are also used in Curve Fitting Tool software included in MATLAB suite.

After the signal is filtered, it must be parameterized by transforming it into a computed function using a mathematical procedure for finding the best-fitting curve to a given set of points by minimizing the sum of the squares of the offsets ("the residuals") of the points from the curve.

The following options are available from the application's main window:

- import measured data from text file;
- eliminate non-significant data from the measured current and save the new data set;
- graphic of the measured polarization current;
- apply one of the 4 fitting methods and the possibility to apply it on a custom interval;
- fitting measured current using 1 to 5 fitting parameters;
- graphic comparison between measured and computed polarization current;
- graphic comparison between computed polarization and depolarization current;
- save the computed current and the fitting parameters.

6. Experimental data

Table 1

Polarisation current

T **[s]**	I_{pol} **[A]**	***T*** **[s]**	I_{pol} **[A]**	***T*** **[s]**	I_{pol} **[A]**	***T*** **[s]**	I_{pol} **[A]**	**T [s]**	I_{pol} **[A]**
1,1	$4414*10^{-9}$	9,6	$3267*10^{-9}$	80,4	$785*10^{-9}$	391,2	$320*10^{-9}$	1900,3	$262*10^{-9}$
1,2	$4401*10^{-9}$	13,1	$3162*10^{-9}$	86,6	$697*10^{-9}$	421,2	$316*10^{-9}$	2000,7	$260*10^{-9}$
1,3	$4384*10^{-9}$	15,1	$3053*10^{-9}$	90,8	$650*10^{-9}$	460,6	$311*10^{-9}$	2200,6	$256*10^{-9}$
1,4	$4363*10^{-9}$	17,2	$2941*10^{-9}$	97,0	$594*10^{-9}$	501,0	$307*10^{-9}$	2400,5	$252*10^{-9}$
1,5	$4339*10^{-9}$	19,3	$2826*10^{-9}$	101,1	$565*10^{-9}$	550,7	$302*10^{-9}$	2600,4	$248*10^{-9}$
1,6	$4305*10^{-9}$	21,3	$2708*10^{-9}$	110,4	$530*10^{-9}$	600,4	$298*10^{-9}$	2800,3	$244*10^{-9}$
1,7	$4264*10^{-9}$	23,4	$2590*10^{-9}$	120,8	$503*10^{-9}$	651,2	$287*10^{-9}$	3000,2	$239*10^{-9}$
1,8	$4216*10^{-9}$	24,5	$2473*10^{-9}$	131,1	$480*10^{-9}$	700,9	$281*10^{-9}$	3301,6	$233*10^{-9}$
1,9	$4162*10^{-9}$	26,5	$2358*10^{-9}$	140,5	$462*10^{-9}$	750,6	$277*10^{-9}$	3602,0	$226*10^{-9}$
1,9	$4102*10^{-9}$	28,6	$2246*10^{-9}$	150,8	$447*10^{-9}$	800,3	$275*10^{-9}$	3900,3	$218*10^{-9}$
2,1	$4036*10^{-9}$	30,7	$2140*10^{-9}$	161,2	$433*10^{-9}$	851,1	$274*10^{-9}$	4201,7	$211*10^{-9}$
2,3	$3968*10^{-9}$	34,8	$1944*10^{-9}$	170,5	$421*10^{-9}$	900,8	$273*10^{-9}$	4601,5	$203*10^{-9}$
2,8	$3894*10^{-9}$	36,9	$1856*10^{-9}$	180,9	$410*10^{-9}$	950,5	$273*10^{-9}$	5001,3	$195*10^{-9}$
3,8	$3817*10^{-9}$	41,0	$1697*10^{-9}$	191,2	$400*10^{-9}$	1000,2	$272*10^{-9}$	5501,5	$183*10^{-9}$

4,8	3736*10^{-9}	43,1	1626*10^{-9}	200,6	391*10^{-9}	1100,7	272*10^{-9}	6001,8	176*10^{-9}
5,8	3651*10^{-9}	47,2	1501*10^{-9}	221,3	377*10^{-9}	1200,1	271*10^{-9}	6501,0	174*10^{-9}
6,8	3561*10^{-9}	51,4	1396*10^{-9}	241,0	365*10^{-9}	1300,6	270*10^{-9}	7001,4	173*10^{-9}
7,9	3466*10^{-9}	56,6	1270*10^{-9}	260,6	355*10^{-9}	1401,0	269*10^{-9}	7500,6	173*10^{-9}
8,9	3368*10^{-9}	60,7	1204*10^{-9}	281,4	346*10^{-9}	1500,5	269*10^{-9}	8000,9	173*10^{-9}
		66,9	1073*10^{-9}	301,0	339*10^{-9}	1600,9	269*10^{-9}	8500,2	173*10^{-9}
		71,1	979*10^{-9}	331,1	331*10^{-9}	1700,4	267*10^{-9}	9000,5	173*10^{-9}
		75,2	894*10^{-9}	361,1	325*10^{-9}	1800,8	264*10^{-9}	9500,8	173*10^{-9}

Stable value of Polarisation current $I_{pol\ t=\infty}$: 173*10^{-9} A

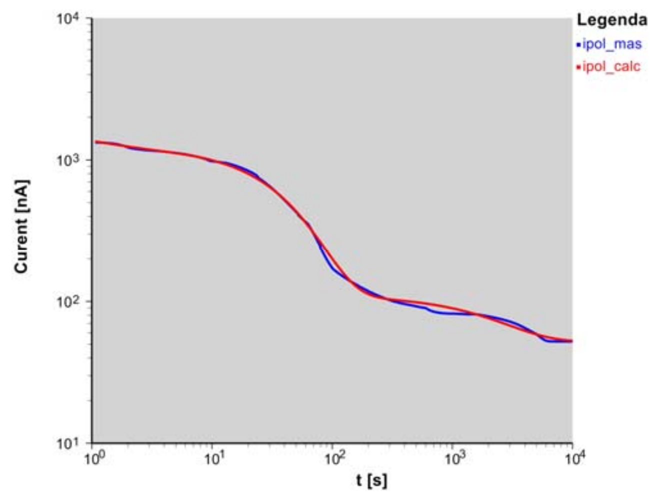

Fig. 1. Measured vs computed polarization current reported at 20 °C

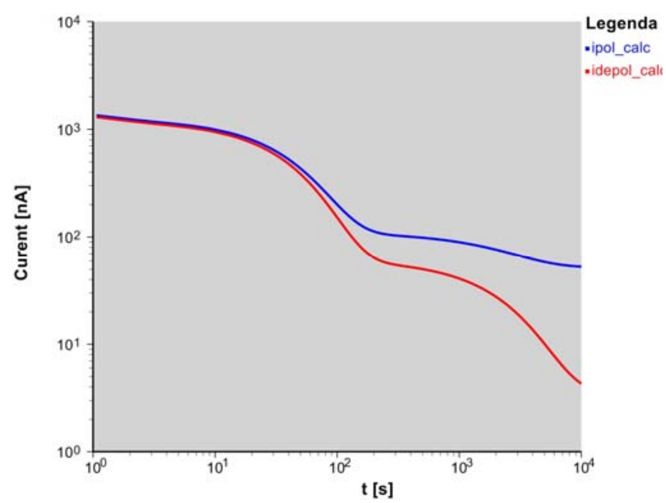

Fig. 2. Computed polarization vs depolarization current reported at 20 °C

After applying the software filtering and fitting, we obtain the parameters of the of computed function $I_{pol}(t) = I_0 + \sum_{i=1}^{5} Ip_i e^{\frac{-t}{Tp_i}}$ where I_0 is the stable value of measured current (in our example: $173*10^{-9}$ A).

Table 2

I_{pi}[A]	T_{pi} [s]
$3,17*10^{-07}$	1,17
$1,13*10^{-06}$	40,10
$5,72*10^{-08}$	2262,04
$7,07*10^{-10}$	10753,28

are given in ([10]).

Conclusion

These data and similar others with the computed parameters obtained as above, are used by the technical experts to diagnose and predict the moisture and life period remained at a power transformer.

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