MODELING WITH THE CHAOS GAME (II). A CRITERION TO DEFINE THE RELEVANT TRANSITION PERIODS IN ROMANIA

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Lucrarea prezintă o metodă de partiţionare a perioadelor relevante pentru tranziţia sistemului economic românesc din ultimii 20 de ani, reflectată în cursul de schimb leu-dolar american. Extinzând rezultatul unei lucrari anterioare, metoda ia în considerare evoluţia în timp a valorilor cuartilelor şi masura în care sub-seriile reale pot genera structuri fractale auto-replicante "Sierpinski gasket" prin procedeul iterativ de tip "Chaos Game". Metoda are corespondent în istoria reală a evenimentelor.

The paper is focusing on a new partition method of the transition époque suffered by the Romanian economic system in the last two decades as revealed by the Romanian Leu-United States Dollar exchange rate time series. By extending the results of a previous work, the method is considering the time evolution of the quartiles and the extent to what the real sub-series could generate the fractal self-similar structure "Sierpinski gasket" by using the iterative procedure of Chaos Game type. The method is supported by the historical evolution of the events.

Keywords: Chaos game, time series, partition criterion, Romanian transition.

1. Introduction

In the last decades Romania experienced an interesting ongoing process from a totalitarian economy toward a functional market one [1-3] and therefore it is a good candidate for reflecting the transition as it appears in the framework of the model we proposed in the previous work [4]. Many attempts have been performed in order to grab the most significant aspects of such transitions that have never occurred before and to find corresponding states in the historical past to make the future predictable in the sense of chaos theory [5].

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While using the Iterated Function Systems (IFS) clumpiness test, particularly the Chaos Game, in the present study we partition the Romanian evolution on the bases of the features exhibited by the exchange rate Romanian Leu – United States Dollar (ROL-USD).

Section 2 presents the relation between data series and IFS plot when playing the Chaos Game, data source and preparation is given in Sec.3, the partitioning criterion is subject of Sec.4, analysis is undertaken in Sec.5, and Sec.6 concludes.

2. Time series plots with Chaos Game

The Chaos Game is extensively presented in literature and its associated plot is based on the assigning of the data points to the quartiles of the series and further to the current number of the corners (or vertices) delimiting a plane square that are labelled 1,2,3,4 in a clockwise direction starting in the lower left corner [6]. Besides the deterministic rule of the plotting of the points, they are plotted in a different order each time, and the order depends on all the points plotted before.

By denoting y_n the returns (or diflog) of the discrete series $x_n=\{x_1, x_2,\ldots x_N\}$ of length N defined as the derivative of the log of the current values

$$y_n = \log x_{n+1} - \log x_n \,, \qquad n=1,\ldots, N-1, \tag{1}$$

for even N the first quartile Q_1 is equivalent to the median of the $N/2$ smallest elements in series, where the median gives the centre element in the sorted version of the halved series if $N/2$ is odd, or the average of the two centre elements if $N/2$ is of even length. For odd N, the first quartile is equivalent to the average of the median of the $(N-1)/2$ smallest elements and the median of the $(N+1)/2$ smallest elements in the series. The third quartile is defined as for the first, but with the largest rather than smallest elements; besides, $y_{\min}$ and $y_{\max}$ are understood as the minimum and maximum value in the series respectively.

The dynamic model relates the current number of the vertex to the quartile number. For every $n=1,\ldots, N-1$ a fixation series y_n^* is generated by replacing every data point y_n on its current position with the label 1, 2, 3, or 4 according to the interval it belongs:

$$y_n^* = 1 \quad \text{if} \quad y_{\min} \le y_n < Q_1 \tag{2}$$

$$y_n^* = k \quad \text{if} \quad Q_{k-1} < y_n \le Q_k \,, \; k=2, 3, \tag{2'}$$

$$y_n^* = 4 \quad \text{if} \quad Q_3 \le y_n \le y_{\max}. \tag{2''}$$

Hereafter there are two examples of 6000 data points series as follows: *i*) a periodic sequence with doubled 3 {1,2,3,3,4,…} whose quartiles are Q_1=2, Q_2=3, Q_3=3, consequently the fixation series is containing only three values in the sequence {1,2,4,4,4,…} because Q_3 generates an empty set, and *ii*) a continuously sequence {1,2,3,4,…6000} whose quartiles are Q_1=1500.5, Q_2=3000.5, Q_3=4500.5, consequently the new series is consisting in four steps of sequences of 1500 constant values each i.e. {1,…1,2,…2,3,…3,4,…4}. The pictures are shown below: Figs.1*a*-2*a* for the genuine series and Fig.1*b*-2*b* for the shuffled fixation series.

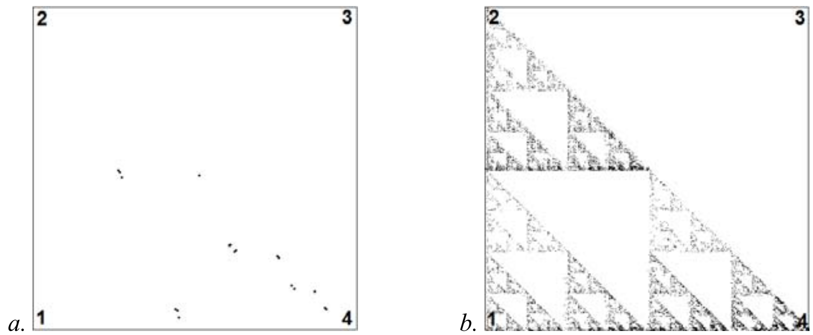

Fig.1 Example of a series with three vertices: the genuine sequence (*a*) and the shuffled sequence (*b*) - the points are spreading for random data and are more dense toward the corner labelled 4.

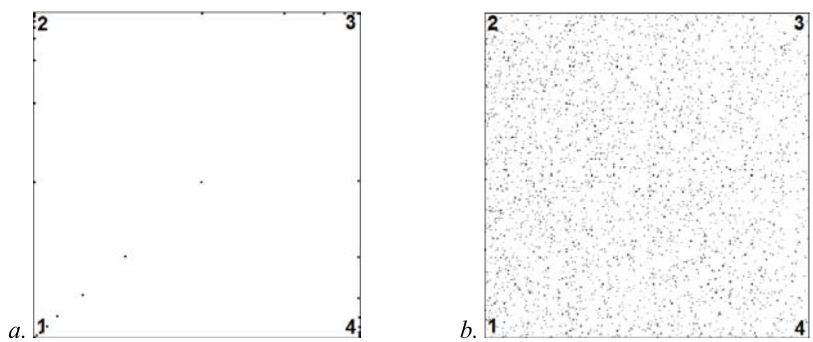

Fig.2 Example of a series with four vertices: the genuine sequence (*a*) and the shuffled sequence (*b*) - for random data the figurative points are uniformly distributed

Some remarks are emerging: *i)* periodic sequences give rise to very localised plots; *ii)* the more correlated the successive values, the smaller the area the points are spreading over and only when the vertices are non correlated and equally likely the points seem to be uniformly distributed in the plane; *iii)* the fractal Sierpinski triangle becomes visible when one of the vertices has negligible likelihood to occur with respect to the others. The underlying correlations in the series are recognized to the extent to different structures could be insulated as distinct clusters. A skewed distribution is good prerequisite for distinguishing a Sierpinski gasket from the pixelated background. Other patterns are exhibiting symmetries or could arise from composed gaskets when the correlation properties are changing in time [7]. When using Chaos Game is therefore mandatory to detect such changes by different methods and this paper suggests such a method.

3. Data

The series is the ROL-USD exchange rate between 1 Jan. 1990 and 31 Mar. 2009 therefore 6980 points [8]. The interval is covering interesting époques of the transition of the Romanian economy from the command type toward the free market one. As common feature for controlled economies or, at least, for controlled exchange rate of national currency is to preserving the same value for several consecutive days. Typical case is the case of the North Korean Won. The immediate consequence is a significant number of zero values in the returns. The Romanian Leu is not an exception: around 3100 values are zero in the period subjected to analysis (44,4%), but there are different weights of such values starting from 85% for the first two years 1990-1991 and decaying to 24% in the last two 2007-2008. This property is speculated hereafter when plotting the figurative points according to the Chaos Game rules.

4. Criterion for partitioning the Romanian transition

Differing from previous studies [9], here we adopted the criterion of the maximum length when “switching the quartile” for partitioning the whole period 1 Jan. 1990-31 Mar. 2009.

The criterion works as follows. Starting with minimum lengths of 1000 points, an interesting partition could be found when repeatedly computing the quartiles of the sorted return series when prolonging the return series point by point. The minimum length of 1000 points is somehow arbitrary but an acceptable compromise between obtaining relevant results when processing enough number of data points in the series and the historical evolution. Using this procedure the initial values of quartiles are $Q_1=Q_2=Q_3=0$ out of Q_3 will firstly switch to a certain positive value and the first period is thus defined. The Romanian exchange rate has the particular feature (mainly in the first years of transition) the intervals of constant values are separated by stepwise discontinuities causing many zero data points in the series of returns disrupted by huge spikes (Fig.3).

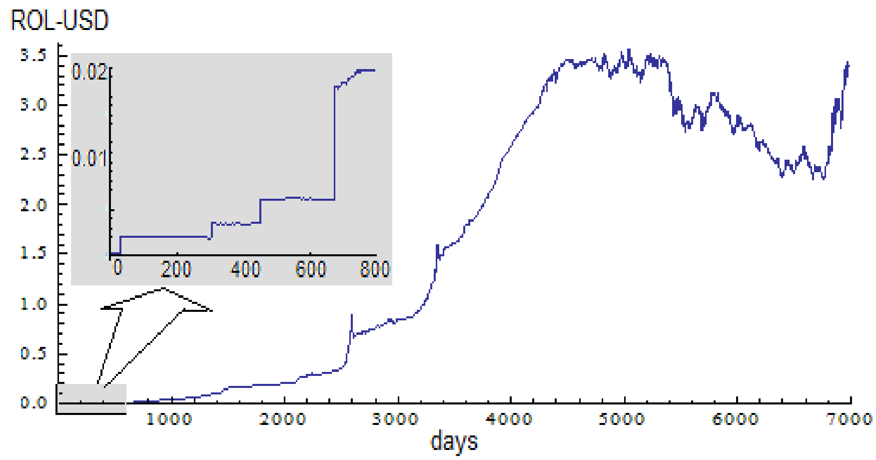

Fig.3 ROL-USD exchange rate and detail of the first 800 days.

The same procedure is further used starting again with the following 1000 points but in this case the initial computing gives $Q_1=Q_2=0$, $Q_3\neq0$; extending the interval step by step the next quartile that switches is Q_1, therefore, the second period is chosen. The third part is entirely taken because of the limited number of the remaining data points (Table 1).

Table 1

Interval partition according to the "switching quartile" criterion

Sub-interval	Time (days), date	Quartiles of ROL-USD returns		
		Q_1	Q_2	Q_3
I	1-2851, 1 Jan 1990-25 Oct.1997	0	0	0
II	2852-5548, 26 Oct.1997-14 Mar.2005	0	0	Change to 0.0006157
III	5549-6980, 15 Mar.2005-31 Mar.2009	Change to –0.001088	0	0.0009176

Apart from a short back-and-forth oscillation between zero and positive (negative) value of Q_3 (Q_1) lasting for several days at most after the first triggering, the quartile is definitely preserving the sign whatever the subsequent time length.

5. Computational results

When applying the rule the following results are obtained i.e. three relevant periods arise. The corresponding partition in sub-intervals I, II, and III assigned to the relevant periods is shown in Fig.4.

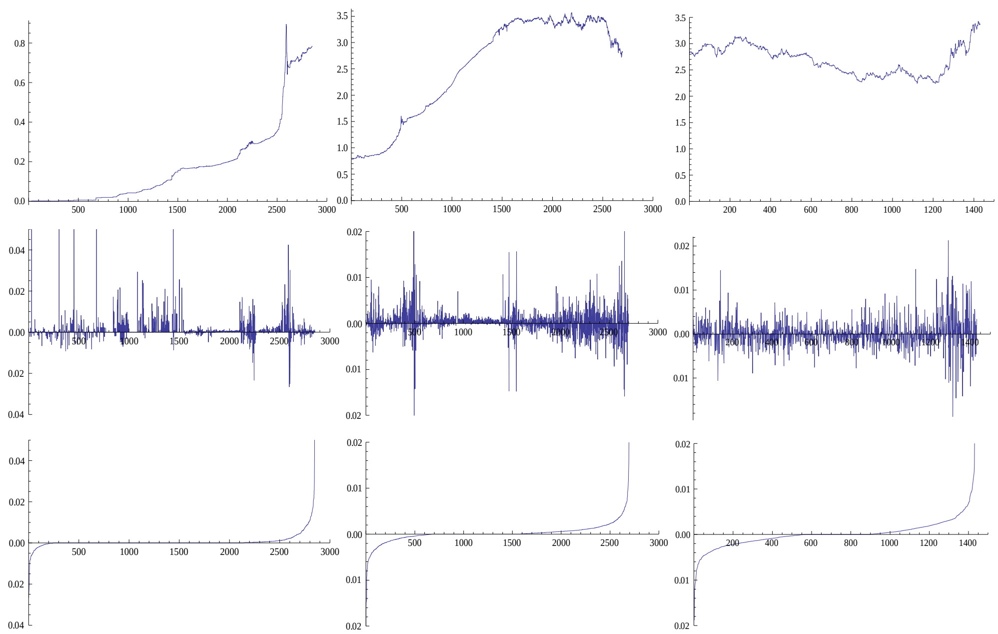

Fig.4 ROL-USD exchange rate: log values (upper row), returns (middle), and sorted values (lower row). From left to right are the partitioned sub-intervals I, II, and III.

It is easy to remark that for the sub-interval I there is no vertex labelled 3 and consequently the Sierpinski gasket arise with a weak additional structure due to the excess of the positive values in the fixation series that are not balanced by the number of the negative values (Fig.5*a*). ROL behaves like the Chinese Yuan [4,10] in terms of fixation series y. This is the effect of the two-digit (in some years three) inflationary process that imprinted the increasing trend of the ROL-USD exchange rate.

The middle sub-interval II exhibits the same unbalance of the positive values vs. negative ones that comes to end by the beginning of 2005, but the structure of the fixation series is a compromise between the Sierpinki triangle and the emergent trend to randomness (Fig.5*b*).

Finally, the remaining sub-interval III exhibits three different values for quartiles and therefore all vertices are involved in the Chaos Game algorithm with approximately the same weight (Fig.5*c*); however, the residuals of some structure are still visible. One can use ROL – sub-interval I – for replicating some series in the same manner like the Chinese Yuan [4]. One should remind that there could be different pictures according to the particular assignation of the order of labelling the vertices used in different software packages [7].

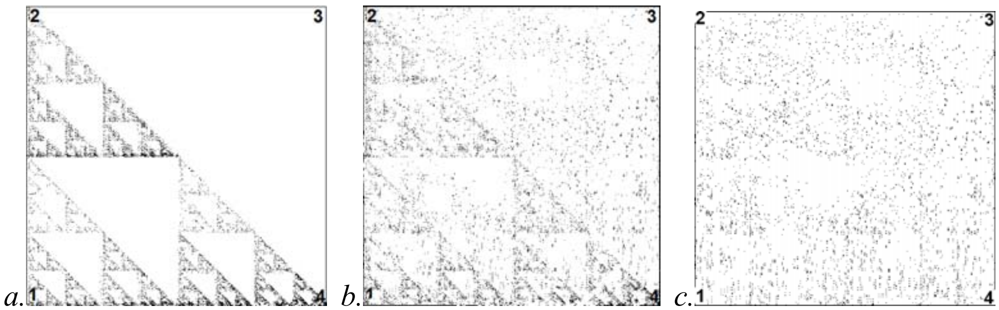

Fig.5 Chaos Game plots for the partitioned sub-intervals I (*a*), II (*b*), and III (*c*) of ROL-USD exchange rate

6. Conclusions

An alternate criterion for partitioning the evolution of the Romanian economic system is presented. Based on the specific features of ROL-USD exchange rate that contains a significant number of zeros, the criterion accounts for the moment when the values of the quartiles switch to positive or negative values respectively. These thresholds are profoundly modifying the Chaos Game plot.

The first sub-interval 1 Jan 1990-25 Oct.1997 exhibits strong correlations such that one quartile give rise to empty set while the remaining are almost randomly distributed with a significant shift toward larger values. The Sierpinski gasket is a clean one. The correlations are the consequences of the limited convertibility of ROL whose exchange rate was artificially kept at constant values under the huge inflationary pressure that exploded from time to time in stepwise changes.

The second sub-interval 26 Oct.1997-14 Mar.2005 is the beginning of stability of a fully convertible ROL; the long run correlations seem to be also of deterministic type since the small negative corrections are lacking. Consequently the Sierpinski fractal structure is clearly defined with clustering regions also marking the lower part of the plot.

The last sub-interval 15 Mar.2005-31 Mar.2009 is the closest to randomness for all four vertices; however, a weak structure is still visible. A weak de-clustering is preserving long range correlations but the exchange rate is by far a less predictable one.

This criterion might be considered as a practical method for evaluating the complexity of the series and a useful quantitative tool for measuring the qualitative changes of the correlation properties.

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