U.P.B. Sci. Bull., Series A, Vol. 72, Iss. 1, 2010 ISSN 1223-7027

OPTICAL COMMUNICATION METHODS BASED ON CHAOTIC LASER SIGNALS

Paul STERIAN[1], Valerică NINULESCU[2], Andreea-Rodica STERIAN[3], Bogdan LAZĂR[4]

În lucrare se efectuează simulări numerice a două variante de implementare a transmisiei de informație bazată pe o purtătoare haotică deterministă. În prima situație se consideră cazul unei cavități optice în inel cu un mediu neliniar, după care comunicarea este realizată prin sincronizarea dinamicilor a două lasere cu semiconductoare. Aceste metode pot fi utilizate pentru transmisia pe un canal de comunicație clasic sau cuantic, cu un înalt nivel de mascare a informației.

Numerical investigations of two methods for the information transmission based on a chaotic deterministic carrier are performed. The first is represented by a ring optical cavity with a nonlinear medium inside, while the second refers to the synchronization of two laser diodes. These methods can be used for the transmission on a classical or quantum channel with a high level of information masking.

Keywords: chaotic lasers, synchronization, chaos masking, laser diode, quantum channel

1. Introduction

The paper focuses on transmission of information by masking it in a chaotic signal, the amplitude of the message being added to that of the carrier.

Two chaotic systems can be synchronized if they have similar parameters [1, 2]. This phenomenon has a very good application potential in transmission coding. The transmission is made by using an emergent wave from a laser as a chaotic transmitter. The signal is attached to a carrier wave which is of a chaotic nature and has much higher amplitude. This ensures a higher degree of difficulty in intercepting and decoding. For attaching the signal to the carrier chaotic masking, modulation and translation can be used.

[1] Prof., University POLITEHNICA of Bucharest, Faculty of Applied Sciences, Bucharest, Romania
[2] Lecturer, University POLITEHNICA of Bucharest, Faculty of Applied Sciences, Bucharest, Romania, vninulescu@physics.pub.ro
[3] Lecturer, University POLITEHNICA of Bucharest, Faculty of Applied Sciences, Bucharest, Romania
[4] PhD, University POLITEHNICA of Bucharest, Faculty of Applied Sciences, Bucharest, Romania

The properties of transmission and reception of the data as well as the synchronization of the lasers can be studied by using a pair of master-slave lasers. Modeling of the lasers is made by use of the rate equations [3] that are integrated based on fourth order Runge-Kutta method in MATLAB.

2. Pecora-Caroll synchronization scheme

Synchronization of periodic signals is a common phenomenon in many scientific areas. On the other hand, deterministic chaotic systems present the property of sensitivity with respect to initial conditions: two identical autonomous chaotic systems starting at very close initial conditions evolve so that the trajectories in the phase space start diverging exponentially and for large times they are uncorrelated. It seems that synchronization can not be reached in such systems. Nevertheless, it was recently proven [2] that certain chaotic systems may be linked such that their chaotic motions synchronize. The research in this direction is greatly motivated by the possibility of using chaotic signals as broadband carriers of analog and digital information [4-11]. Tests were performed using electronic circuits. For example, the message was added to a chaotic carrier signal and transmitted to a system which is a copy of that one creating the chaotic signal. The receiver synchronizes with the carrier signal and the message is recovered by a simple substraction of the receiver signal from the total transmitted one in an adequate electronic block.

It was a widespread idea that deterministic chaos will not have practical applications. The ability to design synchronizing chaotic systems may open opportunities for the use of chaotic carrier signals in private communication, taking advantage of the unique features of chaotic signals. More than that, synchronization is structurally stable in this case and using chaotic signals may be preferable to periodic signals in certain cases where robustness is important.

Pecora-Caroll “scheme” is the most known method for synchronization subsystems [2]. An autonomous n-dimensional dynamical system given in the form of a flow

$$\dot{u} = \boldsymbol{f}(\boldsymbol{u}) \tag{1}$$

is decomposed into two subsystems,

$$\dot{\boldsymbol{v}} = \boldsymbol{f}_{\boldsymbol{v}}(\boldsymbol{v},\boldsymbol{w}), \tag{2a}$$

$$\dot{\boldsymbol{w}} = \boldsymbol{f}_{\boldsymbol{w}}(\boldsymbol{v},\boldsymbol{w}), \tag{2b}$$

with $\boldsymbol{v} = (u_1,...,u_m)$, $\boldsymbol{w} = (u_{m+1},...,u_n)$ and $\boldsymbol{f} = (\boldsymbol{f}_{\boldsymbol{v}}, \boldsymbol{f}_{\boldsymbol{w}})$. Now create a new $\boldsymbol{w}'$ subsystem driven by the $\boldsymbol{v}$ subsystem,

$$\dot{\boldsymbol{w}}' = \boldsymbol{f}_{\boldsymbol{w}}(\boldsymbol{v},\boldsymbol{w}'), \tag{3}$$

i.e., given by the same vector field $\boldsymbol{f}_{\boldsymbol{w}}$. Subsystem $\boldsymbol{w}'$ synchronizes with subsystem *w*, that is, $\|\boldsymbol{w}-\boldsymbol{w}'\| \to 0$ as $t \to \infty$, if the conditional Lyapunov exponents of subsystem ***w*** are all negative.

3. Proposed communication systems

A communication scheme compatible with the above syncronization method is presented in Fig. 1. In the transmission area, the subsystem ***u*** is called master system, while the *w* and $\boldsymbol{w}'$ subsystems are referred to as slave systems; the link between the two subsystems in the transmission area is unidirectional. The encryption is done by using the chaotic signal of the slave system at the transmitter as carrier for the message. At the receiver, the slave system synchronizes with its replica at the transmitter through the one linking drive signal. This allows the extraction of the information from the total transmitted signal.

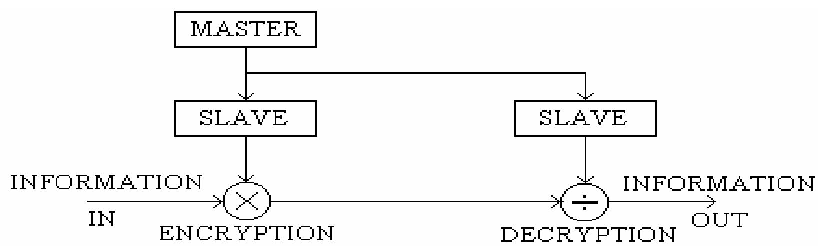

Fig. 1. Block scheme of a communication based on the synchronization of two chaotic systems.

3.1. Optical ring cavities with a nonlinear medium

As optical fibers have already become a very important transmission medium and they still have a great perspective, all-optical systems are advantageous compared to electrical ones [14-16]. As building blocks of the communication scheme in Fig. 1 there are proposed ring optical cavities with a nonlinear optical medium inside [8].

The nonlinear medium is a Kerr material whose response is described by the Debye relaxation equation. When the relaxation time constant of the medium is much longer than the delay time of the feedback of light and the medium is thin enough so that the phase shift of the electric field and the dissipation are small, the cavity is ruled by the set of ordinary differential equations [7]

$$\dot{e} = a - be + \mathrm{i}(\phi - \phi_0)e\,, \qquad (4a)$$

$$\dot{\phi} = -\phi + |\,e\,|^2\,. \qquad (4b)$$

In the above, *e* is proportional to the slowly varying envelope of the electric field inside the cavity at $z=0$, ϕ is proportional to the phase shift of the electric field across the Kerr medium and time is expressed in Debye relaxation time units. a is a measure of the incident electric field amplitude, b characterizes the dissipation and ϕ_0 is the mistuning parameter of the cavity. See Ref. [7] for precise definitions. Eqs. (4) are valid when the ratio of the transit time of the cavity and the time relaxation constant of Kerr medium, denoted as $\rho\varepsilon$ ($\rho = 1/b$), is $\rho\varepsilon << 1$, with $\varepsilon << 1$.

The output electric field of the cavity, which is proportional to e, exhibits not only bistability [8,9], but also a sequence of periodic and chaotic-like dynamics, as shown in Fig. 2 for the parameters $b=2$, $\phi=4$ and a varying in both directions.

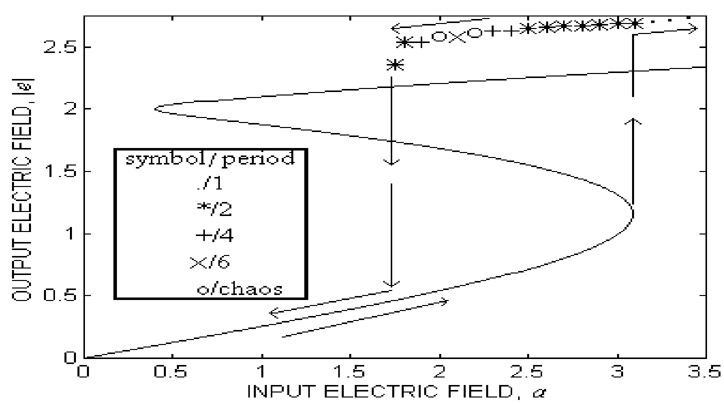

Fig. 2. Stationary states curve (solid curve) and actual dynamics (follow the arrows) of the optical cavity for $b = 0.2$, $\phi_0 = 4$ and a slowly sweeping in both directions. It can be seen that the system do not follow at all the upper branch of stationary states, but another one, composed of periodic orbits and chaotic ones. For the latter case, the temporal average of $|e|$.

Below, the variables e and ϕ, as well as the parameters characterizing the master system will be written with the index 1. Indices 2 and 3 are reserved for the slave systems at the transmitter and receiver, respectively. Hence, the equations describing the communication scheme are

$$\dot{e}_1 = a_1 - b_1 e_1 + \mathrm{i}(\phi_1 - \phi_{01})e_1, \tag{5a}$$

$$\dot{\phi}_1 = -\phi_1 + |e_1|^2, \tag{5b}$$

$$\dot{e}_2 = \exp(\mathrm{i}\rho\varepsilon\phi_1)c_{12}e_1 - b_2 e_2 + \mathrm{i}(\phi_2 - \phi_{02})e_2, \tag{5c}$$

$$\dot{\phi}_2 = -\phi_2 + |e_2|^2, \tag{5d}$$
$$\dot{e}_3 = \exp(i\rho\varepsilon\phi_1)c_{13}e_1 - b_3e_3 + i(\phi_3 - \phi_{03})e_3, \tag{5e}$$
$$\dot{\phi}_3 = -\phi_3 + |e_3|^2. \tag{5f}$$

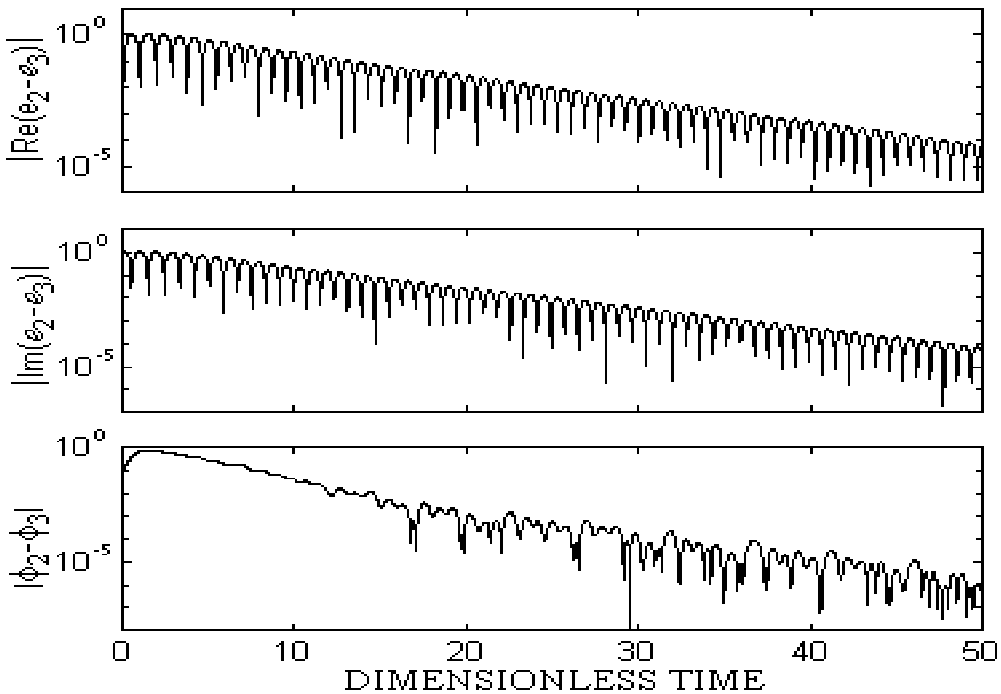

Fig. 3. Synchronization of optical cavities 2 and 3 both driven by the output signal from cavity 1. The values of the parameters are $a_1 = 2$, $b_1 = 0.2$, $\phi_{01} = 4$, $\rho\varepsilon = 0.01$, $b_2 = b_3 = 0.2$, $\phi_{02} = \phi_{03} = 4$ and $c_{12} = c_{13} = 0.2\exp(0.2i)$.

The coupling of cavities 1 and 2 and 1 and 3, respectively, is introduced through the complex quantities c_{12} and c_{13} that include both the attenuation and the phase shift of the signals during propagation between cavities. To be in agreement with Pecora and Caroll synchronization method, the two slave systems are caracterized by identical parameters.

The choice $b_2 = b_3 = 0.2$, $\phi_{02} = \phi_{03} = 4$, $c_{12} = c_{13} = 0.2\exp(0.2i)$ and $\rho\varepsilon = 0.01$ gives synchronization as shown in Fig. 3. On the average, the synchronization errors decay exponentially.

Numerical simulations prove that synchronization holds for any initial conditions and large ranges of parameter values. Besides, the synchronization is robust with respect to deviations from the identity of the slave systems up to about $10\,\%$ in the parameter values. Parameters that may have significant different values are the coupling coefficients c_{12} and c_{13}.

A good encryption is enabled by a chaotic in a high degree of the carrier signal. This is given, for instance, by high values of the largest Lyapunov exponent and a high entropy [10]. Of special interest here is the mutual information of signal driving the slave system at transmitter and carrier signal. Taking the modulus of the electric field wave as signal, the plot $|e|_2$ versus $|e|_1$ shows a small correlation of the two signals. The mutual information calculated is about 2 bits, leading to a tough interception of the information.

The form of the carrier wave signal makes it suitable for an analog information signal. Frequency bandwidth of the carrier signal is about the reciprocal of time relaxation constant of Kerr medium. For instance, in case of carbon sulphide this quantity is of the order of 10^{12} Hz.

A specific model [Eqs. (4)] of the optical ring cavity has been investigated. For an improved modelling of the interaction processes in the system, see Refs. [15,16] for quantum formalisms of the interaction of an electromagnetical (optical) field with atomic media.

Upper bound limit of transmission capacity is determined by quantum noise [15] establishing the theoretical limit of performances. Additional difficulties appear for the noise treatement in case of a carrier optical originating from a source with both optical and atomical coherence weights dependent on the operation conditions of the source.

Masking digital information needs random sequences of pulses separated through large time intervals where the amplitude signal is very small. This kind of chaotic signals is available from other optical systems like semiconductor lasers with injection current modulation [17], where a modulation frequency in the GHz range ensures a good transmitting speed for the bits. A semiconductor laser with external cavity is expected to be used in information encoding at much higher bit rate.

3.2. Synchronization between two self-pulsating laser diodes

A simplified master-slave synchronization scheme is presented below. The two systems are represented by identical semiconductor lasers, unidirectionally coupled through the laser field. The following rate equations describe a self-pulsating laser diode (SPLD), driven by an injection current $I^{\mathrm{T,R}}(t)$. The system (6)-(8) refers to the transmitter laser [10,17,18]. The receiver is governed by the same set of equations in which T$\to$R, and (8) is replaced by (9):

$$\dot{N}_1^T = \frac{I^T}{qV_1} - \frac{k_1\xi_1}{V_1}(N_1^T - N_{g1})S^T - \frac{N_1^T}{\tau_s} - \frac{N_1^T - N_2^T}{T_{12}}, \tag{6}$$

$$\dot{N}_2^T = -\frac{k_2\xi_2}{V_2}(N_2^T - N_{g2})S^T - \frac{N_2^T}{\tau_s} - \frac{N_2^T - N_1^T}{T_{21}}, \tag{7}$$

$$\dot{S}_2^{\mathrm{T}} = [k_1\xi_1(N_1^{\mathrm{T}} - N_{\mathrm{g1}}) + k_2\xi_2(N_2^{\mathrm{T}} - N_{\mathrm{g2}}) - G_{\mathrm{th}}]S^{\mathrm{T}} + C\frac{N_1^{\mathrm{T}}V_1}{\tau_s}, \tag{8}$$

$$\dot{S}_2^{\mathrm{R}} = [k_1\xi_1(N_1^{\mathrm{R}} - N_{\mathrm{g1}}) + k_2\xi_2(N_2^{\mathrm{R}} - N_{\mathrm{g2}}) - G_{th}]S^{\mathrm{R}} + \delta S^{\mathrm{T}} + C\frac{N_1^{\mathrm{R}}V_1}{\tau_{\mathrm{s}}}, \tag{9}$$

where S is the photon density, N_1 is the electron density in the active region, N_2 is the electron density in the saturable absorption region, δ is the coupling factor of transmitted field into receiver SPLD. $I^{\mathrm{R,T}}$ are given by

$$I^{\mathrm{R,T}} = I_{\mathrm{bias}}^{\mathrm{R,T}} + I_{\mathrm{A}}^{\mathrm{R,T}}\sin(2\pi ft), \tag{10}$$

where I_{bias} is the continuous component of the injection current, while I_{A} and f are the amplitude and frequency of a sinusoidal current superimposed over the continuous injection bias component. The lasers parameters can be found in Table 1 [10].

Table 1

Parameters for self-pulsating laser diodes described by eqs. (6)-(9)

Parameter	Description	Value
k_1	linear approximation constant for the gain curve in the active region	3.08×10^{-12} m^3/s
k_2	linear approximation constant for the gain curve in the saturable absorption region	1.232×10^{-11} m^3/s
ξ_1	confinement factor in the active region	0.2034
ξ_2	confinement factor in the saturable absorption region	0.1449
N_{g1}	transparent level of electron density in the active region	1.4×10^{24} m^{-3}
N_{g2}	transparent level of electron density in the saturable absorption region	1.6×10^{24} m^{-3}
V_1	active layer volume	72 μm^3
V_2	saturable absorption region volume	102.96 μm^3
T_{12}	carrier time diffusion constant between layer 1 and 2	2.65 ns
T_{21}	carrier time diffusion constant between layer 2 and 1	4.425 ns
G_{th}	threshold gain level	3.91×10^{11} s^{-1}

C	coupling ratio between spontaneous field and the lasing mode	1.573×10^{-23} μm^{-3}
τ_s	carrier lifetime	3 ns
q	elementary electric charge	1.6×10^{-19} C
$k=\dfrac{\sqrt{\delta}(1-R)}{\tau_{in}\sqrt{R}}$	optical coupling	variable
δ	coupling factor transmitter - receiver	variable
R	reflectivity coefficient	0.36
τ_{in} τ_{in}	laser cavity round trip time	6 ps

A SPLD is an active optical device able to produce a continuous train of pulses with repetition rate dependent of the injection current (Fig. 4).

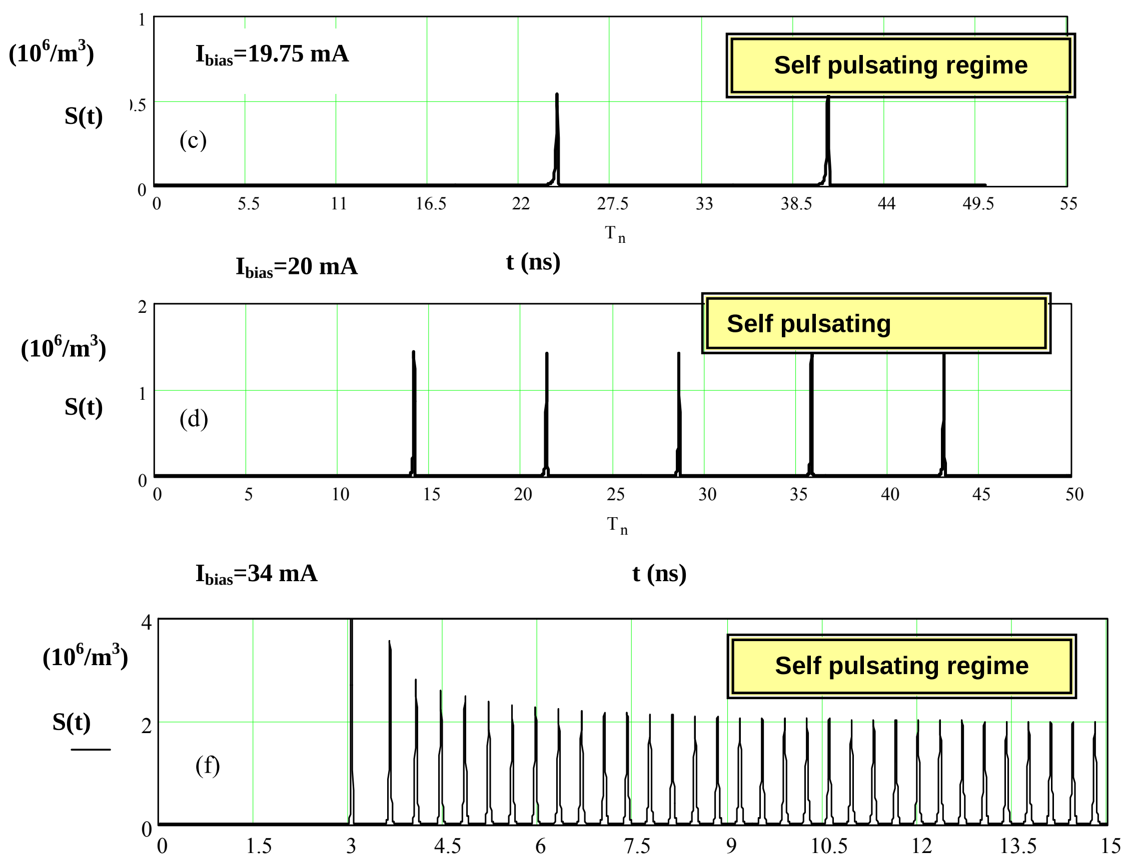

Fig. 4. SPLD temporal dynamics for three values of the injection current I_{bias}.

If a sinusoidal current is superimposed over I_{bias}, then the self-pulsations became chaotic either in repetition interval or amplitude (Fig. 5) for $19.75\ \text{mA} < I_{\text{bias}} < 34\ \text{mA}$; two identical SPLDs have similar outputs but uncorrelated (Fig. 6)

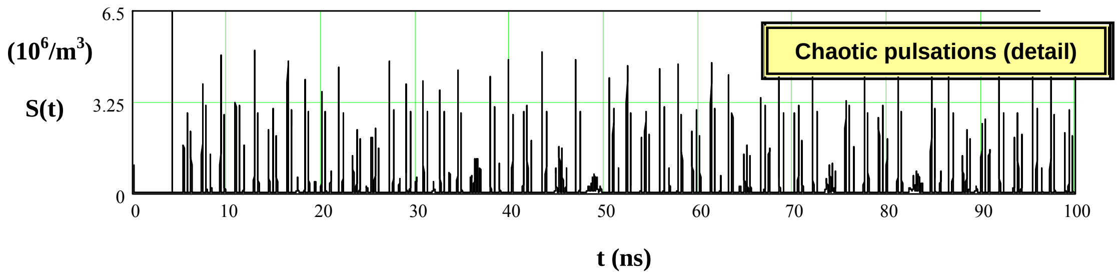

Fig. 5. Chaotic pulsations of the SPLD described by equations (6)-(8) with parameters from Table 1. I_{bias}=28 mA, I_A=7 mA, f=0.33f_0, where f_0=1.687 GHz is the autopulsation frequency corresponding to the continuous injection current $I_{\text{bias}} = 28\ \text{mA}$.

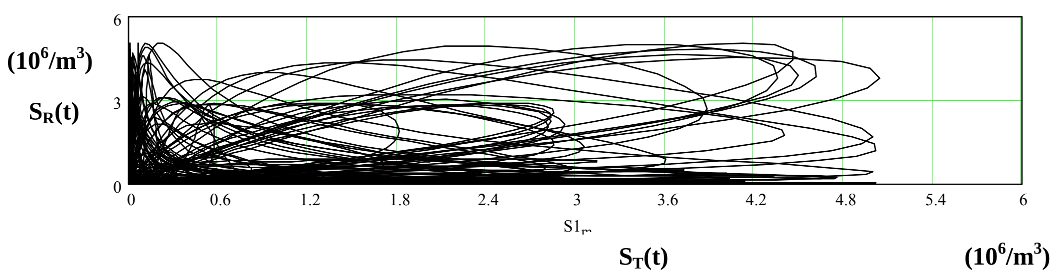

Fig. 6. The output of SPLD-R versus that of SPLD-T. The two photon densities are not synchronized.

When a small part of the transmitted chaotic carrier is coupled into the receiver device, synchronization between the two lasers becomes possible (Fig. 7, 8) independently of the initial conditions of the two systems.

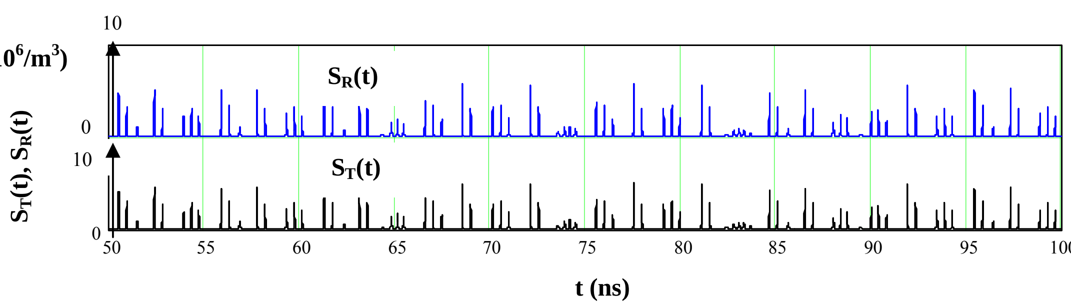

Fig. 7. $S_R(t)$ and $S_T(t)$ versus time for $\delta = 0.04$. The two photon densities are synchronized.

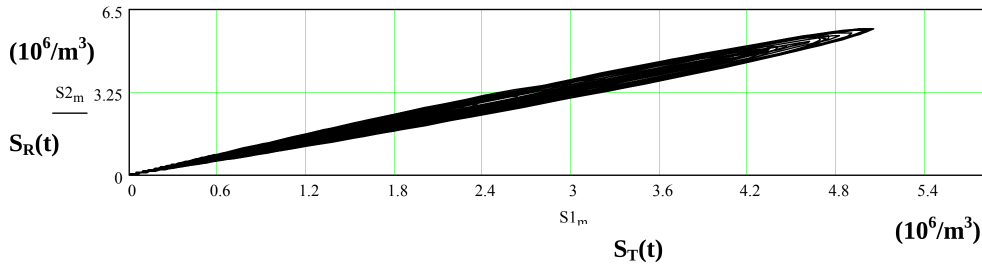

Fig. 8. The output of SPLD-R versus that of SPLD-T. The two photon densities are closely synchronized.

To investigate the possibility of transmission information, we introduce the information as an amplitude modulation of the sinusoidal injection current into the transmitter laser. This is a more convenient method compared to the variable modulation of the transmitter output used in [20, 21, 22]. For simplicity, we consider below a sinusoidal message signal $I_{\mathrm{m}}(t) = I_{\mathrm{m,A}} \sin(2\pi f_{\mathrm{m}} t)$, where $I_{\mathrm{m,A}} << I_{\mathrm{bias}}$ and $f_{\mathrm{m}} << f$ for secrecy and small modulation distortion reasons. Numerical simulations shows that receiver output synchronizes with the carrier field, rather than to the transmitted signal; the decoded signal is obtained as $S_d(t) = S_T(t) - S_R(t)$. Figure 9 presents a typical result of the information recovering.

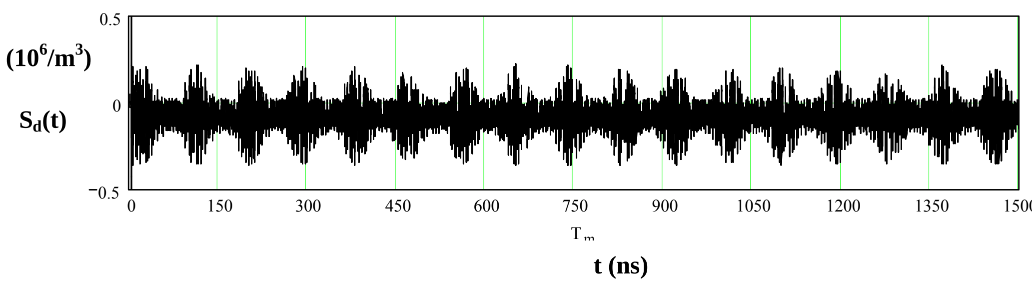

Fig. 9. The decoded signal $S_d(t) = S_T(t) - S_R(t)$ versus time for $I_{\mathrm{m}}(t) = (7/50)\sin[2\pi(f/50)t]$ mA.

4. Conclusions

Two transmitter-receiver models for chaotic communication are presented and their performances are analyzed.

We have firstly demonstrated that masking the information in a chaotic optical wave from a nonlinear ring cavity is theoretically possible. The output of such a system makes it suitable for analog information transmission. Some problems arise in technical realization of two identical systems.

Secondly it was shown that an amplitude modulator-demodulator with chaotic carrier can be implemented using a pair of SPLDs with closely matched parameters. If a sinusoidal injection current is superimposed over a polarization current for which self-pulsations occurs, then the pulses become chaotic both in amplitude and repetition interval. When the sinusoidal current is modulated with an information signal having amplitude and frequency much smaller than the disorder maker current, then the transmitted signal spectrum does not present a clear component having the frequency of the message, and the filtering operation is of no use in recovering the transmitted information. If a small part of the transmitted chaotic signal is coupled into the SPLD-R, the information can be recovered based on the property that the receiver output field synchronizes only with the chaotic carrier and not with the whole transmitted optical field. The useful message can be recovered by low filtering the quantity $[S_T(t) - S_R(t)]/S_R(t)$. As SPLDs are widely available, cheap and compact, their use in a private communication system could be a good choice for digital information transmission.

Further steps in the analysis of synchronization include the robustness with respect to parameter mismatches, random noise and coupling factor.

REFERENCES

[1] *S. Hayes*, *C. Grebogy*, and *E. Ott*, "Communicating with Chaos", Phys. Rev. Lett. **70**, no. 20, pp. 3031-3034, 1993.

[2] *L. M, Pecora* and *T. L. Caroll,* "Driving Systems with Chaotic Signals", Phys. Rev. A **44**, no. 4, pp. 2374-2383, 1991.

[3] *K. M. Cuomo*, and *A. V. Oppenheim*, "Circuit Implementation of Synchronized Chaos with Applications to Communications", Phys. Rev. Lett. **71**, no. 1, pp. 65-68, 1993.

[4] *P. Colet* and *R. Roy*, "Digital Communication with Synchronized Chaotic Lasers", Opt. Lett. **19**, no. 24, pp. 2056-2058, 1994.

[5] *U. Parlitz, L. Kocarev, T. Stojanovski, and H. Preckel*, "Encoding Messages Using Chaotic Synchronization", Phys. Rev. E **53**, no. 5, pp. 4351-4361, 1996.

[6] *P. M. Alsing*, *A Gavrielides*, *V. Kovanis*, *R. Roy*, and *K. S. Jr. Thornburg*, "Encoding and Decoding Messages with Chaotic Lasers", Phys. Rev. E **56**, no. 6, pp. 6302-6310, 1997.

[7] *K. Ikeda* and *O. Akimoto*, "Instability Leading to Periodic and Chaotic Self-Pulsations in a Bistable Optical Cavity", Phys. Rev. Lett. **48**, no. 9, pp. 617-620, 1982.

[8] *P. E. Sterian*, "Communication Based on Chaotic Signals", Proceedings of The Romanian Academy, Series A, vol. **3**, no. 1-2, pp.45-48, 2002.

[9] *R. M. Mirică*, *P. E. Sterian*, and *R. Bercia*, "Optic System for Transmission with Chaotic Signals", Romanian Journal of Optoelectronics, vol. **11**, issue 1, pp.1-11, 2003.

[10] *P. E. Sterian* and *B. Lazăr*, "Self-Pulsating Laser Diodes with Applications in Optical Chaotic Communications", Proceedings of SPIE, vol. **5581**, pp.143-150, 2004.

[11] *P. E. Sterian*, *O. Rancu*, and *I. M. Popescu*, "Complex Dynamics of Radiation Field in the Transition Area of Optical Bistability", Int. Conf. on Trends in Quantum Electronics, Proceedings of SPIE, vol. **1033**, pp. 154-157, 1988.
[12] *B.-A. Ştefănescu*, *D.-A. Anghel*, *O. Danilă*, *P. E. Sterian*, and *Andreea-Rodica Sterian*, "Quantum Cryptology Applications Based on BB84 Protocol for Student Training", 9th Int. Balkan Workshop on Applied Physics, Constanța, Romania, July 5-7 2008.
[13] H. Weinfurter and A. Laubereau, "Experimental Quantum Cryptography", http://xqp.physik.uni-muenchen.de/publ/henning-diplom.pdf, 2003.
[14] *P. E. Sterian*, Fotonica, Editura Printech, Bucureşti, 2000.
[15] A. *M. Fraser* and *H. L. Swinney*, "Independent coordinates for strange attractors from mutual information", Phys. Rev. A **33**, no. 2, 1134-1140, 1986.
[16] *E. Ştefănescu* and *P. E. Sterian*, "Exact Quantum Master Equations for Markoffian Systems", Opt. Eng. **35**, no. 6, pp. 153-155, 1996.
[17] *P. E. Sterian*, Transmisia optică a informației, vol. I, II, Editura Tehnică, Bucureşti, 1983.
[18] *G. D. VanWiggeren* and *R. Roy,* "Optical communication with chaotic waveforms", Phys. Rev. Lett. **81**, no. 16, pp. 3547-3550, 1998.
[19] *A. Sanchez-Diaz*, *C. Mirasso*, *P. Colet*, and *P. Garcia-Fernandez*, "Encoded Gbit/s Digital Communications with Synchronized Chaotic Semiconductor Lasers", IEEE Journal of Quantum Electronics, vol. **35**, no. 3, pp. 292-297, 1999.
[20] *S. Garcia-Revilla*, *A. Valle*, *P. Spencer*, *R. Jones*, *P. Rees*, *J. Revuelta*, *L. Pesquera*, and *A. Shore*, "Theoretical Analysis of Syncronization of Chaotic Self-Pulsating Semiconductor Lasers", SPIE Proceedings (Physics and Simulation of Optoelectronic Devices, X), vol. **4646** , pp. 505-516, 2002.
[21] *B. Lazar, P. E .Sterian, Andreea Sterian*, "A 2D numerical model for analysing the evolution of phenomena in laser structure", U.P.B., Sci. Bull. Series A, Applied Matematics and Physics, **Vol**.65 No. 1-4, pp.36 -48, 2003.
[22] *V. Iancu, M.R. Mitroi, L. Fara,*" Modelling of multi-layered quantum well photovoltaic cells" U.P.B. Scientific Bulletin, Series A, **Vol.** 71, Iss. 4, pp.53-63, 2009.