MATHEMATICAL MODELING OF SELECTIVE WITHDRAWAL FROM HELIOTHERMIC STRATIFIED LAKES. CASE STUDY

Virgil PETRESCU[1], Liviu IANUŞ[2], Nicolai SÎRBU[3]

Heliothermal phenomenon has been found and investigated for closed seas, great lakes in Canada and United States of America, and for some glacial lakes. In Romania, heliothermal properties have been identified and studied at Sovata and Ocna Sibiu lacustrine basins by a research team of the National Research Institute for Environmental Engineering (ICIM), Bucharest, based on field measurements made in 2000-2004. The paper presents the mathematical modeling of the selective withdrawal of warm salted water from thermocline in order to optimize hydraulic parameters, so heliothermal phenomenon at Ursu Lake, Sovata basin is preserved.

Keywords: heliothermal, stratified system, mathematical modeling, numerical simulation

1. Introduction

Heliothermal phenomenon is a physical one which occurs in some continental salt lakes. It consists of a seasonal storage of heat in salt water body due to the presence at the surface of an insulating thin layer of fresh water, which prevents heat transfer in the atmosphere. The phenomenon was identified in closed seas, great lakes (Canada, USA, etc.), and in some glacial lakes [1].

In Romania, a research team from the National Research Institute for Environmental Engineering (ICIM) Bucharest has investigated this phenomenon by field measurements between 2000 and 2004. They have demonstrated that only relatively deep lakes with high concentrations of natural salt dissolved manifests heliothermal properties. Water temperature increases with depth, until a maximum is reached (thermocline), followed by a slight decrease and then the temperature remains almost constant until the lake bottom [2-4].

So far, in Romania heliothermal phenomenon was identified in Sovata and Ocna Sibiu salt lakes, and is used for spa and therapeutic purposes.

Salty and relatively deep lakes are thermal and chemical stratified, as is Ursu Lake (Fig. 1), with an upper layer, called *epilimnion*, a lower layer, *hypolimnion*, and in between *metalimnion* layer or *thermocline* is found.

[1] Prof., Technical University of Civil Engineering of Bucharest, Romania, e-mail: virgil.petrescu@gmail.com

[2] Senior Advisor, PhD Student, SC KPMG Romania SRL, e-mail: lianus@kpmg.com

[3] Lecturer, Technical University of Civil Engineering of Bucharest, Romania, e-mail: nsirbu@utcb.ro

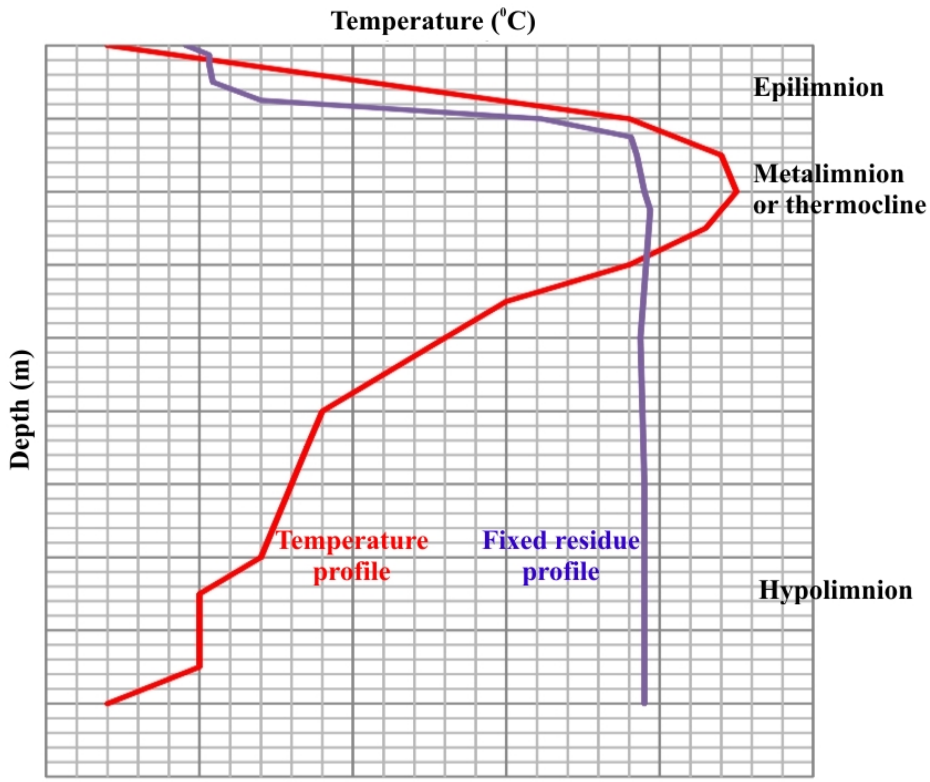

Fig. 1. Thermal and chemical stratification in Ursu Lake.

Epilimnion is relatively high oxygenated, with variable temperature from one season to another and well-heated in summer season by solar radiation. In calm conditions, the termocline appears, and in windy conditions vertically uniform heating occurs, leading to thermal homogenization.

Hypolimnion is a deep layer of a thermally stratified lake, with lower temperature in summer and higher in winter as compared to the atmospheric temperature.

Metalimnion or termocline is situated between epilimnion and hypolimnion, and has the peak of high lake temperature.

Thermal stratification exists simultaneously with chemical one, which is quantified by measuring the fixed residue or dissolved salt concentration.

2. Mathematical modeling of heliothermal phenomenon

After identifying *in situ* the heliothermal phenomenon in Romanian salt lakes, the mathematical modeling and forecasting the evolution of the phenomenon after selective withdrawal of warm water from the termocline, fully optimized and controlled, were done [4]. A software package for calculating 2D free surface water in turbulent, sub-critical flow, and adjacent processes, named Surface Water Modeling System (SMS v.10.1.7) [5], and an extension ArcGis v.9.1 [6-7] has been used.

2.1. RMA2 Module

Part of SMS soft-package, RMA2 module calculates the solution of Navier-Stokes equations system in Reynolds form for free surface turbulent sub-critical motion [8]. The bed friction is computed with Manning/Chézy formula, and the turbulent characteristics are considered using the eddy viscosity coefficients.

RMA2 module can solve engineering problems of dynamic and static state of water as: water levels, local velocity distributions around islands, water flowing under bridges, hydrodynamics at the rivers and channels confluences, internal and external motion of the hydro power plant channels, flow in rivers with wetland areas, hydrodynamics in rivers, lakes, deltas, estuaries, etc.

Being a two-dimensional horizontally model (vertically averaged), RMA2 module operates on the assumption that the acceleration in the vertical direction is neglected.

The equations used by RMA2 module consists of two motion equations in Cartesian coordinates x and y (1 and 2), together with continuity equation (3) for unsteady flow of incompressible fluids:

$$h\frac{\partial u}{\partial t}+hu\frac{\partial u}{\partial x}+hv\frac{\partial u}{\partial y}-\frac{h}{\rho}\left(E_{xx}\frac{\partial^2 u}{\partial x^2}+E_{xy}\frac{\partial^2 u}{\partial y^2}\right)+gh\left(\frac{\partial z}{\partial x}+\frac{\partial h}{\partial x}\right)+\frac{gun^2}{\left(h^{1/6}\right)^2}\times$$
$$\times\left(u^2+v^2\right)^{1/2}-\zeta V_a^2\cos\psi+2h\omega v\sin\phi=0 \tag{1}$$

$$h\frac{\partial v}{\partial t}+hu\frac{\partial v}{\partial x}+hv\frac{\partial v}{\partial y}-\frac{h}{\rho}\left(E_{yx}\frac{\partial^2 v}{\partial x^2}+E_{yy}\frac{\partial^2 v}{\partial y^2}\right)+gh\left(\frac{\partial z}{\partial y}+\frac{\partial h}{\partial y}\right)+\frac{gvn^2}{\left(h^{1/6}\right)^2}\times$$
$$\times\left(u^2+v^2\right)^{1/2}-\zeta V_a^2\sin\psi+2h\omega u\sin\phi=0 \tag{2}$$

$$\frac{\partial h}{\partial t}+h\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+u\frac{\partial h}{\partial x}+v\frac{\partial h}{\partial y}=0 \tag{3}$$

where:

h - is water depth (m);
u, v – local velocities on x and y directions (m/s);
t - time (s);
ρ – fluid density (kg/m^3);
E - eddy viscozity coefficients (Pa.s sau kg/m/s);
g – acceleration due to gravity (m/s^2);
z – talweg level (m);
n - Manning roughness coefficient (-);
ζ – wind friction coefficient (-);
V_a – wind velocity (m/s);
ψ – wind direction;
ω – Earth angular velocity (s^{-1}) ;
φ - latitude of the place.

Equations (1-3) are solved by finite element method (FEM), using the Galerkin weighted residues. Elements are of two-dimensional type (quadrilaterals and triangles), and interpolation functions are of grade 2 (field velocities) and grade 1 (depths). Spatial integrals are Gauss type and derivatives with respect to time are approximated by linear finite differences.

2.1. RMA4 Module

Using RMA2 hydrodynamics results, RMA4 module solve convection-diffusion processes, being used in dispersion analysis of any conservative or non-conservative pollutants, by computing the concentration field based on the transport equation (4) [9]:

$$h\left(\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}-\frac{\partial}{\partial x}D_x\frac{\partial C}{\partial x}-\frac{\partial}{\partial y}D_y\frac{\partial C}{\partial y}-\sigma+kC+\frac{R(C)}{h}\right)=0 \tag{4}$$

where:

C – pollutant concentration (mg/l, ppm sau %);
D_x, D_y - diffusion coefficients on x and y directions (m^2/s);
k - reaction/decay coefficient (s^{-1});
σ - local pollutant source (concentration unit/s);
$R(C)$ - rainfall/evaporation (concentration unit$\times$m/s).

Equation (4) is solved also with the help of FEM, Galerkin weighted residues method. The RMA4 module admits the same structure as RMA2 finite element, and the integration is done by Gaussian quadrature.

3. Case study. Numerical simulation of selective withdrawal of salt warm water

The conservation of heliothermal phenomenon in lakes which have this phenomenon is an important goal for the spa and therapeutic operations. One of the basic aspects of optimizing the operations of these types of lakes with therapeutic properties is the selective withdrawal of warm and salt water from the termocline so that thermal stratification is minimal disrupted.

The case study refers to the Ursu Lake, Sovata basin (Fig. 2), lake extensively used due to its therapeutic properties. Thermal/saline stratification is influenced by the withdrawn discharge, the freshwater input from upstream, but especially the number of direct bathing in the lake. The effect of bathing was virtually eliminated by separating the warm water withdrawn area of the bathing area.

Fig. 2. Aerial view of the Ursu Lake.

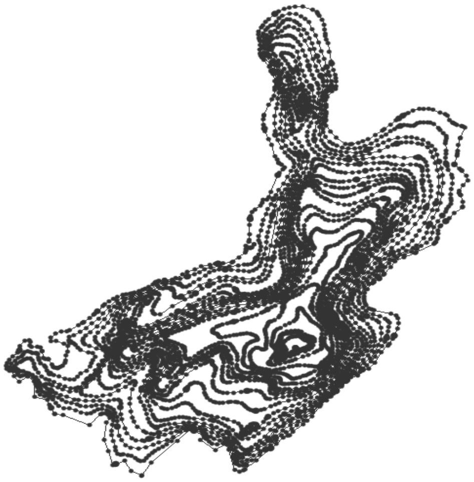

Fig. 3. Raw input data (bathymetry).

Fig. 4. 3D vertex pattern in SMS.

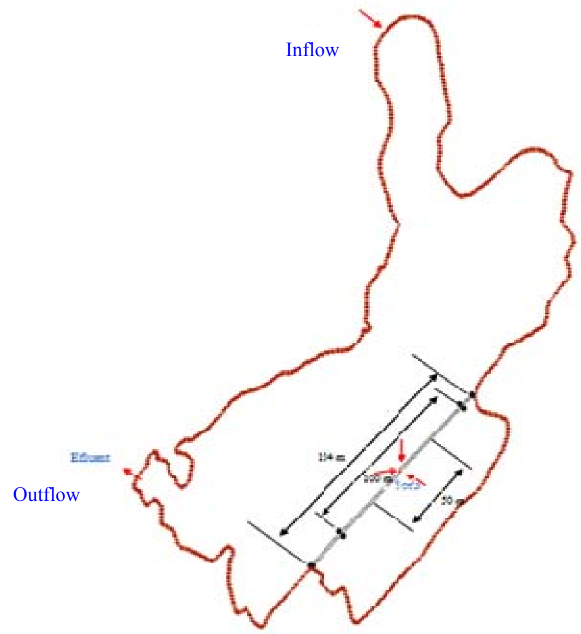

Fig. 5. Horizontal computing scheme for Ursu Lake.

Due to the fact that the density is influenced by temperature, the warm water tends to rise and the cold water to descend into the deep region. Under the influence of motion generated by uncontrolled extraction of water, it is possible to mix the water at the suction hood and in the surrounding area.

Therefore, the main objective of this paper is to identify by mathematical modeling a maximum warm water flow rate extracted from the termocline, within the required operation time, so that the termocline temperature does not fall below a certain value.

However, there is water motion in a heliothermal lake even without selective withdrawal, due to the natural environmental factors: freshwater input and output, infiltration, precipitation, evaporation, condensation, solar radiation, wind condition, etc.

In order to mathematical modeling of selective withdrawal, the first thing to be done is to get numerical model of the terrain. This requirement was made in GIS (using ArcGis v.9.1), after importing data from software acquisitions, by nodes and arcs that make bathymetric lines (Fig. 3). Then, it was exported these data in SMS, obtaining closed curves with uniformly distributed vertex pattern (Fig. 4).

After this stage, a horizontally mash model of the lake was generated. According to the computing scheme of Fig. 5, an optimized mesh in vertical section (Fig. 6) with an observation window in length of 50 m (Fig. 7) was obtained.

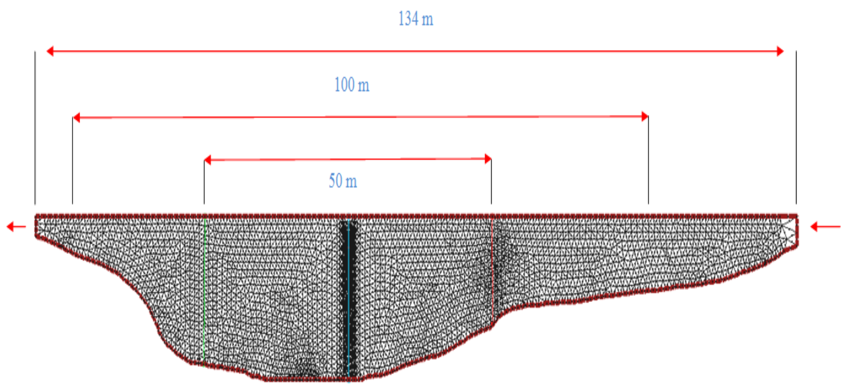

Fig. 6. Vertical cross-section of the optimized mash (L = 134 m).

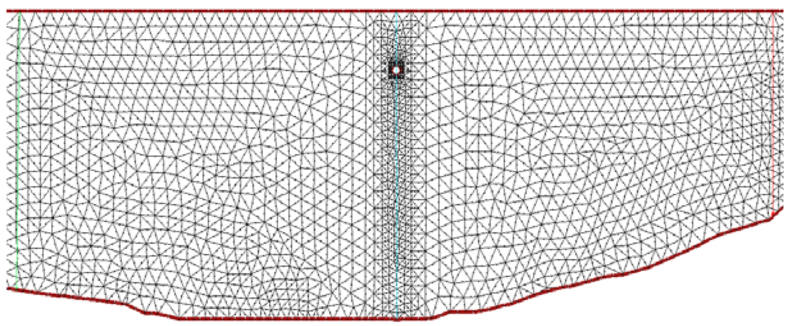

Fig. **7.** Observation window (50 m)
with a detail of optimized mash, including the suction hood.

After obtaining the numerical model of the terrain, horizontally and vertically, we have proceeded as follows:

a) Using RMA2 module, with an inflow discharge of 0.027 m^3/s, it was computed the horizontal field of local velocities with and without selective withdrawal. The values of these velocities did not exceed 0.004 m/s. We can say that the influence of selective withdrawal on the entire lake hydrodynamics is insignificant. Therefore, to compute the water motion in detail, a region around the suction hood was chosen, because only there the influence of withdrawal occurs and it is possible to appear thermal destratification.

b) Since the gravitational field has a limited influence on the size of the local velocity near extraction zone, we proceeded to rotate by 90° the computed plane, so that we can use RMA2 module for different flow rates withdrawn.

c) Using the measured temperature distribution, we obtained initial thermal stratification (hot-start) in the modeled vertical cross-section (Fig. 8):

- Surface temperature T_s = 20° C;
- Thermocline depth H_t = 4 m;
- Thermocline temperature T_t = 45° C;
- Temperature jump ΔT_t = 25° C.

In order to ensure numerical stability, we used dimensionless values for temperature distribution by dividing with the average temperature of 23.33° C multiplied by 10. Thus, dimensionless thermocline temperature is 0.193.

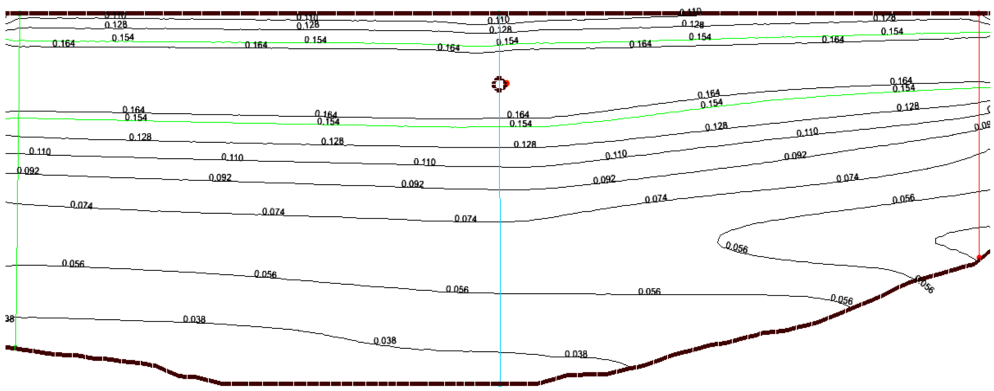

Fig. 8. Initial thermal stratification (hot-start) in the modeled vertical cross-section.

d) Once the local velocity field was obtained using RMA2 module for different flow rates of warm water extracted, RMA4 module for unsteady regime was applied (dimensionless temperature replaced the concentration *C* in equation 4). In this way, thermal stratification was computed as a function of time. So, we succeed to calibrate and then validate the mathematical model based on field measurements in different operating options.

For calibration, the data of ’80 year was used. The extracted discharge was of 2 l/s for 20 minutes time period. In numerical simulation, isothermal of 0.154 (corresponding to a water temperature of 36° C) reached the suction hood after 21 minutes (Fig. 9).

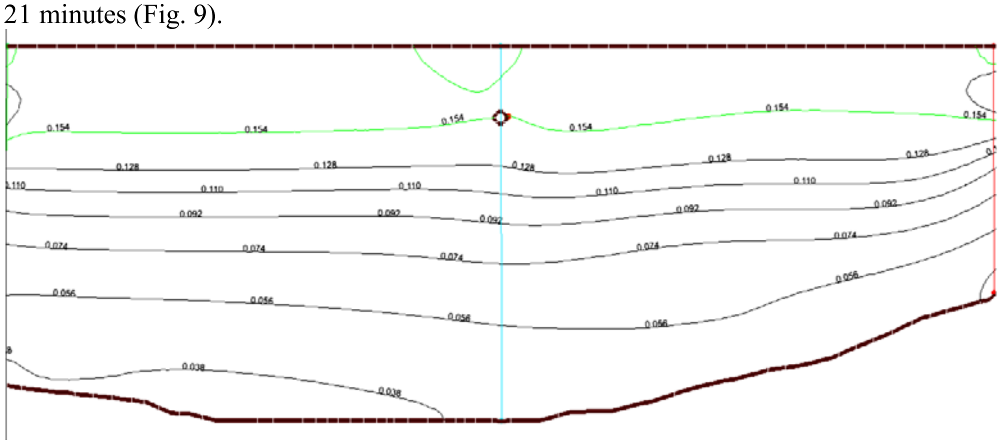

Fig. 9. Thermal stratification after 21 minutes, flow rate of 2 l/s (calibration).

The data of 2003 for mathematical model validation was used. The warm water flow rate recomanded was of 0.1 l/s during 35 minutes time period, which is in good agreement with computed period of 39 minutes (Fig. 10).

At present, for spa and therapeutic procedures extension a discharge of 1 l/s is required. The computed time period until the water temperature at thermocline drops from 45° C to 36° C (isothermal of 0.154) is of 33 minutes. A larger discharge shortens the time period, so: for 5 l/s – 16 minutes and for 10 l/s – 6 minutes.

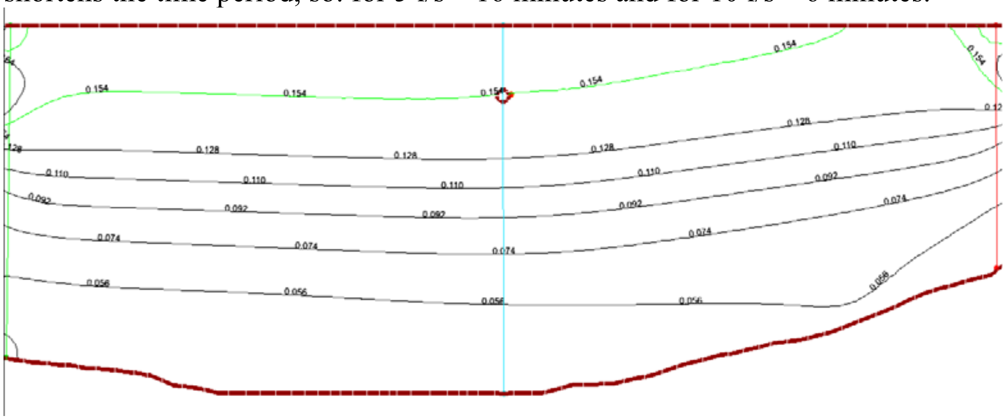

Fig. 10. Thermal stratification after 39 minutes, flow rate of 0.1 l/s (validation).

3. Conclusions

Found in Canada and in the United States of America, heliothermal phenomena have been identified and studied also in Romania, at Sovata and Ocna Sibiu areas, by field measurements done in 2000-2004.

In the paper we tried to simulate numerically the selective withdrawal of warm salt water from thermocline (the place with a maximum temperature and salinity), in order to optimize the discharge and extraction time period, so that heliothermal phenomenon at Ursu Lake, Sovata basin is preserved.

This work was based on extensive field data, which allowed mathematical model calibration and validation. The numerical results enable to optimize hydraulic parameters at spa and therapeutic operations such that no lake destratification occurs.

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