B-SPLINE COLLOCATION BASED APPROXIMATIONS FOR STRUCTURAL PROBLEMS OF FOURTH ORDER

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Application of B-splines as approximating polynomials in collocation technique improves its accuracy and makes it suitable for numerical solutions of complex governing equations. In this paper a mathematical model of an available fourth order boundary value problem (governing equation of a beam) subjected to mechanical and thermo-mechanical load is considered. Also material non-linearity is taken into account through which behavior of functionally graded beam is analyzed. The approximate solution of the various problems addressed is approached using B-spline Collocation Technique and results are compared with those in available literature. Through this work, suitability of B-Spline Collocation Technique for approximate solution of complex higher order boundary value problems has been reported.

Keywords: boundary value problems, b-spline basis functions, greville abscissa, thermo-mechanical load, functionally graded materials

1. Introduction

The application of B-splines in Computer Aided Design and Manufacturing in the recent era has been tremendous. B-spline curves exhibit superior characteristics regarding the local control and adaptability. Owing to their accuracy in handling minute variations over specific regions without effecting the overall geometry of the curve makes it very elegant in CAD applications. Researchers feel that the characteristic of B-spline makes it suitable for wide application areas and not just limited to CAD/CAM.

In recent era owing to technological advancements and high-performance computing, the demand for more precise predictions to match real life conditions is on rise. In this sequence, Hughes [1] has applied the concept of Non-uniform Rational B-splines (NURBS) in analysis procedure so as to update the classical Finite Element Techniques to match with modern CAD. Traditionally the concept of B-splines has been successfully implemented in curve fitting and several CAD applications. Its applications in linear regression models and applications in data mining, image processing etc. has also been of considerable impact.

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B-spline regression analysis is used along with traffic flow theory for data mining application by Sun et al [2]. Aguilera et al [3, 4] have reported the B-spline approaches for the analysis of functional data. Zakaria et al [5] have used the fuzzy B-spline surface model based on fuzzy set theory to represent fuzzy data/ control points and thus modeled the data point uncertainty. Guo [6] has considered the splines for functional data analysis while Valenzuala and Pasadas [7] have used the cubic splines for fuzzy data approximation. Lehmann et al [8] in their work has reported the use of B-splines interpolation technique in image processing and author has also showed that the present technique gives very good interpolation results and Fourier properties within a reasonable time for computation. The concept of B-spline interpolation has been also used in signal processing by Unser and Blu [9]. From the above discussion it is clear that B-splines as interpolation functions have wide applications and it may be good choice for data analysis as well.

As B-splines have excellent properties regarding the local control and smoothness at the interfaces which makes it highly suitable for curve-fitting applications. This also makes such curves suitable for numerical techniques in finding approximate solutions of complex governing equations (with non-linearities). Collocation method is one of the simpler and fast methods to arrive at approximate solutions. However the accuracy of collocation is not high which limits its applicability to a relatively narrow area. However the accuracy can be improved substantially by using B-splines as approximating polynomials. The superior accuracy and smoothness of B-spline basis functions and low computational cost of collocation makes it very attractive. The collocation method using B-splines can prove efficient as compared to the traditional Finite Element (FEM) and Finite Difference Method (FDM) and can give results in piecewise continuous and closed form and it has much simpler application process to get results with satisfactory accuracy. A comparison of the FEM, FDM and B-spline collocation has been reported by Kadalbajoo and Yadaw [10] to show that the present technique gives better approximations and convergence and hence it may be useful for applications in simpler geometries like beams and plates.

A number of works pertinent to application of B-spline collocation has been published. Fairweather [11] and Rashidiniya [12] have compiled a number of related works in their survey on B-spline collocation. It is seen that the effectiveness of the present method has been largely explored in thermal problems particularly to heat transfer and fluidics; however this method may be justifiably applied to structural problems as well. The authors Chawla et al [13] and Chawla [14] have successfully applied the collocation using b-splines to heat or mass transfer problem and radiation-conduction problem. Botella [15] has presented the numerical solution of Navier Stokes equation using B-spline collocation to obtain efficient results of desired accuracy. Kadalbajoo [16] used the collocation method

to solve boundary value problems of convection and diffusion; thus, obtain an approximate solution with sufficient accuracy level.

In 1995, Bert [17] has applied the method of spline collocation to find approximate solution in statics of beams and plates. Wu [18] has used the Spline collocation technique to solve generalized problems of beam structures. The dynamics of non-uniform beams supported on elastic foundation and of pre-twisted beams using Spline collocation with uniform knot span has been reported by Hsu [19, 20].

The technique of B-spline collocation may also be extended to cover the variation in material behavior as in case of functionally graded materials (FGMs). These are advanced composites (mixture of different materials, arranged in certain form) whose parent materials are mixed in a desired form so as to obtain required properties. Hence the material anisotropy inherits material non-linearity in the defined problem. Shankar [21] studied the behavior of FG Euler-Bernoulli beam using elasticity approach. Chakraborty et al [22] have developed a new beam element that can be applied to a bi-material shear deformable beam with functionally graded intermediate layer and hence to determine its mechanical response to static and dynamic loads in a thermal environment. Li [23] reported a unique unified formulation for FG Timoshenko beam by reducing the three differential equations of displacement variables into a single fourth order equation.

Recently, Isogeometric (IG) analysis [1] using NURBS, which is a refined formulation of B-splines, as shape functions, is increasingly attracting researchers for finding approximate solutions to complex problems. Auricchio et al [24] have reported the applications of IG collocation technique and applied it to theoretical analysis of many mathematical problems. Reali [25] has applied for the first time IG collocation to slender beams and plates and have illustrated the inherent potential of the method.

Patlashenko et al [26] have applied the cubic B-spline collocation technique to study the behavior of panels subjected to mechanical and thermal loading. Patlashenko [27] extended his work to nonlinear analysis of laminated panels using two dimensional spline collocation methods. Mizusawa et al [28, 29] have used the concept of spline finite strip method to study the vibration characteristics of cross-ply and thick laminated cylindrical panes. Akhras et al [30] have applied the spline finite strip method to investigate the stability and vibration behavior of piezoelectric composite plates while Loja et al [31] have applied the B-spline finite strip technique to study the static and dynamic behavior of FG sandwiched plates with piezoelectric skins. In a recent paper, Provaditis [32] has used the B-spline collocation technique to obtain the natural frequencies of thin plates in bending. The results of [32] report the effectiveness of B-spline collocation technique over the cubic B-spline Galerkin-Ritz formulation.

From the above review of literature it is felt that though the technique is being used for last 3-4 decades but its application area is largely limited to thermal problems of interest. In structural problems its application is quite lean; considering the level of smoothness provided by this technique, its applications areas may further be explored to a wider scale. As mentioned above, in the present era, isogeometric analysis using NURBS (Non Uniform Rationalized B-splines) has been considered as a powerful tool in approximation theory. It is felt that application of B-splines in approximations can render results with satisfactory accuracy, at least for relatively simpler 1d- or 2d-problems of structural mechanics.

In this paper the governing differential equation of a beam (fourth order boundary value problem) with known boundary conditions is solved numerically using B-spline collocation technique to find an approximate solution. Two types of loading are considered- purely mechanical load and thermo-mechanical load. The results are then verified with the known results from literature to obtain a perfect match. The effect of material non-linearity as in FGMs has also been addressed.

2. Formulation

2.1 B-Spline Basis Functions

B-splines are piece-wise polynomials made up of linear combinations of b-spline basis functions that can be used to approximate a solution to a mathematical problem. It is drawn in a parametric space 't'. They are defined in terms of order/ degree of the curve and also depend upon a set of non-decreasing coordinates in parametric space which is called a knot-vector. It is assumed that the knot vector is of open uniform type given by:

$$T = \left[t_0, t_1, t_2 \ldots \ldots, t_{n+k+1} \right] \tag{1}$$

where, $n+1$ = no. of control points and 'k' is order of the polynomial spline. The b-spline basis of 'k_{th}' order is defined recursively using the relation:

$$N_{(i,k)} = \frac{t - t_i}{t_{i+k-1} - t_i} N_{(i,k-1)} + \frac{t_{i+k} - t}{t_{i+k} - t_{i+1}} N_{(i+1,k-1)} \tag{2}$$

In order to use the above relation we need to define the smallest basis function i.e. of first order:

$$N_{i,1} = \begin{cases} 1, t_i < t < t_{i+1} \\ 0, otherwise \end{cases} \tag{3}$$

Using $(n+1)$ control points $B_0, B_1 \ldots B_n$, a B-spline function is defined as:

$$B(t) = \sum_{i=0}^{n} N_{i,k}\left(t \right).B_i \tag{4}$$

2.2. Collocation Technique

In order to simplify the procedure we may assume (and in a general sense) that $t_0 = 0$ and $t_{n+k+1}=L$ so as to merge the parametric space with real space; and thereby we may assume $t = x$ as reported in [33]. Let the entire length of the beam be divided into 'l' spans. Let 'k' be the order of the spline function that is taken as the approximating polynomial in the present case. Each span is represented by a single spline function as such the total number of spline functions is equal to the no. of spans. As the continuity at the knots equals C^{k-s-1}, the number of smoothness conditions at the knots is (k-s) and hence depends upon the multiplicity 's'. Let ϑ_i denote the number of smoothness conditions at each knot. Hence the no. of coefficients of 'l' piecewise polynomials which is equal to the no. of control points, $\mathfrak{N}$, can be expressed as:

$$N = k.l - \sum \vartheta_i \tag{5}$$

This leads to $\mathfrak{N}$ unknowns that are to be determined. If there are 'm' boundary conditions, then collocation points required will be given by $\mathfrak{N}$-m. In this work collocation points are calculated using the method of Greville abscissa as discussed in [33], defined for a knot vector, $T = [t_0, t_1, t_2 ……, t_{n+k+1}]$ as:

$$x_i = \frac{1}{n}\left(t_i + t_{i+1} + … + t_{i+n-1}\right) \tag{6}$$

In Equation (6) 't' represents the knots in B-spline and 'n' is equal to (k-1) and is the degree of b-spline curve used. The above leads to $\mathfrak{N}$ values of x_i, the first and the last of which represent the boundaries which have been already used in boundary conditions hence must be omitted. Out of the remaining, points which are located towards centre are selected as collocation points. Thus we have $\mathfrak{N}$ algebraic equations with $\mathfrak{N}$ unknowns. These can be solved to obtain a unique solution.

2.3. Isotropic Beam Problem

An isotropic Euler beam is considered for analysis. Its governing equation is a fourth order boundary value problem [34] given by:

$$EI\frac{\partial^4 w}{\partial x^4} + q = 0 \tag{7}$$

where EI is the flexural rigidity of the beam for bending, 'w' is transverse deflection of beam and 'q' is constant mechanical pressure applied at the top surface of the beam with known boundary conditions. The following boundary condition of the beam may be considered:

a)   Clamped-Free

$$(i)w(0) = 0, (ii)\frac{\partial w}{\partial x}(0) = 0, (iii)\frac{\partial^2 w}{\partial x^2}(L) = 0, (iv)\frac{\partial^3 w}{\partial x^3}(L) = 0 \qquad (8)$$

b)   Clamped-Simply supported

$$(i)w(0) = 0, (ii)\frac{\partial w}{\partial x}(0) = 0, (iii)\frac{\partial^2 w}{\partial^2 x}(L) = 0, (iv)w(L) = 0 \qquad (9)$$

c)   Clamped-Clamped

$$(i)w(0) = 0, (ii)\frac{\partial w}{\partial x}(0) = 0, (iii)\frac{\partial w}{\partial x}(L) = 0, (iv)w(L) = 0 \qquad (10)$$

d)   Simply supported-Simply supported

$$(i)w(0) = 0, (ii)\frac{\partial^2 w}{\partial x^2}(0) = 0, (iii)\frac{\partial^2 w}{\partial x^2}(L) = 0, (iv)w(L) = 0 \qquad (11)$$

When the aspect ratio (l/h) of the beam is less ($l/h<10$) there is substantial deformation owing to shear strain, under these conditions the above theory underestimates the actual deformation. Under such conditions Timoshenko beam theory is suitable that accounts for shear deformation also. The governing equation for Timoshenko beam is given by [34]:

$$EI\frac{\partial^3 \phi}{\partial x^3} + q = 0 \qquad (12)$$

where 'ϕ' is the slope of elastic curve. The boundary conditions are modified accordingly for the cases in which at least one of end condition is of clamped type. In all such cases $\frac{\partial w}{\partial x} \neq \phi$ as the shear deformation given by:

$\gamma = \left(\phi - \frac{dw}{dx}\right) \neq 0$. The solution of the above equation will calculate the value of ϕ

that can be used to back calculate 'w'. However we can simplify the process by a simple assumption as follows. We assume an independent parameter 'F' that relates the slope ϕ and transverse deflection 'w' [23] as:

$$\left.\begin{aligned} w &= F - \frac{EI}{k_s GA}\frac{d^2 F}{\partial x^2} \\ \phi &= -\frac{dF}{\partial x} \end{aligned}\right] \qquad (13)$$

Hence when substituted for 'ϕ' in Equation (12) we get:

$$EI\frac{\partial^4 F}{\partial x^4} + q = 0 \qquad (14)$$

Once the parameter 'F' is calculated the slope and deflection terms for Timoshenko beams can be easily determined. The boundary conditions for a Timoshenko beam are:

a) Clamped-Free

$$(i)w(0)=0,(ii)\phi(0)=0,(iii)\frac{\partial^2 w}{\partial x^2}(L)=0,(iv)\frac{\partial^3 w}{\partial x^3}(L)=0 \tag{15}$$

b) Clamped-Simply supported

$$(i)w(0)=0,(ii)\phi(0)=0(iii)\frac{\partial^2 w}{\partial x^2}(L)=0,(iv)w(L)=0 \tag{16}$$

c) Clamped-Clamped

$$(i)w(0)=0,(ii)\phi(0)=0,(iii)\phi(L)=0,(iv)w(L)=0 \tag{17}$$

d) Simply supported-Simply supported

$$(i)w(0)=0,(ii)\frac{\partial^2 w}{\partial x^2}(0)=0,(iii)\frac{\partial^2 w}{\partial x^2}(L)=0,(iv)w(L)=0 \tag{18}$$

It can be observed through Equation (15) that if the value of 'G' is infinite then 'w' is reduced to 'F', which mean Equation (15) and Equation (7) become identical. Thus only by assuming a finite value for 'G' we get Timoshenko deflection and correspondingly Euler deflection is obtained for its infinite value.

2.4. Anisotropic Beam problem

In this section B-spline collocation technique is used to numerically find the approximate solution of a beam with material non-linearity. The complexities arising due to such non-linearity makes the problem quite complex and it is very difficult to obtain a direct solution. The materials with such non-linearity in material behavior are called composite materials and the one considered in this work are called functionally graded materials.

Functionally Graded Materials are advanced composite materials whose properties are tailored in a desired manner to obtain specific properties. The composition of the parent materials is varied in a specified manner to achieve such variation in the properties. In this work the variation of material properties is assumed to be according to power law function. The expression for power law is given by:

$$P(z)=P_b+(P_t-P_b).\left(\frac{z}{h}+\frac{1}{2}\right)^\beta \tag{19}$$

where 'P' is any material property, 'z/h' represent the normalized distance from geometrical center. Here 'h' is the height of beam cross-section and the subscripts 'b' and 't' represents the bottom and top layer of the beam.

Li [23] has derived the governing equation of a beam made up of functionally graded material such that its properties vary across the cross-section of the beam

given by power law function, Equation (20). The governing equation for a functionally graded beam as reported in [23] is given by:

$$D^* \cdot \frac{\partial^4 F}{\partial x^4} + q = 0 \qquad\qquad (20)$$

Equation (21) is the governing equation for a functionally graded beam. It is very similar to the equation for isotropic beams and can be numerically solved using the B-spline collocation method. (D^* is material property term and 'F' is a parameter that resembles transverse deflection of beam; both are explained in appendix-1).

It can be noted that above Equation (21) is governing equation for a Timoshenko beam made up of functionally graded material. From the appendix-1 it can be observed that the term 'F' depends on D^* and K_{55} which are numerically constants for a particular beam. It reduces to 'w' when the value of K_{55} is infinite. Hence Equation (21) becomes the governing equation for Euler beams only by assuming the rigidity modulus to be infinite. Hence Equation (21) is applicable for both Euler and Timoshenko beams made up of functionally graded materials by constraining a suitable value to rigidity modulus.

The term 'β', used in Equation (20), is called power law index and it represents the composition of a functionally graded material. When its value is zero then the material becomes isotropic, and constant terms reduce as $D^*=EI$ (flexural rigidity), $K_{55}=k_sGA$ (shear rigidity), where 'I' is the moment of inertia of cross-section and 'A' is cross-section area. This means Equation (21) simplifies to isotropic Timoshenko beam if ($\beta=0$), and to an Euler beam equation if ($\beta=0$, $G=infinite$). In the next section the deflections of Euler and Timoshenko beams have been compared for both isotropic as well as functionally graded materials.

3. Numerical Experiments

3.1. Case 1: Isotropic Beam subjected to only mechanical load

A rectangular beam of uniform dimensions throughout the span is considered for analysis. Its length is assumed to be 0.5m, its height is 0.125m and width is assumed to be unity. Let the material of the beam be steel whose Young's modulus is taken as 210GPa. The beam is loaded with a uniformly distributed load of 1 kN/m. Poisson's ratio is assumed to be $\mu=0.3$. The shear correction factor, k_s is calculated from $k_s = \frac{5(1+\mu)}{(6+5\mu)}$ while rigidity modulus is calculated using $G = \frac{E}{2(1+\mu)}$. For Euler deflection a very large value (G=10^{100}) is substituted in Matlab code.

In order to solve Equation (15) a sixth order B-spline basis function is considered as approximating polynomial. For simplicity the knot vector is selected so as to have a single span *(0-1)* given by:

$$T = \begin{bmatrix} 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1 \end{bmatrix} \qquad (21)$$

In this case $\mathfrak{R} = 6$, and since we have four boundary conditions, we need two collocation points. The collocation points are calculated using Greville abscissa, i.e. Equation (6) :

$$X = \begin{bmatrix} 0,\ 0.2000,\ 0.4000,\ 0.6000,\ 0.8000,1 \end{bmatrix} \qquad (22)$$

From the above we may select the middle points as collocation points (i.e. $X = 0.4, 0.6$). Using the above boundary conditions and collocation points we have six linear equations that can be easily solved to obtain the unknown parameters or the coefficients (B_i, $i = 1,2,...\mathfrak{R}$) of Equation (4). A MATLAB code is developed to generate the basis functions and then use them in collocation to find the approximate solution.
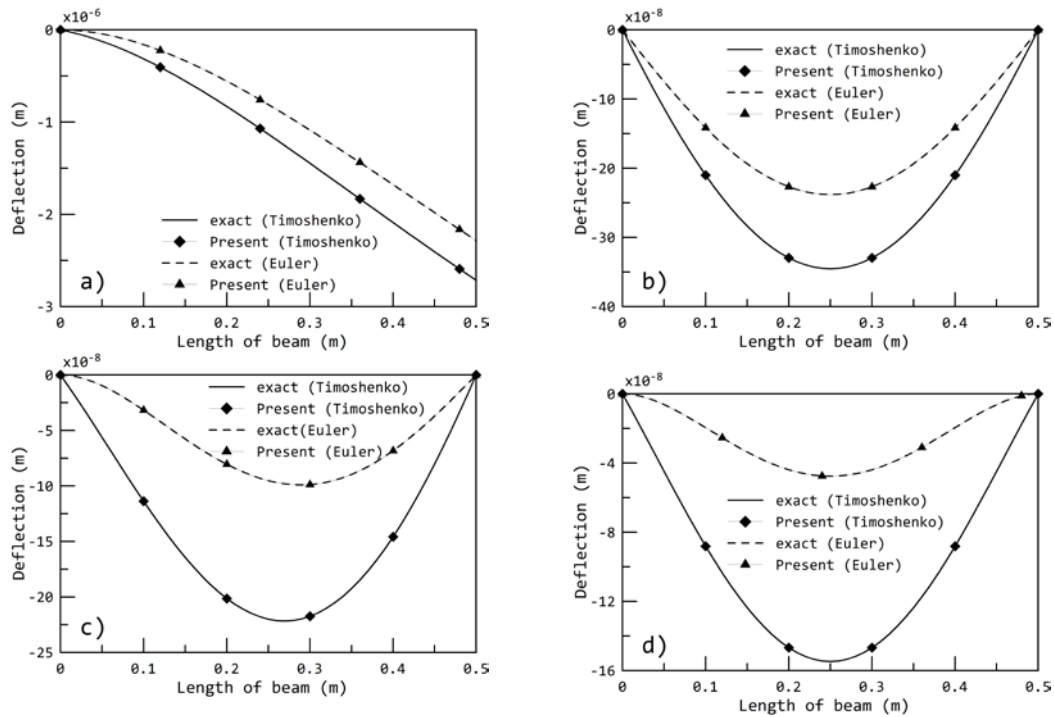
Fig. 1 Deflection of beam (for l/h=4) using B-spline collocation and its validation
a)C-F, b)S-S, c)C-S, d)C-C

Using the boundary conditions i.e. Equations (8-11) corresponding to different beam types, the governing Equation (15) is solved using the MATLAB code and results are plotted in figure 1(a-d). The results are then verified with the standard results available in the literature [34].

An important observation in the figure 1(a-d) is that for smaller aspect ratio (*l/h=4*) there is a significant difference in the Timoshenko and Euler deflection. This is due to appreciable shear deformation observed in Timoshenko beams with a finite value of shear modulus. However, this deformation due to transverse shear decreases as the aspect ratio increase. For aspect ratio more than 10, the difference between the two is very small and can be ignored. The same can be verified from figure 2 in which the aspect ratio has been increased to 40, other conditions remaining same. The elastic curves for both Timoshenko and Euler beam are practically undistinguishable for higher aspect ratio (*l/h*)of 40.
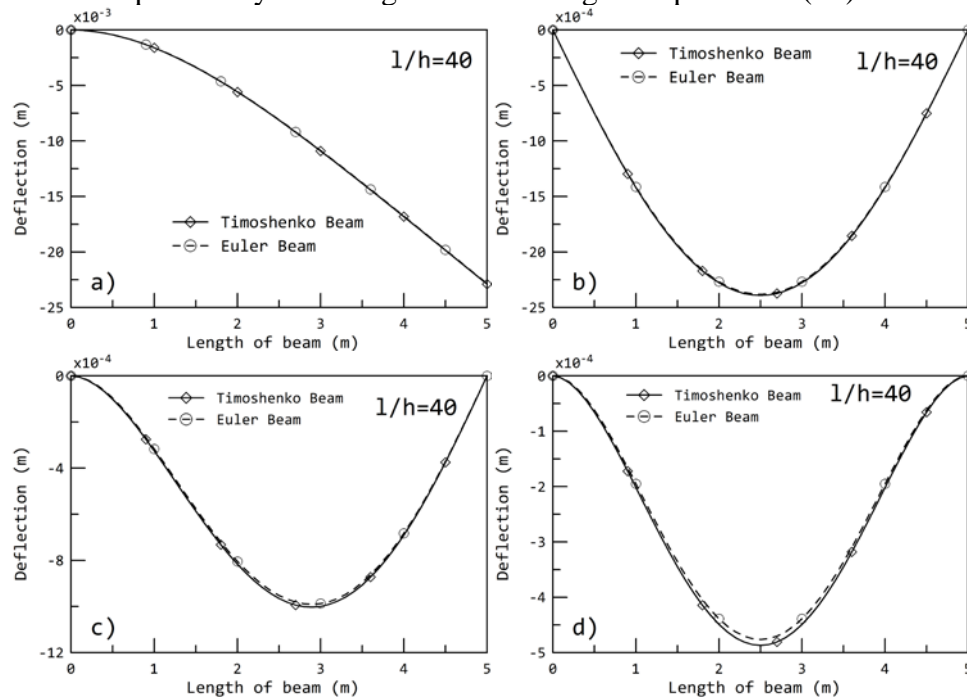
Fig. 2 Comparison of Euler and Timoshenko deflection using B-spline collocation and its validation a)C-F, b)S-S, c)C-S, d)C-C

Deflections under mechanical load for different end conditions, their comparisons with standard results in the available literature and the error in the results have been reported in Table-1. It is clear from the figure 1 & 2 and Table-1 that B-spline collocation technique can be very effective in getting approximate solutions to a high level of accuracy (with error between the obtained and standard results differ in the order of 10^{-13}). The level of accuracy achieved by this technique indicates that it can handle complex geometries and non-linearities as well. Hence beams subjected to thermo-mechanical loading and non-linearities associated with material properties have been considered in the successive sections.

Table-1

Error in the deflections obtained using B-spline collocation

Position on the beam, x (m)	Type of support	Deflection (m)		Error
		Present Work	Reference	
0.1	C-F	-2.11e-08	-2.11e-08	1.38e-12
	S-S	-1.41e-08	-1.41e-08	1.33e-13
	C-S	-3.17e-09	-3.17e-09	6.00e-13
	C-C	-1.95e-09	-1.95e-09	5.88e-13
0.2	C-F	-6.47e-08	-6.47e-08	7.77e-14
	S-S	-2.27e-08	-2.27e-08	1.11e-13
	C-S	-8.05e-09	-8.05e-09	2.89e-13
	C-C	-4.39e-09	-4.39e-09	3.55e-13
0.3	C-F	-1.21e-07	-1.21e-07	2.89e-13
	S-S	-2.27e-08	-2.27e-08	4.44e-14
	C-S	-9.87e-09	-9.87e-09	2.66e-13
	C-C	-4.39e-09	-4.39e-09	3.00e-13
0.4	C-F	-1.81e-07	-1.81e-07	2.89e-13
	S-S	-1.41e-08	-1.41e-08	8.88e-14
	C-S	-6.83e-09	-6.83e-09	1.78e-13
	C-C	-1.95e-09	-1.95e-09	4.44e-14

3.2. Case 2: Isotropic Beam subjected to thermo-mechanical load

In this section the case of thermal load being applied to the beam is considered. In order to validate the results, a problem from [35] is taken. A propped beam (i.e. clamped at left end and simply supported at the right end) is uniformly loaded with intensity 'q'. The beam is also subjected to temperature distribution:

$$T = T_b + (T_t - T_b)\left[\frac{z}{h} + \frac{1}{2}\right] \tag{23}$$

where T_b and T_t are temperature of bottom and top surface respectively which is defined initially, and 'h' is beam depth.

It is observed that the governing differential equation remains unchanged even after the incorporation of the thermal load and only boundary conditions are modified. The thermal load significantly changes the moment distribution across the beam length. The boundary condition for this problem will be obtained by modifying Equation (9) to incorporate thermal moment with mechanical moment as shown in Equation (25). Similarly, Equation (26) shows the modified boundary condition for propped beams loaded under temperature gradient.

a)   Clamped-Free (C-F)

$$(i)\,w(0) = 0, (ii)\,\frac{\partial w}{\partial x}(0) = 0, (iii)\,\frac{\partial^2 w}{\partial x^2}(\text{L}) + T_{moment} = 0, (iv)\,\frac{\partial^3 w}{\partial x^3}(\text{L}) = 0 \tag{24}$$

b)   Clamped-Simply supported (C-S)

$$(i)w(0)=0,(ii)\frac{\partial w}{\partial x}(0)=0,(iii)\frac{\partial^2 w}{\partial x^2}(L)+T_{moment}=0,(iv)w(L)=0 \qquad (25)$$

The behavior of such beam is studied under thermo-mechanical load and Equation (7) is solved using B-spline collocation method and boundary conditions given by Equation (25-26) are substituted. A uniform mechanical pressure of 10kN/m and thermal load (temperatures of top and bottom surfaces) is varied. Two types of boundary conditions i) Clamped-Free and ii) Clamped-Simply supported have been considered. Different types of thermal loading have been considered as given in Table-2. The coefficient of thermal expansion (α) for steel has been taken as 24e10^{-6}/^{0}C. The other data and procedure is similar to the previous case. The results are then verified with the results in [35] and found exact match as shown in fig. 3. In fig. 3(a-b), the thermal moment is opposite to the mechanical moment. Due to this with the increase in temperature gradient the deflection of beam changes sharply. This shows the pronounced effect of thermal gradient load in the behavior of isotropic beams.
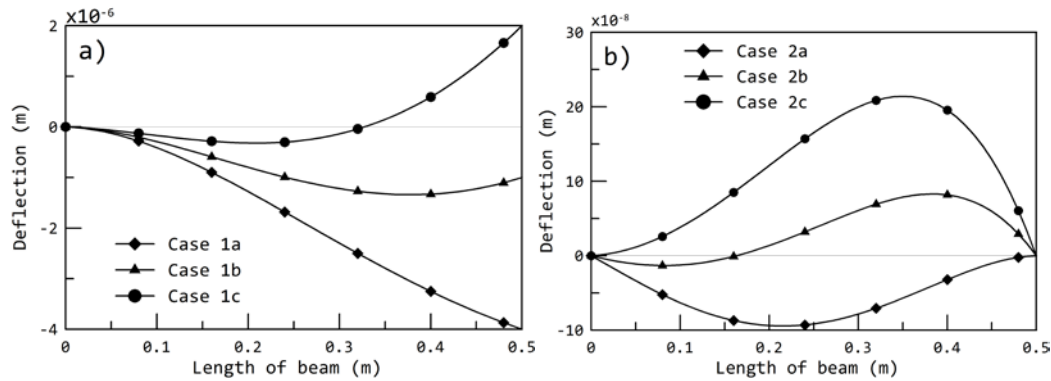

Fig. 3: Deflection under thermo-mechanical load a)C-F b)C-S

3.3. Case 3: Beam made up of functionally graded material and subjected to purely mechanical load

The governing equation for a FG beam given by Equation (13) is solved using B-spline collocation technique using the same procedure followed in example of Case 1. The material properties (Young's modulus) vary according to power law given by Equation (20). The functionally graded material for the beam is assumed to be a mixture of steel and aluminum, the bottom fibers are steel in rich while the top surface is enriched with aluminum. Their Young's modulus is taken as: $E_b = E_{steel} = 210$ GPa, $E_t = E_{Al} = 70$ GPa. The value of Poisson's ratio is assumed to be a constant and taken as 0.3. The other parameters are same as taken in Case 1. In order to find an approximate solution a fifth degree/ sixth order B-spline basis function is used. The results are validated using the results of [23] to

find an exact match as shown in figure 4(a-d). The deflection and corresponding error using present technique have been given in Table 3. It is observed that the computed root mean squared error is $5.62e10^{-13}$. Hence the method gives accurate results for complex cases of material inhomogeneity and anisotropy and may be extended to include different applications of non-linear behavior and critical loading conditions also.

Table-2 **Temperature distribution in beam**

Case	Temperature (^{0}C)		
	Top Surface	Bottom Surface	Ambient Temp.
Case1a	31	33	30
Case1b	31	35	30
Case1c	31	37	30
Case2a	33	31	30
Case2b	35	31	30
Case2c	37	31	30

Table-3 **Error in deflection of FG beam (C-F)**

Position on the beam, x (m)	Deflection (m)		Error
	Present Work	[Li, 2008]	
0.05	-1.11e-08	-1.11e-08	1.49e-12
0.1	-3.38e-08	-3.38e-08	2.22e-14
0.15	-6.58e-08	-6.58e-08	2.66e-13
0.2	-1.05e-07	-1.05e-07	3.55e-13
0.25	-1.49e-07	-1.49e-07	3.89e-13
0.3	-1.96e-07	-1.96e-07	4.33e-13
0.35	-2.45e-07	-2.45e-07	4.11e-13
0.4	-2.95e-07	-2.95e-07	4.33e-13
0.45	-3.45e-07	-3.45e-07	4.33e-13
0.5	-3.95e-07	-3.95e-07	4.33e-13

4. Conclusions

B-splines are synthetic curves which have very good properties related to smoothness, local control and adaptability. These curves are used in different CAD applications. These curves, due to their flexibility, have high potential to be used in other applications also. The other application areas include curve fitting, numerical solution of differential equation, interpolations, image processing, data mining, fuzzy and many more. Recently, the concept of NURBS is used in mechanical analysis and design to give better results. In the present paper the concept of B-splines is used for numerical approximate solution of differential equation using collocation technique. Collocation is very fast and computationally efficient process, but it is also accompanied by poor accuracy at non-collocation points. The use of B-spline basis function in collocation improves its accuracy substantially.

In this work a fourth order boundary value problem of a beam is numerically solved using B-spline collocation method. Three cases have been reported- a) Isotropic beam subjected to purely mechanical load, b) Isotropic beam subjected to thermo-mechanical load and c) functionally graded beam subjected to only mechanical load. Four different boundary conditions are

considered. The results obtained are compared and verified with standard results in literature. The following conclusions can be enumerated:

- B-spline collocation method provide piece-wise closed form solution at very less computational cost and in effective time.

The accuracy of the technique is also quite high and the method can be useful for complex cases also.
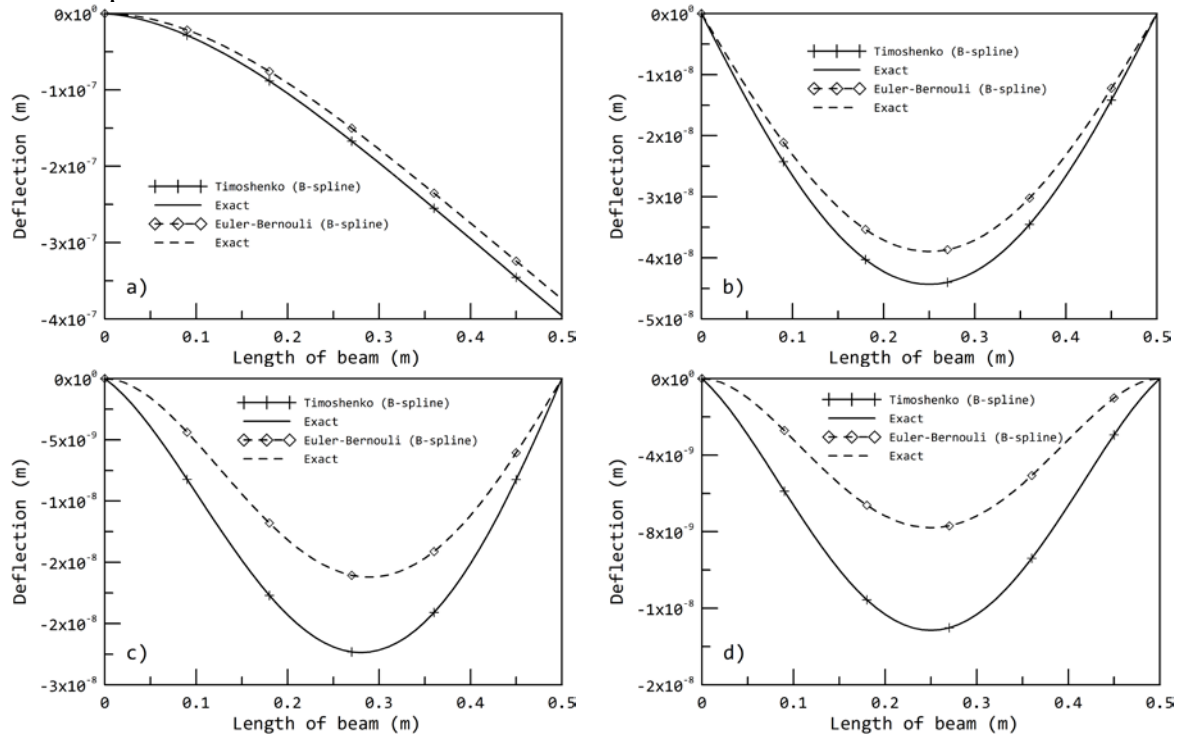

Fig.4 Deflection of FG beam under purely mechanical load a)C-F, b) S-S, c)C-S, d)C-C

R E F E R E N C E S

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Appendix 1

In the derivation of Equation (13) the following assumptions have been made-

$$A_{11} = \int_{h/2}^{-h/2} E(z).1.dz \qquad\qquad B_{11} = \int_{h/2}^{-h/2} E(z).z.dz$$

$$D_{11} = \int_{h/2}^{-h/2} E(z).z^2.dz \qquad\qquad K_{55} = \int_{-h/2}^{h/2} k_s.G(z).dz$$

$$D^* = \frac{B_{11}^2}{A_{11}} - D_{11} \qquad w = F - \frac{D^*}{K_{55}}\frac{\partial^2 F}{\partial x^2} \qquad \phi = -\frac{\partial F}{\partial x}$$

where $E(z)$ is the Young's modulus and $G(z)$ is the modulus of rigidity of the parent materials of FG beam while k_s is shear correction factor which is a constant. 'ϕ' and 'w' are slope and transverse deflection